

Turbulent Pipe-flow Computation Using the Rough Model Method (RMM)

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Abstract-A new approach is presented to solve common straight pipe-flow problems, namely, computation of the discharge Q , computation of the internal diameter D and computation of the energy slope J . The theoretical approach is based on a referential rough pipe model characterized by an arbitrarily assigned relative roughness value, taken in the fully turbulence flow regime. Thus, the friction factor of Colebrook-White remains constant whatever the Reynolds number value. Hence, applying the Darcy-Weisbach formula, all parameters of the flow in the chosen model, such as the flowing discharge \bar{Q} , the internal diameter \bar{D} and the energy slope \bar{J} , are then well defined. These allow a direct determination of the required value of Q , D and J by the use of a non-dimensional correction factors. The efficiency of the proposed approach is put forward through a practical application.

Keywords-Rough Model Method; Turbulent Flow; Pipe; Friction Factor; Discharge; Energy Slope

I. INTRODUCTION

In the field of turbulent pipe-flow, three great categories of problems are encountered. The first problem is to find the discharge capacity of the pipe, the second one is to evaluate the internal diameter in order to design the pipe and the third one is to determine the value of the energy slope in order to adjust the slope if necessary. To answer these problems, three fundamental relations are commonly used, namely Darcy-Weisbach, Colebrook-White and Reynolds number relationships. Among these three parameters, only discharge can be explicitly computed when combining Darcy-Weisbach and Colebrook-White relationships that express the energy slope and the friction factor respectively. Reynolds number is a dimensionless number that expresses the ratio of inertial forces to viscous forces and quantifies then the relative importance of these two forces under a given flow conditions. Two others parameters influence turbulent pipe-flow, namely the absolute roughness which characterizes the state of the inner wall of the pipe and the kinematic viscosity. These last two parameters are measured in practice and rarely cause any particular problem. When it comes to answer the last two categories of problems, the difficulty lies in assessing the friction factor since the Colebrook-White is implicit, as it can be seen in Eq.(2). Moreover, equations (1), (2) and (3) do not allow to express the diameter in an explicit form. Currently, the form of the system of equations (1), (2) and (3) cannot lead to the resolution of the last two categories of the problems of turbulent pipe-flow. For all these reasons, some authors have proposed approximate relations for friction factor, diameter of the pipe and for energy slope as well.

Turbulent pipe flow is governed by the following functional relationship $\phi(Q, J, D, \varepsilon, \nu) = 0$, where Q is the

discharge, J is the hydraulic energy slope, D is the internal pipe diameter, ε is the average roughness height and ν is the kinematic viscosity. Among these, only Q , D and J are of practical interest.

Turbulent pipe flow is modelled by the well known Darcy-Weisbach formula, thereby accounting for the friction factor f according to Colebrook-White relationship. These are expressed respectively as:

$$J = \frac{8f}{g\pi^2} \frac{Q^2}{D^5} \quad (1)$$

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{R\sqrt{f}} \right) \quad (2)$$

where g is the acceleration due to gravity and R is the Reynolds number defined by:

$$R = \frac{4Q}{\pi D \nu} \quad (3)$$

Relation (2) is valid for $R > 2300$.

When Q , D , ε and ν are given, equation (1) allows calculation of the energy slope J , once the friction factor f is determined from Eq.(1) by the use of an iterative procedure. References [1]-[3] gave the following approximate solution:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{R^{0.9}} \right) \quad (4)$$

Equation (4) was established for $5 \times 10^3 < R < 10^8$ and $10^{-6} < \varepsilon/D < 10^{-2}$. The deviation between Eqs.(2) and (4) depends on both R and ε/D as it can be seen in figure 1 for some values of the relative roughness, taken as an example. Combining Esq. (1) and (4), the energy slope J is expressed as follows :

$$J = \frac{2Q^2}{g\pi^2 D^5} \left[-\log \left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{R^{0.9}} \right) \right]^2 \quad (5)$$

Provided J , D , ε and ν are given, a direct determination of the discharge Q is possible when eliminating f between Eq. (1) and Eq. (2). Thus, the following improved relationship is obtained [4, 5]:

$$q = -\frac{\pi}{\sqrt{2}} \log \left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{N\sqrt{2}} \right) \quad (6)$$

where the non-dimensional parameters q and N are expressed as :

$$q = Q / \sqrt{gJD^5}, \quad N = \sqrt{gJD^3} / \nu$$

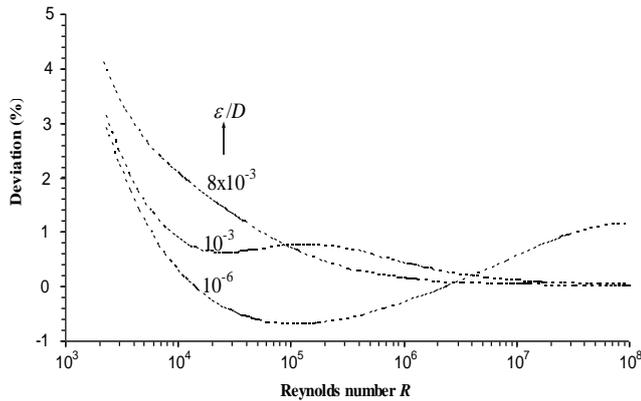


Fig. 1 Deviation between Eqs.(2) and (4) for some values of ϵ/D

On the other hand, computation of the diameter D is more complex. It requires an iterative procedure according to Eqs.(1) to (3). These were reduced to a single equation presented in terms of non-dimensional parameters as follows [1]-[3]:

$$\nu^* = \left[10^{-\sqrt{2}/(\pi D^{*5/2})} - \frac{\epsilon^*}{3.7D^*} \right] \frac{D^{*3/2}}{1.776} \quad (7)$$

Where $\nu^* = \nu D_o / Q$ is a kinematic parameter, $D_o = [Q^2 / (gJ)]^{1/5}$ is the so-called characteristic diameter, $D^* = D / D_o$ is the relative diameter and $\epsilon^* = \epsilon / D_o$ is the relative roughness. As it can be seen, Eq.(7) is implicit in D^* and its solution is not yet available.

For the practically smooth flow regime, corresponding to $\epsilon^* \rightarrow 0$, Eq.(7) becomes :

$$\nu^* = \frac{D^{*3/2}}{1.776} 10^{-\frac{\sqrt{2}}{\pi D^{*5/2}}} \quad (8)$$

Equation (8) is also implicit in D^* and the following approximate solution is proposed [4, 5]:

$$D^* = \frac{2}{5} \log \left[-\frac{54.64}{\log(\nu^*)} \right] \quad (9)$$

which is valid for $10^{-9} < \nu^* < 10^{-3}$ and $10^4 < R < 4 \times 10^9$. The maximum deviation involved in Eq.(9) is 1.5%.

For the rough flow regime, corresponding to $\nu^* \rightarrow 0$, Eq.(7) is reduced to :

$$\frac{\epsilon^*}{3.7D^*} = 10^{-\frac{\sqrt{2}}{\pi D^{*5/2}}} \quad (10)$$

The obtained $D^* (\epsilon^*, \nu^* \rightarrow 0)$ relationship is implicit and [4] gave the following approximate explicit solutions, depending on the relative roughness range:

$$D^* = \frac{\epsilon^{*0.03}}{1.853}, \quad 10^{-8} < \epsilon/D < 7 \times 10^{-4} \quad (11)$$

$$D^* = \frac{\epsilon^{*1/16}}{1.422}, \quad 10^{-4} < \epsilon/D < 7 \times 10^{-2} \quad (12)$$

The most relevant and attractive equations for turbulent pipe flow computation are certainly those we have reviewed. Among the five parameters which govern the flow, only the discharge Q can be explicitly computed by the use of Eq.(6). The diameter D and the energy slope J are both given either by an iterative procedure or by approximate equations with respect to the flow regime. These approximate equations are not founded upon a theory. Moreover, most of the parameters that appear in these relationships have often no physical meaning, such as D^* and ν^* of Eq.(7). Furthermore, these approximate equations are not valid in the entire domain of Moody diagram because their applicability depends strongly on the range of the relative roughness and Reynolds number. In addition, deviation involved in these relationships is sometimes important, depending on both R and ϵ/D , and may be unacceptable in some practical cases. For all these reasons, the main objective of this study is to propose a new approach for turbulent pipe-flow computation. The basic Eqs.(1) to (3) are applied to a rough pipe-flow characterized by an arbitrarily assigned relative roughness value. This is a referential rough model from which the current pipe-flow characteristics are directly deduced. Explicit relevant equations are presented for $R > 2300$ and for $0 \leq \epsilon/D \leq 0.05$, covering the entire domain of Moody diagram [6].

II. REFERENTIAL ROUGH PIPE MODEL

The referential rough pipe-model we consider is a pipe characterized by $\epsilon/D = 0.037$ as the arbitrarily assigned relative roughness value. The prevailed flow regime is fully rough and the friction factor is $f = 1/16$ according to Eq.(2) for $R = \bar{R}$ tending to infinitely large value. Thus, applying Eq.(1), the energy slope \bar{J} and the diameter \bar{D} are expressed as :

$$\bar{J} = \frac{\bar{Q}^2}{2g\pi^2 \bar{D}^5} \quad (13)$$

$$\bar{D} = (2\pi^2)^{-1/5} \left(\frac{\bar{Q}^2}{g\bar{J}} \right)^{1/5} \quad (14)$$

We thus deduce from Eq.(14) that the characteristic diameter in [1] corresponds, to within a constant, to the referential fulfilled rough pipe diameter for $\bar{Q} = Q$ and $\bar{J} = J$. This is the physical meaning of the characteristic diameter. Applying Eq.(3) for the referential rough pipe model, results in :

$$\bar{R} = \frac{4Q}{\pi D \nu} \quad (15)$$

Eliminating \bar{D} between Eqs. (14) and (15) leads to :

$$\bar{R} = (2048 / \pi^3)^{1/5} \frac{(gJ\bar{Q})^{1/5}}{\nu} \quad (16)$$

As it can be observed, the inverted kinematic parameter ν^{*-1} in [1] corresponds in fact, to within a constant, to the Reynolds number \bar{R} governing the flow in the referential rough pipe model for $\bar{Q} = Q$ and $\bar{J} = J$. On the other hand, eliminating \bar{Q} between Eqs.(14) and (15) gives :

$$\bar{R} = 4\sqrt{2} \frac{\sqrt{gJ\bar{D}^3}}{\nu} \quad (17)$$

III. TURBULENT PIPE-FLOW COMPUTATION

A. Q Is the Unknown Parameter

1) Reynolds Number R

In this section, we assume $\bar{J} = J$ and $\bar{D} = D$. These identities imply $\bar{Q} \neq Q$ and obviously $\bar{R} \neq R$. Combining Eqs.(1) and (13) leads to :

$$Q = \psi_Q \bar{Q} \quad (18)$$

where

$$\psi_Q = \frac{1}{4\sqrt{f}} \quad (19)$$

In view of (18), Q is equal to \bar{Q} corrected for effect of ψ_Q which can be then considered as a non-dimensional correction factor of discharge. From Eqs.(3), (15) and (18) and bearing in mind that $\bar{D} = D$, we thus deduce :

$$R = \psi_Q \bar{R} \quad (20)$$

Substituting Eqs.(19) and (20) into (2), one may obtain :

$$\psi_Q = -\frac{1}{2} \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right) \quad (21)$$

Inserting Eq.(21) into (20), we derive the following explicit Reynolds number relationship:

$$R = -\frac{1}{2} \bar{R} \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right) \quad (22)$$

Equation (22) permits a direct computation of the Reynolds number R provided J, D, ε and ν are given. The Reynolds number \bar{R} is expressed by Eq.(17) for $\bar{J} = J$ and $\bar{D} = D$, whence :

$$\bar{R} = 4\sqrt{2} \frac{\sqrt{gJ\bar{D}^3}}{\nu} \quad (23)$$

Notice that in Eq.(23) $\bar{R} = \bar{R}(Q \neq \bar{Q}, D = \bar{D}, J = \bar{J})$.

2) Friction Factor f

The explicit friction factor relationship is obtained when combining Eqs.(19) and (20), whence :

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right) \quad (24)$$

Although the discharge Q is unknown, Eq.(24) permits a direct calculation of the friction factor f for $R > 2300$, provided J, D, ε and ν are given. The Reynolds number \bar{R} must be computed using Eq.(23).

3) Discharge Q

Inserting Eq.(21) into Eq.(18) results in :

$$Q = -\frac{\bar{Q}}{2} \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right) \quad (25)$$

where \bar{Q} is given by Eq.(13) for $\bar{J} = J$ and $\bar{D} = D$, whence :

$$\bar{Q} = \pi \sqrt{2} \sqrt{gJ\bar{D}^5} \quad (26)$$

On the other hand, eliminating R between Eqs.(3) and (22), the discharge Q can also be expressed as :

$$Q = -\frac{\pi D \nu}{8} \bar{R} \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right) \quad (27)$$

Equation (27), along with Eq.(23), permits calculation of the discharge Q provided J, D, ε and ν are given.

Introducing the kinematic parameter $\aleph = Q/(\nu D)$, Eq.(27) becomes :

$$\aleph = -\frac{\pi}{8} \bar{R} \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{\bar{R}} \right) \quad (28)$$

B. D Is the Unknown Parameter

1) Reynolds Number R

In this section, we assume $Q = \bar{Q}$ and $J = \bar{J}$. Thus, one may write $D \neq \bar{D}$ and obviously $R \neq \bar{R}$. When considering Eqs.(14) and (16), \bar{D} and \bar{R} can be then expressed as follows :

$$\bar{D} = (2\pi^2)^{-1/5} \left(\frac{Q^2}{gJ} \right)^{1/5} \quad (29)$$

$$\bar{R} = (2048/\pi^3)^{1/5} \frac{(gJ\bar{Q}^3)^{1/5}}{\nu} \quad (30)$$

On the other hand, Eqs.(1) and (13) lead to :

$$D = (16f)^{1/5} \bar{D} \quad (31)$$

which can be simply rewritten as :

$$D = \psi \bar{D} \quad (32)$$

where

$$\psi = (16f)^{1/5} \quad (33)$$

In view of Eq.(32), D is equal to \bar{D} corrected for effect of ψ which can be then considered as a non-dimensional correction factor of diameter. Combining Eqs.(3), (15) and (32) results in :

$$R = \psi^{-1} \bar{R} \quad (34)$$

Introducing Eqs.(32), (33) and (34) into Eq.(2), we thus derive :

$$\psi^{5/2} \log\left(\frac{\varepsilon/\bar{D}}{3.7\psi} + \frac{10.04}{R\psi^{3/2}}\right) = -2 \quad (35)$$

Due to the implicit form of Eq.(35), ψ must be graphically estimated or computed with the aid of an iterative procedure. One way to avoid this is to use the following derived explicit relationship [7]:

$$\psi \cong 1.35 \left[-\log\left(\frac{\varepsilon/\bar{D}}{4.75} + \frac{8.5}{\bar{R}}\right) \right]^{-2/5} \quad (36)$$

A comparison was made between Eqs.(35) and (36), varying ε/\bar{D} from 0 to 0.02 (Fig.2). As it can be seen in figure 2, maximum deviation is less than 0.4% for $\bar{R} > 2200$ corresponding to $R > 2300$.

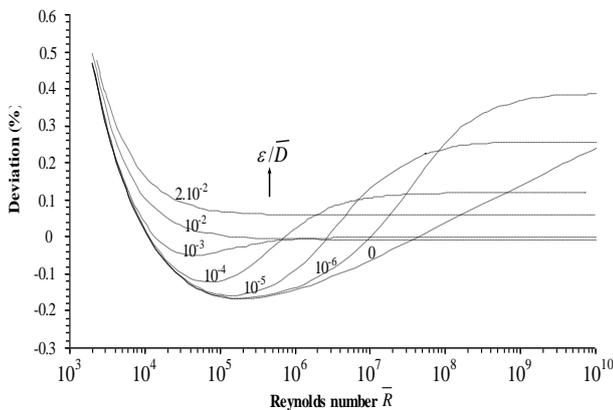


Fig. 2 Comparison between Eqs.(35) and (36) for some values of ε/\bar{D} .

Assuming Eq. (36) and inserting it into Eq.(34) results in :

$$R \cong \frac{\bar{R}}{1.35} \left[-\log\left(\frac{\varepsilon/\bar{D}}{4.75} + \frac{8.5}{\bar{R}}\right) \right]^{2/5} \quad (37)$$

Equation (37) permits calculation of the Reynolds number R even though the diameter D is unknown. The diameter \bar{D} and the Reynolds number \bar{R} are given by Eqs.(29) and (30) respectively. Notice that in Eq.(37) $\bar{R} = \bar{R}(Q = \bar{Q}, D \neq \bar{D}, J = \bar{J})$. The maximum deviation involved in Eq.(37) should not exceed 0.4%, since $(\Delta R/R)_{\max} = (\Delta\psi/\psi)_{\max} < 0.4\%$ according to Eq.(34). If a better accuracy is sought, it is possible to establish a more suitable formulation for R , when assuming what follows. Taking into account Eq.(32), Eq.(23) can be written as :

$$\bar{R} = 4\sqrt{2} \psi^{3/2} \frac{\sqrt{gJ\bar{D}^3}}{\nu} \quad (38)$$

or as :

$$\bar{R}(Q \neq \bar{Q}, D = \bar{D}, J = \bar{J}) = \psi^{3/2} \bar{R}(Q = \bar{Q}, D \neq \bar{D}, J = \bar{J}) \quad (39)$$

On the other hand, combining Eqs.(19) and (33), one obtains :

$$\psi^{-1} = \psi^{2/5} \quad (40)$$

With the aid of Eqs.(21),(32), (39) and (40), Eq.(34) becomes :

$$R = \bar{R} \left[-\frac{1}{2} \log\left(\frac{\varepsilon/\bar{D}}{3.7\psi} + \frac{10.04}{R\psi^{3/2}}\right) \right]^{2/5} \quad (41)$$

Equation (41) was compared to Eq.(34) along with Eq.(35), for $\bar{R} > 2200$ and $0 \leq \varepsilon/\bar{D} \leq 0.05$. It found that the deviation is negligible since it is equal or less than 0.05%. Equation (41) allows then a direct determination of R , with respect to the following three steps:

Knowing Q, J and ν , Eqs.(29) and (30) give \bar{D} and \bar{R} respectively. Notice that \bar{R} can also be computed from Eq.(15) for $\bar{Q} = Q$.

Knowing ε, \bar{D} and \bar{R} , the non-dimensional correction factor ψ follows then immediately from Eq.(36).

With $\varepsilon, \bar{D}, \bar{R}$ and ψ , Eq.(41) gives finally the required value of R .

2) Friction Factor F

Although the diameter D is the unknown parameter, the friction factor f can be computed using an explicit relationship, provided Q, J, ε and ν are given. This can be derived from Eqs.(33) and (36), whence :

$$f = \left[-1,889 \log\left(\frac{\varepsilon/\bar{D}}{4,75} + \frac{8,5}{\bar{R}}\right) \right]^{-2} \quad (42)$$

Both \bar{D} and \bar{R} must be computed using Eqs.(14) and (15) respectively, for $\bar{Q} = Q$ and $\bar{J} = J$.

The maximum error involved in Eq.(42) is less than 1% when compared to Eq.(2). When a better accuracy is needed, a more suitable relationship can be derived from Eq.(33), (34) and (41). One can write :

$$f = \left[-2 \log\left(\frac{\varepsilon/\bar{D}}{3.7\psi} + \frac{10.04}{R\psi^{3/2}}\right) \right]^{-2} \quad (43)$$

In Eq.(43), computation of \bar{D}, \bar{R} and ψ follows the same steps than those previously indicated. For $R > 300$ and $0 \leq \varepsilon/D \leq 0.05$, Eq.(43) allows a direct calculation of the practically exact value of f when compared to Eq.(2).

3) Diameter D

Provided Q, J, ε and ν are given, the diameter D can be computed by the use of one of the two following explicit relations. The first one is deduced from Eqs.(32) and (36), the final result being :

$$D = 1.35 \bar{D} \left[-\log\left(\frac{\varepsilon/\bar{D}}{4.75} + \frac{8.5}{\bar{R}}\right) \right]^{-2/5} \quad (44)$$

The maximum deviation involved in Eq.(43) is 0.4% which is quite satisfactory for practical purposes.

The second one is obtained when eliminating f between Eqs.(1) and (43). It reads:

$$D = (2/\pi^2)^{1/5} \left[-\log\left(\frac{\varepsilon/\bar{D}}{3.7\psi} + \frac{10.04}{R\psi^{3/2}}\right) \right]^{-2/5} \left(\frac{Q^2}{gJ}\right)^{1/5} \quad (45)$$

Eq.(45) is a much better approximation than Eq.(44). It gives practically the exact value of the internal diameter D of the pipe. With the aid of Eq.(29), Eq.(45) can be written as :

$$D = \bar{D} \left[-\frac{1}{2} \log \left(\frac{\varepsilon / \bar{D}}{3.7\psi} + \frac{10.04}{R\psi^{3/2}} \right) \right]^{-2/5} \quad (46)$$

or, in terms of non-dimensional parameters, as :

$$q = -\frac{\pi}{\sqrt{2}} \log \left(\frac{\varepsilon / \bar{D}}{3.7\psi} + \frac{10.04}{R\psi^{3/2}} \right) \quad (47)$$

C. J Is the Unknown Parameter

1) Friction Factor F

In this section, Q , D , ε and ν are given. Thus, the Reynolds number R and the relative roughness ε/D are known. We assume $Q \neq \bar{Q}$, $D = \bar{D}$, implying $R \neq \bar{R}$ in view of Eqs.(3) and (15). Furthermore, the friction factor f can be computed by Eq.(24), once \bar{R} is determined using Eq.(22). The latter is however implicit in \bar{R} and the following suitable $\bar{R}(\varepsilon/D, R)$ explicit relationship is recommended:

$$\bar{R} = 2R \left[-\log \left(\frac{\varepsilon/D}{3.7} + \frac{5.5}{R^{0.9}} \right) \right]^{-1} \quad (48)$$

Assuming Eq.(48), a comparison was made between Eqs.(2) and (24). The result has been plotted in figure 3 from which one can observe that the deviation depends on both R and ε/D , but remains less than 0.4% for $R > 2300$ and $0 \leq \varepsilon/D \leq 0.05$.

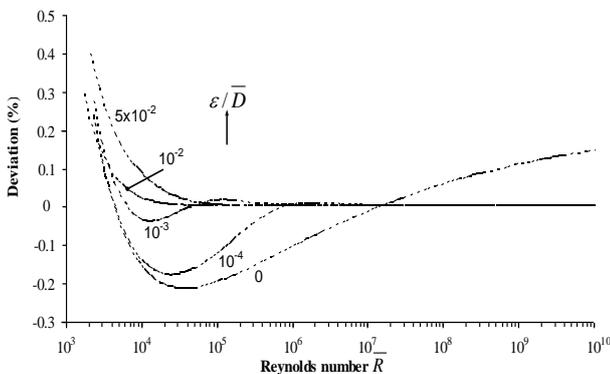


Fig. 3 Comparison between Eqs.(2) and (24), along with Eq.(48) for some values of ε/\bar{D}

2) Energy Slope J

Combining Eqs.(1) and (24) results in the following explicit expression of the energy slope J :

$$J = \frac{2Q^2}{g\pi^2 D^5} \left[-\log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{R} \right) \right]^{-2} \quad (49)$$

where \bar{R} is given by Eq.(48) for the known values of both ε/D and R . Equation (49) is valid for $R > 2300$, covering the entire range of the Moody diagram [6].

Assuming $Q = \bar{Q}$ and $D = \bar{D}$, Eq.(13) gives :

$$\bar{J} = \frac{Q^2}{2g\pi^2 D^5} \quad (50)$$

Combining Eq.(49) and (50), one can write :

$$J = \bar{J} \left[-\frac{1}{2} \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{R} \right) \right]^{-2} \quad (51)$$

Otherwise, the energy slope J can be written as:

$$J = \psi_J \bar{J} \quad (52)$$

where ψ_J is the non-dimensional correction factor of energy slope, given by:

$$\psi_J = \left[-\frac{1}{2} \log \left(\frac{\varepsilon/D}{3.7} + \frac{10.04}{R} \right) \right]^{-2} \quad (53)$$

bearing in mind that \bar{R} is defined by Eq.(48).

IV. PRACTICAL APPLICATION

Determine the internal diameter D of a pressure pipe for:

$$J = 2 \times 10^{-4}, Q = 1.5 \text{ m}^3/\text{s}, \varepsilon = 0, \nu = 10^{-6} \text{ m}^2/\text{s}$$

The problem can be solved under the two following conditions:

$$\bar{Q} = Q, \bar{J} = J$$

The rough pipe model diameter \bar{D} is given by Eq.(29) as :

$$\bar{D} = (2\pi^2)^{-1/5} \left(\frac{Q^2}{gJ} \right)^{1/5} = (2 \times \pi^2)^{-1/5} \times \left(\frac{1.5^2}{9.81 \times 2 \times 10^{-4}} \right)^{1/5} \cong 2.253361 \text{ m}$$

Using Eq.(15), Reynolds number \bar{R} is then :

$$\bar{R} = \frac{4Q}{\pi D \nu} = \frac{4 \times 1.5}{\pi \times 2.253361 \times 10^{-6}} = 847560.3329$$

With the aid of Eq.(36), the correction factor of diameter ψ is obtained as :

$$\begin{aligned} \psi &\cong 1.35 \left[-\log \left(\frac{\varepsilon/\bar{D}}{4.75} + \frac{8.5}{\bar{R}} \right) \right]^{-2/5} \\ &= 1.35 \times \left[-\log \left(\frac{8.5}{847560.3329} \right) \right]^{-2/5} = 0.70923334 \end{aligned}$$

The required value of D is thus:

$$D = \psi \bar{D} = 0.70923334 \times 2.253361 = 1.598 \text{ m} \cong 1.6 \text{ m}$$

For the same pipe, determine the discharge Q .

The problem can be solved under the following conditions:

$$\bar{D} = D, \bar{J} = J$$

The Reynolds number \bar{R} is given by Eq.(23), whence :

$$\bar{R} = 4\sqrt{2} \frac{\sqrt{gJD^3}}{\nu} = 4 \times \sqrt{2} \times \frac{\sqrt{9.81 \times 2 \times 10^{-4} \times 1.6^3}}{10^{-6}} = 507112.6739$$

The discharge \bar{Q} flowing in the rough model is given by Eq.(26), hence :

$$\bar{Q} = \pi\sqrt{2}\sqrt{gJD^5} = \pi \times \sqrt{2} \times \sqrt{9.81 \times 2 \times 10^{-4} \times 1.6^5} = 0.63725658 \text{ m}^3/\text{s}$$

Applying Eq.(21), the non-dimensional correction factor of discharge ψ_Q is then :

$$\psi_Q = -\frac{1}{2} \log\left(\frac{10.04}{\bar{R}}\right) = -\frac{1}{2} \times \log\left(\frac{10.04}{507112.6739}\right) = 2.35168538$$

Finally, Eq.(18) gives the required discharge Q as :

$$Q = \psi_Q \bar{Q} = 2.35168538 \times 0.63725658 = 1.4986 \text{ m}^3 \text{ s}^{-1} \cong 1.5 \text{ m}^3/\text{s}$$

Consider the same pipe and evaluate the energy slope J .

The problem can be solved under the following conditions:

$$\bar{Q} = Q, \quad \bar{D} = D$$

Eq.(3) allows calculation of Reynolds number R , whence:

$$R = \frac{4Q}{\pi D \nu} = \frac{4 \times 1.5}{\pi \times 1.6 \times 10^{-6}} = 1193662.07$$

Furthermore, by the use of Eq.(48), the Reynolds number of the flow in the rough model is then:

$$\begin{aligned} \bar{R} &= 2R \left[-\log\left(\frac{5.5}{R^{0.9}}\right) \right]^{-1} \\ &= 2 \times 1193662.07 \times \left[-\log\left(\frac{5.5}{1193662.07^{0.9}}\right) \right]^{-1} = 504844.508 \end{aligned}$$

Using Eq.(49), the energy slope J is finally :

$$\begin{aligned} J &= \frac{2Q^2}{g\pi^2 D^5} \left[-\log\left(\frac{10.04}{\bar{R}}\right) \right]^{-2} \\ &= \frac{2 \times 1.5^2}{9.81 \times \pi^2 \times 1.6^5} \times \left[-\log\left(\frac{10.04}{504844.508}\right) \right]^{-2} = 2 \times 10^{-4} \end{aligned}$$

V. CONCLUSIONS

Turbulent flow-pipe characteristics are computed by a new approach based on a referential rough pipe model. This is characterized by the arbitrarily assigned relative roughness value $\bar{\varepsilon}/\bar{D} = 0.037$. The flow is supposed to be fully rough so that the friction factor is $\bar{f} = 1/16$ according to Colebrook-White equation. Applying the Darcy-Weisbach relation [Eq.(1)], it is then possible to express the characteristics of the

rough pipe-flow model, in particular the energy slope \bar{J} , the diameter \bar{D} and the Reynolds number \bar{R} . These are given by Eqs. (13) to (17).

Furthermore, computation of the three pipe-flow parameters of practical interest namely, $Q(D, J, \varepsilon, \nu)$, $D(Q, J, \varepsilon, \nu)$ and $J(Q, D, \varepsilon, \nu)$ is worked out conformably to the following steps :

Computation of the discharge $Q(D, J, \varepsilon, \nu)$ is conducted by assuming the three following conditions, namely $Q \neq \bar{Q}$, $D = \bar{D}$ and $J = \bar{J}$. The Reynolds number \bar{R} is then well defined by Eq.(23) and the required discharge Q follows then from Eq.(24) which is the exact solution of the original implicit Colebrook's equation. Otherwise, the discharge Q can be obtained once the friction factor $f(\varepsilon/D, \bar{R})$ is determined by using Eq.(26) and putting the result in the Darcy-Weisbach equation.

Computation of the diameter $D(Q, J, \varepsilon, \nu)$ is conducted by assuming the following conditions, namely, $Q = \bar{Q}$, $D \neq \bar{D}$ and $J = \bar{J}$. The diameter \bar{D} and the Reynolds number \bar{R} are determined by using Eq.(27) and (28) respectively. Thus, the required diameter D is computed by Eq.(44) which is of simple formulation, involving only 0.4% of maximum deviation. Otherwise, D can be computed with a better accuracy by the use of Eq.(45) once the correction factor $\psi(\varepsilon/\bar{D}, \bar{R})$ is determined from Eq.(34).

Computation of the energy slope $J(Q, D, \varepsilon, \nu)$ is conducted under the same conditions than those in (i), i.e. $Q \neq \bar{Q}$, $D = \bar{D}$ and $J = \bar{J}$. The Reynolds number \bar{R} is then well defined by Eq.(48) for the known values of ε/D and R . Furthermore, Eq.(49) gives the required value of the energy slope J .

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