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Faculté des Sciences et de la Technologie
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MASTER thesis

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Analysis ECG signals by adaptive wavelets

The: jeudi 7 juillet 2022

Présenté par :

-BAHRI Mounira

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Université Mohamed Khider de Biskra
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Analysis ECG signals by adaptive wavelets

Presented and supported by:

-Beddiaf Seif Eddine -Bahri Mounira

On:

07/07/2022

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Master's thesis

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Theme:

Analysis ECG signals by adaptive wavelets

Presented by:

- **BAHRI Mounira**
- **BEDDIAF Seif Eddine**

Favorable opinion of the supervisor:

Signature

Favorable opinion of the President of Jury:

Signature

Dedications:

This work is for my parents, my brothers, sisters, my partner, my family, relations, friends, and all who support me.

Beddiaf seif eddine

Thank you my parents, my brothers, sisters, my partner, my family, relations, friends, and all who supported me to finish this work, thank you for supporting me.

Bahri Mounira

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Abstract

This work aims to determine the cardiac pathologies which are distinctive with their own special ECG signals, this study analyses these signals by discrete wavelet transform which is based on decompositions into approximations and details and adaptive lifting technique based on designing wavelet filter by adaptive lifting scheme , and these methods are blended with fast Fourier transform algorithm to determine pathologies by using the variations between frequencies in each case, and also making comparison between discrete wavelet transform and adaptive lifting and how much each one of them goes far in this type of studies specially in case where we want to determine as much possible pathologies using the same transform at the same time.

Keywords : ECG, Wavelet transform, DWT, CWT, FFT, Adaptive lifting.

Résumé

Ce travail vise à déterminer les pathologies cardiaques qui se distinguent par leurs propres signaux ECG spéciaux, cette étude analyse ces signaux par transformée en ondelettes discrètes qui est basée sur des décompositions en approximations et détails et technique de levage adaptatif basée sur la conception d'un filtre à ondelettes par schéma de levage adaptatif, et ces méthodes sont combinées avec un algorithme de transformée de Fourier rapide pour déterminer les pathologies en utilisant les variations entre les fréquences dans chaque cas, et en faisant également une comparaison entre la transformée en ondelettes discrète et le levage adaptatif et combien chacun d'eux va loin dans ce type d'études spécialement dans cas où l'on veut déterminer un maximum de pathologies possibles en utilisant la même transformée en même temps.

Mots clés : ECG, Transformée en ondelettes, TOD, TOC,TRF, Lifting adaptatif.

ملخص

يهدف هذا العمل إلى تحديد أمراض القلب التي تتميز بإشارات تخطيط القلب الخاصة بها ، وتحلل هذه الدراسة هذه الإشارات عن طريق التحويل المويجي المنفصل الذي يعتمد على التحلل إلى تقديرات تقريبية وتفاصيل وتقنية الرفع التكيفية القائمة على تصميم مرشح المويجات عن طريق الرفع التكيفي المخطط ، ويتم دمج هذه الطرق مع خوارزمية تحويل فورييه السريعة لتحديد الأمراض باستخدام الاختلافات بين الترددات في كل حالة ، وكذلك إجراء مقارنة بين التحويل المويجي المنفصل والرفع التكيفي ومدى تقدم كل منهما في هذا النوع من الدراسة ، خاصة في الحالات التي تريد فيها تحديد الحد الأقصى من الأمراض المحتملة باستخدام نفس التحويل في نفس الوقت.

الكلمات المفتاحية: التحويل الموجي ,الرفع التكيفيDWT, FFT,CWT , . ECG , .

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General introduction

The development of applied sciences forced scientists to develop and improve new mathematical methods(algorithms, functions, transformations…etc.) to address and solve various intractable problems that are still up to the present time in a variety of research frameworks and in multi-dimensional fields, and one of these applied sciences, is the signal processing, which has undergone a great development in the last 30 years, starting from Eighties, this science it is a subfield in electrical engineering that focuses on analysing, modifying, and synthesizing signals such as sounds, images, and scientific measurements, also its techniques can be used to improve transmission, storage efficiency and subjective quality and to also emphasize or detect components of interest in a measured signal.

One of the most developed methods in this field is the wavelet analysis which is based on wavelet transform**.**The wavelet is a wave like oscillation with an amplitude that begins at zero, increases or decreases, and then returns to zero one or more times. Wavelets are termed a "brief oscillation". Taxonomy of wavelets has been established, based on the number and direction of its pulses. Wavelets are imbued with specific properties that make them useful for signal processing, the wavelets are classifying according 2 principals conceptions thedimension(1D,2D,3D) and the type of signal processed by it (continues or discrete).

From 1807 the field of signal processing and that was by Joseph Fourier who developed a method for representing a signal with a series of coefficients based on an analysis function. He laid the mathematical basis from which the wavelet theory is developed. The first to mention wavelets was Alfred Haar in 1909 in his PhD thesis. In the 1930’s, Paul Levy found the scale-varying Haar basis function superior to Fourier basis functions. The Transformation method of decomposing a signal into wavelet coefficients and reconstructing the original signal again is derived in 1981 by Jean Morlet and Alex Grossman. In 1986, StephaneMallat and Yves Meyer developed a multi resolution analysis using wavelets. They mentioned the scaling function of wavelets for the first time, it allowed researchers and mathematicians to construct their own family of wavelets using the derived criteria. Around 1998, Ingrid Daubechies used the theory of multi-resolution wavelet analysis to construct her own family of wavelets. Her set of wavelet orthonormal basis functions have become the cornerstone of wavelet applications today. With her work the theoretical treatment of wavelet analysis is as much as covered.

Application of wavelet:

Wavelets are used in a large number of fields. Let us just mention geophysics, astrophysics, quality control, biology and aural signals in medicine, imagery in all its aspects and medical imagery in particular, compressed representation of fingerprints or photographs, satellite imagery, coding of video signals, modelling of traffic in communication networks like the Internet and analysis of atmospheric or wind tunnel turbulence. Even after years of work we remain surprised by the variety of domains concerned and the problems dealt with.

Advantages of wavelet transform:

A WT application is shown, as well as its benefits over the Fourier transform. One of the primary advantages of wavelets is that they provide simultaneous localisation in the time and frequency domains. The second major benefit of wavelets is that they are computationally very quick when utilizing the fast wavelet transform. Wavelets have the distinct benefit of being able to discern small features in a signal. Very small wavelets can be used to isolate very fine features in a signal, whereas very big wavelets can detect coarse details. A wavelet transform can be used to decompose a signal into component wavelets. When compared to the Fourier transform, it is typically possible to produce a fair approximation of the provided function for using only a few coefficients in wavelet theory. Wavelet theory can reveal data characteristics that other signal analysis approaches overlook, such as trends, breakdown points, discontinuities in higher derivatives, and self-similarity. It may frequently compress or de-noise a signal without causing significant deterioration. The Fourier transform appears in a surprising number of domains other than traditional signal processing. Even with this in mind, we believe it is reasonable to conclude that the mathematics of wavelets is far more extensive than that of the Fourier transform. In fact, the mathematics of wavelets encompasses the Fourier transform. The size of wavelet theory is matched by the size of the application area. Initial wavelet applications involved signal processing and filtering. However, wavelets have been applied in many other areas including nonlinear regression and compression. An offshoot of wavelet compression allows the amount of determinism in a time series to be estimated The Fourier transform shows up in a remarkable number of areas outside classic signal processing. Even taking this into account, we think that it is safe to say that the mathematics of wavelets is much larger than that of the Fourier transform. In fact, the mathematics of wavelets encompasses the Fourier transform. Initial wavelet applications involved signal processing and filtering. However, wavelets have been applied in many other areas including nonlinear regression and compression. An offshoot of wavelet compression allows the amount of determinism in a time series to be estimated.

Biomedical signals:

Biomedical signals are observations of organisms' physiological processes, such as gene and protein sequences, brain and cardiac rhythms, and tissue and organ pictures. The goal of biomedical signal processing is to extract useful information from biological signals. Biologists can uncover novel biology and clinicians can monitor specific ailments with the help of biomedical signal processing.And the most important biomedical signals are classified into two types such as action potential and eventrelated potential. Electromyogram (EMG), electroneurogram (ENG), electrocardiogram (ECG) and electroencephalogram (EEG) are existing action potential. The event-related potentials (ERPs) are electrogastrogram (EGG), phonocardiogram (PCG), carotid pulse (CP), signals from catheter-tip sensors, speech signal, vibromyogram (VMG), vibroarthrogram (VAG), oto-acoustic emission signal . The various image modalities are widely used in the biomedical field, i.e., functional magnetic resonance imaging (fMRI), computed tomography (CT), ultrasound imaging and positron emission tomography (PET) , in our work we focuses in studying ECG signals in (4 cases will be detailed).

The objective of our work:

In our research, we are going to study the analysing of signals by using a technique based on wavelet transform DWT in the discrete time field and the adaptive lifting. The adaptive lifting scheme is a modified version of the classic lifting scheme, the adaptation consists in choosing between several filters, according to the local information of the signal. We try make a comparison between analysis ECG signals with classical wavelet transform and analysis ECG signals by adaptive lifting schemes.

The manuscript is divided in four chapters:

- In the first chapter, we will provide a general understanding of the heart's functioning and the ECG signals that will be the focus of our research, beginning with the anatomic structure of the heart and moving on to explain its conductional system, including the normal ECG waveform and its construction (QRS complex, ST segment, P wave, T wave...etc.), as well as how we can perform the medical process of measuring ECG cardiograms for treatment (medical process conception).

- The second chapter will address the theory of wavelet, different wavelets (haar, symlet … etc.), their algebraic structure and characteristics, types, and an overview about fast Fourier transform algorithm, and the signal processing in time and frequency domains.
- The third chapter will be a practical study by using matlab wavelet toolbox which works on simulation and getting results, this chapter will complete with direct conclusions about cases of ECG like myocardial infraction as an example in this chapter we will compare between wavelets about which one of them is more efficacy in ECG analysis and we will see the evolution of the signal processing between the Fourier analysis and the wavelet theory by comparing the FFT algorithm and different wavelets decompositions .
- In the fourth chapter we introduce the concept of adaptive lifting. Then, we can make a comparison between analysis ECG signals with classical wavelet transform and analysis ECG signals by adaptive lifting schemes.

And at the final conclusion we will give a say to what we're going to study, and we'll give an opinion on the work we've done with a commitment to impartiality and objectivity.

Chapter I: description of ECG signal

I.1 Introduction and overview

In this chapter we will address the beginning of the research, which will be the description of ECG signals, and of course that will be only by looking at the functioning of the heart and its anatomical structure, and then we will look at what these signals are (ECG) and we will talk about their wavelength components (P wave, T wave,… etc.), and at the end of this part we will give the practical part which is a simulation by MATLAB.

I.2 FUNCTION AND STRUCTURE OF THE HEART

I.2.1 Overview

The heart is a structure made up of cardiac muscles that help circulate blood throughout the body. The anatomy and conduction system of the heart are shown in Figure 1.2.1.The heart has four main functions: collecting the blood to be purified from all parts of the body (through veins), pumping this collected blood to the lungs, collecting the refined blood from the lungs, and pumping the refined blood back to all parts of the body (1). As can be seen in Figure I.1, the heart has four chambers - two atria and two ventricles. The atria work in unison, and so do the ventricles. The atrium is separated from the venous system by a valve so that outflow is only possible in one direction. The superior vena cava and the inferior vena cava lead into the right atrium together with the coronary sinus, while the pulmonary veins supply the left atrium. The valve only allows flow from the atrium to the ventricle and not in the opposite direction. This valve is called the atrioventricular valve (2).

The atrioventricular valve between the right atrium and the right ventricle is also called the tricuspid valve because it has a three-leaf structure. The left atrium and left ventricle each have a bicuspid valve, or mitral valve, which helps to separate the two chambers. The ventricles dilate when they fill with pressure from blood flow from the atria. The electrical delay line between the excitation of the atrium and the atrium causes the ventricle to contract out of sync with the atrium. The ventricle pumps blood into an artery. This artery is then separated from the ventricle by a one-way valve. The left ventricle pumps blood into the aorta, which is separated from the ventricle by the aortic valve to prevent backflow. The right ventricle squeezes the blood that needs refining into the arteria pulmonalis, separated by the pulmonary valve. The arteria pulmonalis feeds into the lung circulation (2) (3).

A schematic representation of the blood circulation provided by the heart is illustrated in Figure

The wall of the atrium and the ventricle consists of three main layers. Moving from the inside outward, the inner lining of the heart wall is called the endocardium; it consists of a single cell layer of flat, thin endothelial cells. The second layer is the myocardium; it is the main muscle of the ventricle. The epicardium is the outside lining of the ventricular wall; it consists of a single cell layer made up of flat cells. The left and right ventricles are separated by the septum, which is also a three-layer structure with endocardium, myocardium, and epicardium. The entire heart is suspended in the pericardial sack, which provides free movement in the area of the chest in between the lungs on the left side of the body. The main functionalities of the heart are provided by the structure and characteristics of the cardiac muscle to be described next. (1) (4) (2)

I.2.2 Cardiac muscle:

The main function of the cardiac muscle can be summarized as follows. An action potential that causes the heart muscle cells to contract reduces the atrial and ventricular volume, respectively. This results in an increase in pressure, leading to an outflow through the valve when the pressure before the valve exceeds the pressure behind the valve. This process provides the pressure changes to open and close the valves and therefore perform the pumping role of the heart. The contraction part of the heart is called the systole. Each systolic period is followed by a rest period, which is called the diastole. (5). (1)

In order to see how these contractions are provided by the cardiac muscle, we need to study the characteristics of cardiac muscles more closely. As a smooth muscle type tissue, the cardiac muscle is a hollow muscle that is designed to force blood into a tube to fill the body. The cardiac muscle has many of the typical properties of other muscle cells, except for the fact that cardiac muscle fibres are not excited all at once by one motoneuron. One major difference between skeletal muscle and heart muscle is with regard to the duration of the action potential (6). The cardiac muscle has a considerably longer depolarization and repolarization period. The longer depolarization period (several hundred milliseconds versus only a few milliseconds for skeletal muscle) ensures the maximum contraction from one single impulse. The longer repolarization period in the cardiac muscle ensures that there will be no immediate overlap in contractions. An important fact about the function of the cardiac muscle is that the depolarization period is a function of the frequency in which the cardiac muscle receives the initiation pulses. During this time and the subsequent repolarization period, theoretically, no new depolarization can be initiated. However, both the depolarization and the repolarization period are subject to shortening if the demand is there. The higher the repetition rate becomes, a shorter depolarization period is achieved. This is another fact differentiating the cardiac muscle from skeletal muscle. This characteristic of cardiac muscle allows the heart to adapt the state of physical exercise. Relative locations of the elementary components in the circulation mechanism of the heart. (1)

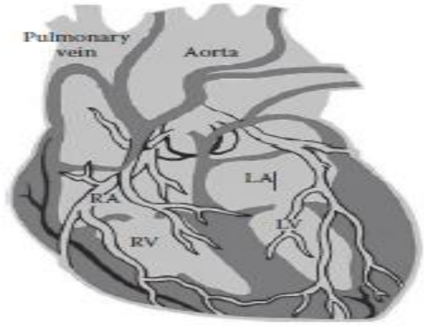


Figure I. 1: Function and structure of the heart

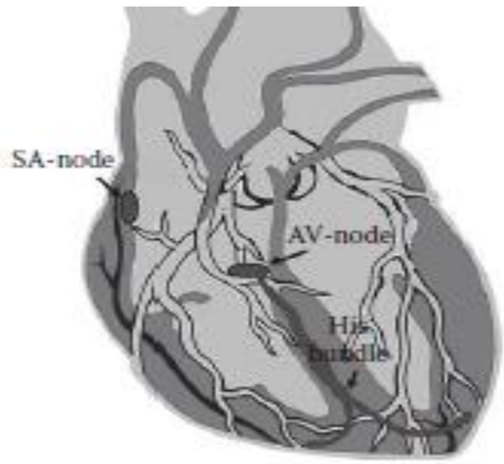


Figure I. 2: Location of natural pacemakers and conduction system of the heart.

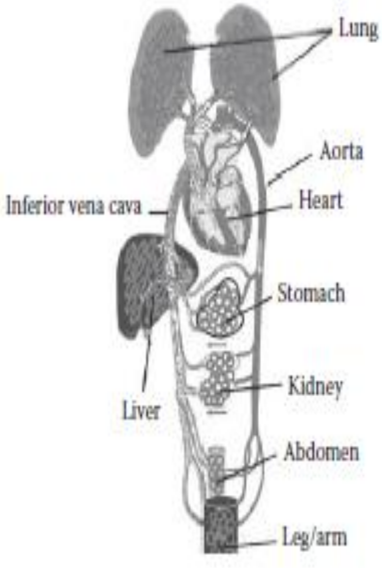


Figure I. 3 : Relative locations of the elementary components in the circulation mechanism of the heart

I.3 What is ECG:

The Electrocardiogram (ECG) is the most commonly used biomedical signal in clinical diagnostics of the heart. The word "electrocardiogram" is a combination of three words: **electro**, pertaining to electric signal; **cardio,** which translates into heart; and **gram**, which stands for recording. The recording of the electric activity of the heart is called ECG. (r biomedical signal N).

The contraction of the cardiac muscle is a direct result of the cellular electric excitation described by the ECG. The depolarization initiates the shortening of each individual muscle cell. The electric activation of each cell is an indication of the functioning of that cell. Therefore, the ECG is the result of depolarization of the heart muscle in a controlled repetitive fashion. By tracking the process of electric depolarization of the cardiac muscle cells, an impression of the heart's functionality can be formed and used to recognize regions in the heart structure that are not functioning to specifications and may require medical attention. Any deviation from the typical ECG observed in the recorded electric depolarization signal is analysed and classified as a certain cardiac disorder (7) (1).

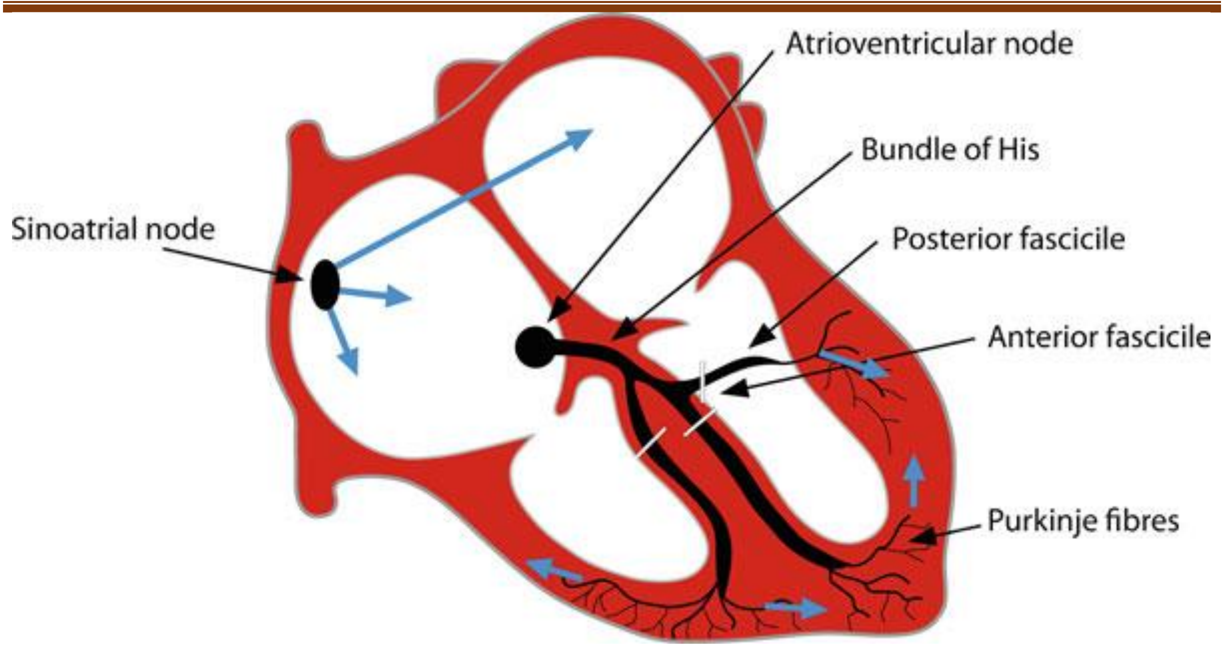


Figure I. 4: The electrical conduction system of the heart

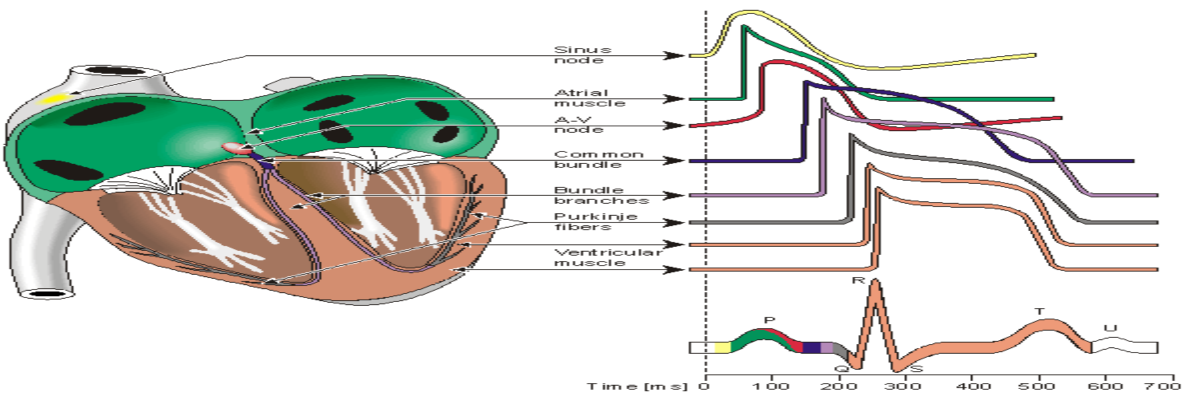


Figure I. 5: Electrophysiology of the heart and different waveforms for each of the specialized cells in the heart

I.4 The Electrical Conduction System of the Heart:

Cardiac muscle is composed of two main cell types : cardiomyocytes, which generate electrical potentials during contraction and cells, specialized in the generation and conduction of the action potentials. These specialized electrical cells depolarize spontaneously. At rest, cardiomyocytes are polarized with an electrical membrane potential of about −90 mV. Excitation by an external stimulus can trigger a rapid reversal of the electrical potential of working myocardial cells (depolarization). The depolarization is usually due to a sudden increase in permeability of the membrane to sodium, which allows positively charged sodium ions to enter the cell. In some cardiac cells the action potential is carried by calcium ions instead of sodium ions. The downward swing of the action potential, or repolarisation phase, is mainly due to the movement of potassium ions out of the cell. After depolarization, the muscle returns to its original electrical state. During the repolarisation, the cardiac muscle is Incapable of being stimulated (i.e., is refractory), which protects it against premature activation (4).

The conduction system of the heart is shown in Fig. 1.4. The Sino atrial node (S-A) has the highest rate of spontaneous depolarization and acts as the primary Pacemaker. At normal condition, the S-A node generates impulses that stimulate the atria to contract. This node is located in the superior wall of the right atrium, close to the opening of the superior vena cava. Other bêlements of the conduction system include the atrioventricular node (A-V), located between the atria and the ventricles, in the lower atrial septum adjacent to the annulus of the mitral valve (1) (4).

and the bundle of His. The bundle of His divides into a right and left branch at the level of membranous part of the interventricular septum. The left branch is further branched into an anterior and posterior bundle. The Purkinje fibres are the final component of the conduction system, which are intertwined with muscle fibres and the papillary muscles.

Their task is to conduct the wave fronts directly to the two ventricles so that they contract simultaneously. The Purkinje fibres have intrinsic automaticity (ventricular escape rhythm) generating approximately 30 bpm (beats per minute). The cells of the A-V node also depolarize spontaneously but at a higher rate (about 40–50bpm) and this automaticity is called (escape) junctional rhythm. In physiological conditions the automaticity of these rescue pacemakers is suppressed by the activity of the S-A node.

If electrical activity appears on the ECG recording later than expected (i.e., the

R-R interval is longer than the R-R interval in sinus rhythm), this means that an action potential originated in one of the lower pacemakers. However, the appearance of activity earlier than expected indicates the presence of a premature ectopic beat. Extra systolic beats are usually followed by compensatory pause. A full compensatory pause occurs after ventricular premature beats, and an incomplete compensatory pause occurs in cases of supraventricular premature beats. This happens because of the depolarization of the A-V node if the activity is supraventricular and its changes firing rate of the S-A node (Fig. 1.4).

The activity of the S-A node is regulated mainly by the autonomic nervous system. An activation of sympathetic fibres causes an increase in the heart rate, while activation of the parasympathetic fibres results in a decrease in this rate.

The normal heart rate at rest is approximately 60–70bpm but is lower at night during sleep. The heart rhythm is normally regular except for minor variations with respiration especially in young individuals (8) (4).

Using the terminology associated with electrical devices, the conduction system of the heart can be described as a pacemaker (S-A node), a resistor that simultaneously acts like a fuse (the A-V node) and two insulated electrical cables (branches of the bundle of His) (Fig. I.1). The term “resistor” for the property of the A-V node is appropriate because it slows down the depolarization (conduction velocity through the A-V node is slower than in other parts of the conducting system −0.05 m/s vs. 4 m/s, respectively). This delay enables the transfer of blood from the atria to the ventricles and is responsible for ensuring that the sequence of ventricular contraction follows atrial contraction. The comparison between the A-V node and a “fuse” is appropriate because the A-V node possesses Wenckebach’s point, which is thought to maintain the ratio of maximum conduction of the supraventricular impulses to ventricles at 1:1. Under normal conditions, this is about 180 impulses/min. In certain situations, such as in atrial fibrillation, Wenckebach’s point provides protection against the propagation of atrial fibrillation to ventricular fibrillation. In the presence of accessory conduction pathways bypassing the atrioventricular A-V node, this mechanism can fail and atrial flutter or fibrillation can therefore progress to ventricular flutter or fibrillation.

A properly functioning conduction system guarantees an appropriate heart rate and sequential contractions of the atria and ventricles. Cardiac electrical dysfunction can be caused by damage to or improper functioning of any of the components of the conduction system separately or in combination with other problems (for example, sinus bradycardia and bundle-branch block). Other causes of cardiac arrhythmias can be a pathological stimulus generation (for example, premature ectopic beats) or pathological conductive loops. Re-entry, which is a recurrence of electrical impulses, is the most common mechanism responsible for the occurrence of paroxysmal and persistent tachycardia. A common arrhythmia that arises in this manner is atrial flutter.

For this phenomenon to occur, a re-entry circuit formed by two conduction pathways is necessary. Impulses in the first pathway travel at a high velocity (the fast pathway) while impulses in the second pathway travel at a considerably lower velocity (the slow pathway). This means there is a delay between the arrivals of the two signals, so that when the second impulse arrives, the cells are no longer refractory. (4) (1)

I.5 Electrical Axis and Orientation of the Heart in the Chest Cavity:

Bipolar limb leads (I, II, III) developed by Willem Einthoven (Fig. 1.4.2) aim to calculate the mean depolarization vector of the heart in the frontal plane (the electrical axis of the heart) (Figs. I .7).

The normal range of the electric axis lies between +30◦ and −110◦ in the frontal plane.

The morphology of the ECG recorded depends on the orientation of the heart.

The unipolar augmented limb leads of Goldberger (aVR, aVL, and aVF) are used in determining the orientation of the heart. If the position of the heart is vertical as, for example, in asthenic women, the net deflection of the QRS complex in lead aVF is positive and it resembles lead V6 in morphology. Meanwhile, the QRS complex in lead aVL is negative and resembles lead V1 (Fig. 1.6 In contrast, when the orientation of the heart is horizontal, for example, in an endomorphic individual or in someone whose diaphragm is in a high position, the QRS complex in lead aVF resembles lead V1 (the net deflection of the QRS complex is negative) and lead aVL resembles lead V6 (Fig. 1.4.3b). (4) (1)

The placement of Wilson's unipolar precordial leads is as follows :

V1: 4th intercostal space, right of the sternum

V2: 4th intercostal space left of the sternum

V3: halfway between V2 and V4

V4: 5th intercostal space, left midclavicular line

V5: anterior axillary line, where it is intersected by a perpendicular line from lead V4

V6: midaxillary line, where it is intersected by a perpendicular line from lead V4

The chest leads sense the ECG in the transverse or horizontal plane. Leads V1 and V2 are placed above the anterior wall of the right ventricle. For this reason they are referred to as right ventricular leads. When the heart is normally oriented along the long axis, leads V5 and V6 are placed above the lateral wall of the left ventricle and are therefore known as left ventricular leads. The transitional zone between the left and right ventricles (interventricular septum) is found at the level of lead V3 and V4 (equal amplitudes of the R-wave and S-wave). In cases where the heart is rotated around its long axis the transitional zone is displaced, for example during an acute pulmonary embolism. The right ventricle is enlarged in this situation and the transitional zone is often shifted to the right toward the leads V5 or even V6 (4).

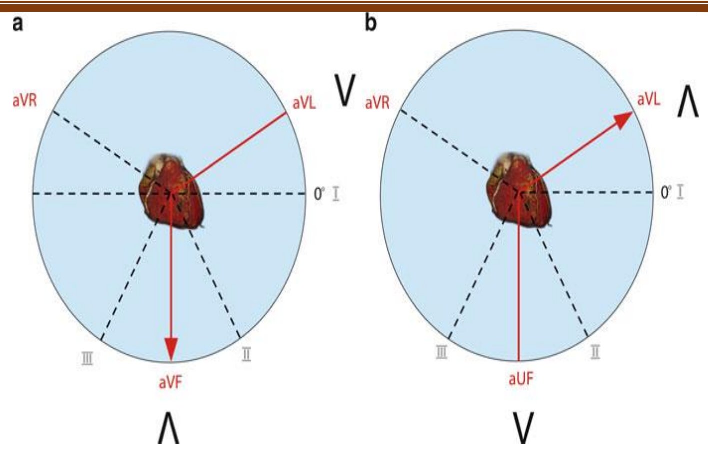


Figure I. 6: The orientation of the heart in chest cavity. Panel (a) – vertical position. Panel (b) –horizontal position of the heart. A fundamental principle of ECG recording is that when the wave of depolarization travels toward a recording lead these results in

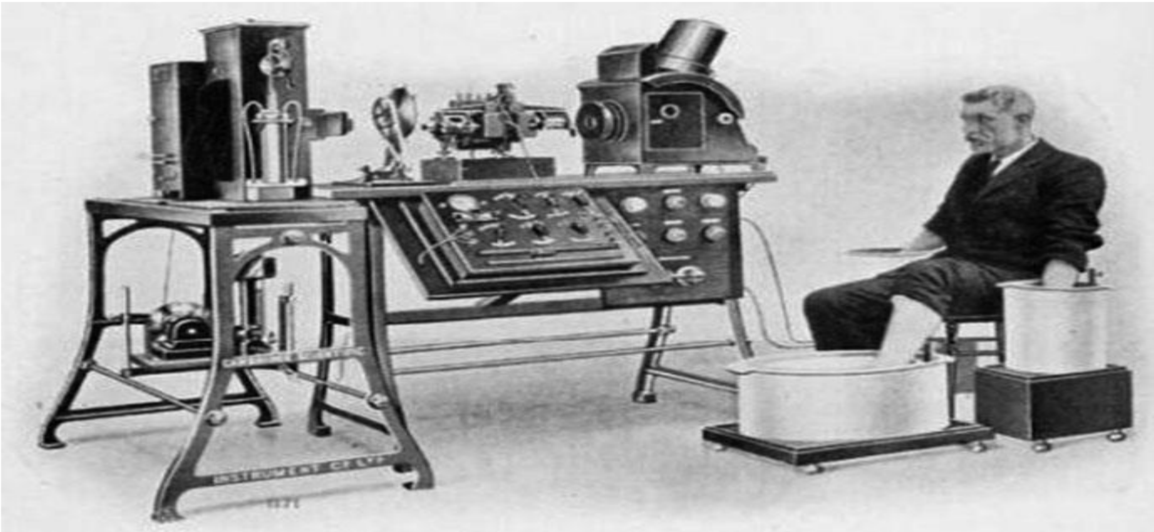


Figure I. 7: The ECG machine constructed by Willem Eindhoven (1860–1927). The patient is connected to the galvanometer and both hands and one leg are immersed in saline containers.

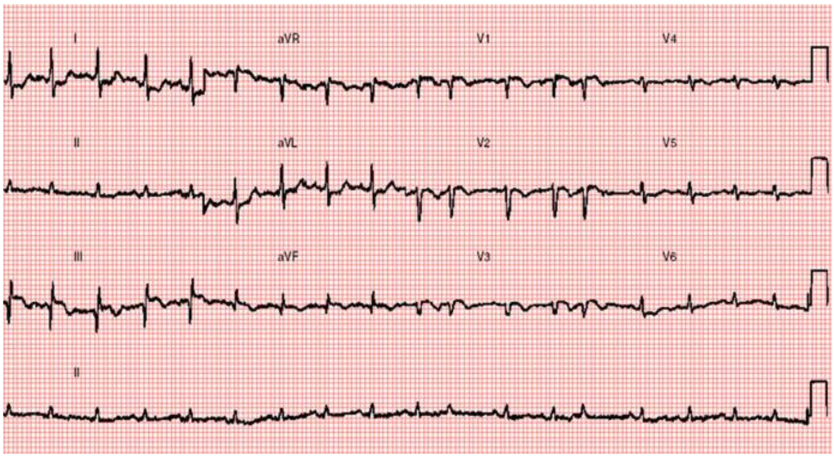


Figure I. 8: acute pulmonary embolism.

The transitional zone in the chest leads is located near lead V5. In this situation the right ventricular leads are V1–V4. The first left ventricular lead is V6. This ECG shows the ST segment elevation in leads III, aVF, and V1–V3 with negative T-waves in these leads (characteristic pattern of ischemia of the right ventricle). The eleventh and fourteenth cardiac cycles are premature supraventricular beats. The compensatory pause is incomplete. The RR interval including premature contractions is shorter than double the regular R-R interval of sinus rhythm (This figure was published in Polo´nski and Wasilewski (2004), p. 8, Fig.1.2.4 Copyright Elsevier Urban & Partner Sp. z.o.o, 2006, Wrocław. (4)

According to the ECG shown in Fig. 1.4.4 the right ventricular leads are V1–V4 and the left ventricular lead is V6. For this reason, the right ventricular leads are best defined as lying to the right of the transitional zone, while the left ventricular leads lie to the left of the transitional zone. In patients with cyanotic congenital heart defect with significant hypertrophy and enlargement of the right ventricle, most or even all of the precordial leads can be found overlying the right ventricle. In these situations the displacement of the transitional area is accompanied by a high voltage wave in V1 and V2, which is typical for right ventricular hypertrophy. (4)

I.6 The components of ECG signal: Waves, Segments, and Intervals

There are certain elements in the ECG waveform (Fig. I.9):

• The isoelectric line: a horizontal line when there is no electrical activity on ECG.

• Segments: the duration of the isoelectric line between waves.

• Intervals: the time between the same segments of adjacent waves.

The P-wave is the first deflection of the ECG. It results from depolarization of the atria.

Atrial repolarisation occurs during ventricular depolarization and is obscured. The QRS complex corresponds to the ventricular depolarization.

The T-wave represents ventricular repolarisation, i.e., restoration of the resting membrane potential. In about one-quarter of population, a U-wave can be seen after the T-wave. This usually has the same polarity as the preceding T-wave. It has been suggested that the U-wave is caused by after-potentials that are probably generated by mechanical–electric feedback. Inverted U-waves can appear in the presence of left ventricular hypertrophy or ischemia (6) (1).

The PQ segment corresponds to electrical impulses transmitted through the S-A node, bundle of His and its branches, and the Purkinje fibres and is usually isoelectric. The PQ interval expresses the time elapsed from atrial depolarization to the onset of ventricular depolarization. The ST-T interval coincides with the slow and rapid repolarisation of ventricular muscle. The QT interval corresponds to the duration of the ventricular action potential and repolarisation. Then TP interval is the period for which the atria and ventricles are in diastole. The RR interval represents one cardiac cycle and is used to calculate the heart rate. (6)

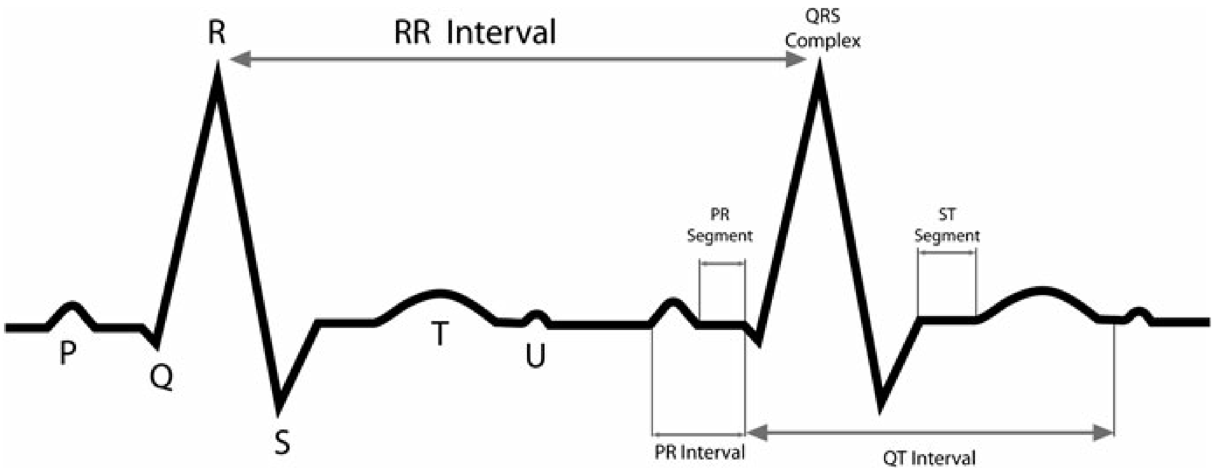


Figure I. 9 : The ECG waves, segments, and intervals/

Table I. 1: Overview of the fundamental parts building a cardiac cycle

P wave	Depolarization of both atria
PR interval	PR interval Time between the depolarization of the atria and the depolarization of the ventricles
QRS complex	QRS complex Depolarization of the ventricles
ST interval	ST interval Time between the end of the ventricle depolarization and the beginning of the ventricle repolarization
T wave	ventricular repolarization
QT interval	QT interval Measure of the total period of ventricular depolarization and repolarization

Feature Normal Value Normal Limit

P Width 110 ms ±20 ms

P Amplitude 0.15mV ±0.05mV

QRS Width 100 ms ±20 ms

QRS Height 1.5mV ±0.5mV

T Amplitude 0.3mV ±0.2mV

PR Interval 160 ms ±40 ms

QTc Interval 400 ms ±40 ms

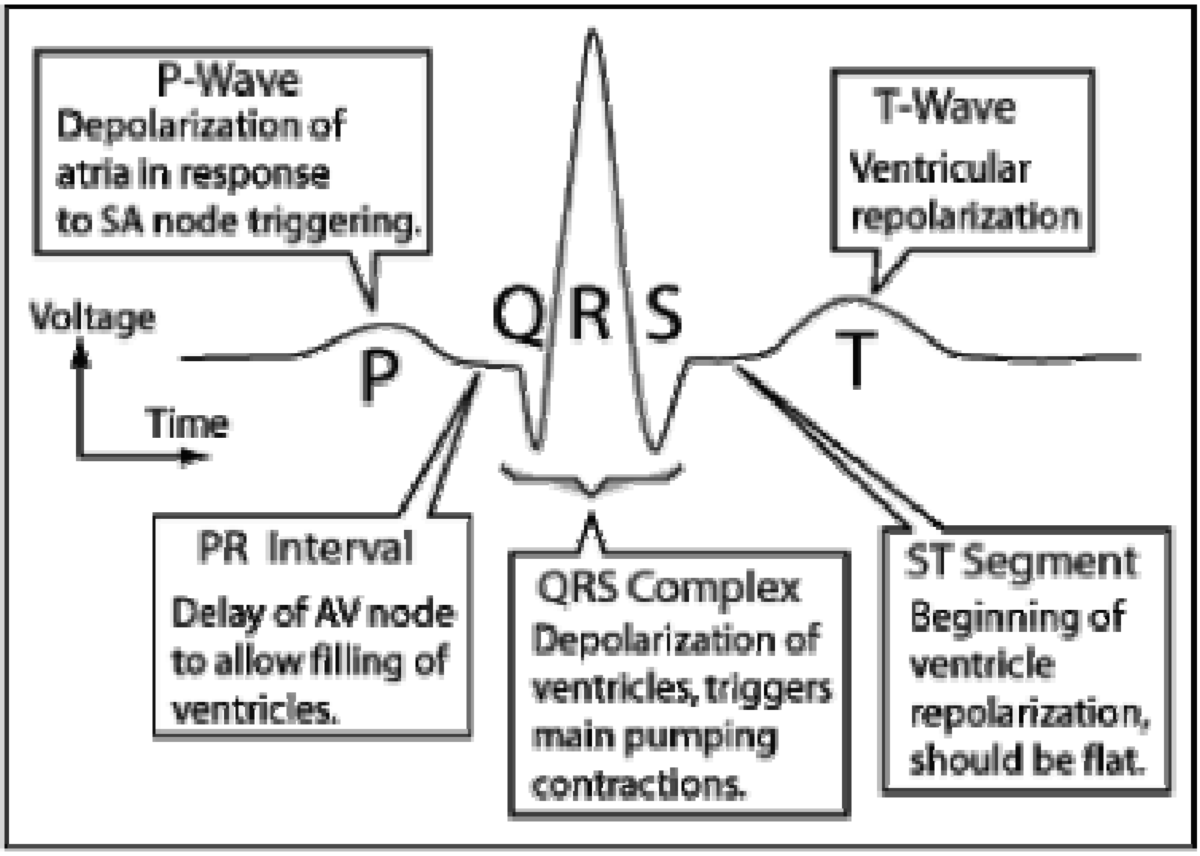


Figure I. 10: The normal ECG waveform

I.7 Conclusion:

In this chapter we had given a general conception about the heart functioning and the ECG signals which will be the axe of our study, starting from the anatomic structure of the heart, and also we explained its conductional system, with showing the normal ECG waveform and its construction (QRS complex, ST segment, P wave, T wave…etc.), of course also we gave how we can do the medical process to measure ECG cardiogram for treating (medical process conception).

Chapter II: wavelet theory and introduction to the lifting scheme

II.1 Introduction and overview

Most signals in the real world are not stationary, and that is precisely in the evolution of their characteristics (statistics, frequency, time, space) where most of the information they contain resides. The voice signals and the images are exemplary in this respect.

However, Fourier's analysis offers an approach of the signal, the integrations are made from minus infinity to plus infinity, and any notion of temporal localization (or spatial for images) disappears in space of Fourier; it is therefore necessary to find a compromise, a transformation that provides information on the frequency content while preserving localization in order to obtain a representation time/frequency or space/scale of the signal and This transformer is the wavelet transformer.

The first section of this chapter, consider generalization about the wavelet transform (Historical, wavelet family, types of wavelet and the most important thing' how we can choose the best wavelet'…).

The second section of this chapter, consider on the conception of lifting technic which is a way for generate wavelet and that's include the phases.

II.2Wavelets Transform:

Historically, the concept of "wavelets" started to appear more frequently only in the early 1980s. This new concept can be viewed as a synthesis of various ideas originating from different disciplines including mathematics (Calderón–Zygmund operators and Littlewood–Paley theory), physics (the coherent states formalism in quantum mechanics and the renormalization group), and engineering (quadratic mirror filters (QMF), sideband coding in signal processing, and pyramidal algorithms in image processing). In 1982, Jean Morlet, a French geophysical engineer, discovered the idea of the wavelet transform, providing a new mathematical tool for seismic wave analysis. In Morlet's analysis, signals consist of different features in time and frequency, but their high-frequency components would have a shorter time duration than their low-frequency components. In order to achieve good time resolution for the high-frequency transients and good frequency resolution for the low-frequency components, Morlet et al. (1982) first introduced the idea of wavelets as a family of functions constructed from translations and dilations of a single function called the "mother wavelet" $\psi(t)$. They are defined by: $\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right), \quad a, b \in \mathbb{R}, a \neq 0$

(II.1)

a: the scaling parameter (it measures the degree of compression).

b: the translation parameter (it determines the time location of the wavelet) (9).

II.2.1 Property of a mother wavelet :

- **Zero moments:** this is the most important property for a wavelet. a wavelet ψhas N zero moments if:

  $$M_k = \int_{-\infty}^{+\infty} t^k \psi(t) dt = 0 \qquad ,0 \leq k \leq N \qquad (II.2)$$

  According to the last equation M_K, any wavelet must have at least one zero moment (the case where$K = 0$).
- **Compact stand:** as long as the wavelet has less zero moments and its support is compact, a more exact high frequency analysis is possible.
- **Regularity:** The regularity of a wavelet is the property allowing to locate singularities in a signal. It can be noted that there is a link between regularity and null moments. As much as we have null moments as much the signal is regular.
- **Symmetry:** like the number of zero moments, the symmetry of the wavelet conditions its regularity over an interval.
- **Orthogonality:** The orthogonality of a wavelet is the property allowing eliminate information redundancy (10).

II.2.2 The wavelet family

There are several mother wavelets used for the calculation of the wavelet transform analysed signals. Each of them has a defined scope of application in the form of the studied signal. Table II.1 contains the most common families

Table II. 1: List of presented wavelets

Names of wavelets families	Abbreviation
Haar wavelet	haar
Daubechies wavelets	Db
Symlets	sym
Coiflets	coif
Biorthogonal wavelets	bior
Meyer wavelet	meyr
Discrete approximation of the Meyer wavelet	dmey

Battle and Lemarié wavelets	btlm
Gaussian wavelets	gaus
Mexican hat	mexh
Morlet wavelet	morl
Complex Gaussian wavelets	cgau
Complex Shannon wavelets	shan
Complex B-spline frequency wavelets	fbsp
Complex Morlet wavelets	cmor

Wavelet families can be characterized by four properties main existence of associated filters, orthogonality or biorthogonality, support compact or non-compact, real or complex wavelets. The table (Tab. 2.2) summarizes these various properties (11) (12).

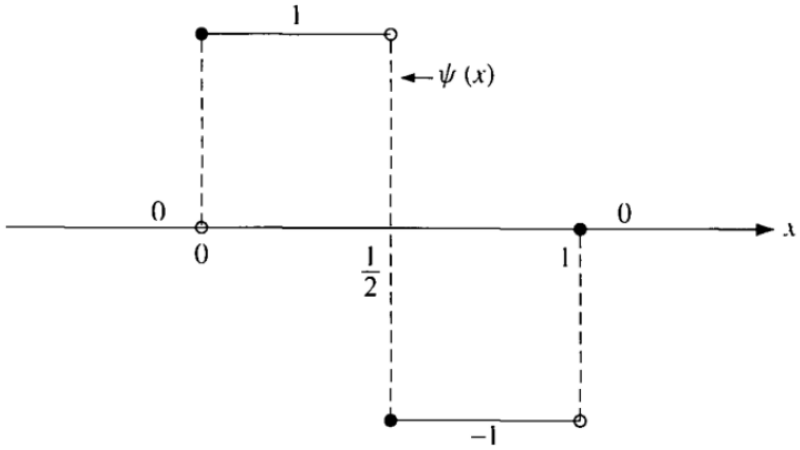
Table II. 2: Principal properties of wavelet families

Wavelets with filters			**Wavelets without filters**	
With compact support		***With non-compact support***	***Real***	***Complex***
Orthogonal	***Biorthogonal***	***Orthogonal***	*gaus, mexh, morl*	*cgau, shan, fbsp, cmor*
dB, haar, sym, coif	*bior*	*meyr, dmey, btlm*		

II.2.2.1 Haar wavelet

Haar functions are used since 1910. They were introduced by Hungarian mathematician Alfred Haar. It's the simplest wavelets, moreover it has compact support, which makes the calculation of the wavelet transform exact and well finished. Let $\psi: \mathcal{R} \longrightarrow \mathcal{R}$ the Haar wavelet function is defined by (13)

$$\psi(t) = \begin{cases} 1, \text{ for } t \in \left[0, \frac{1}{2}\right). \\ -1, \text{ for } t \in \left[\frac{1}{2}, 1\right). \\ 0, \quad \text{ for otherwise.} \end{cases} \tag{II.3}$$



Fgure II. 1 : Haar wavelet ψ(x)

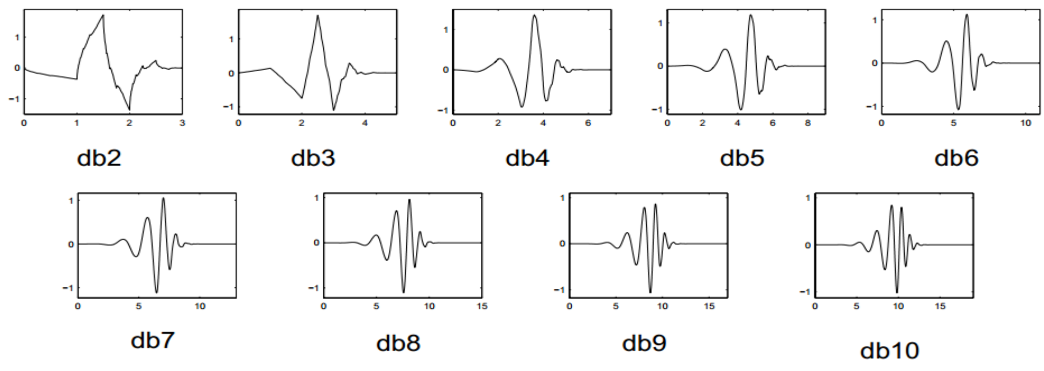
Unfortunately, the Haar Transform has poor energy compaction.

Properties : the basic functions are constructed according to the relation (14):

$$\psi_{j,n}(x) = 2^{\frac{j}{2}} \psi(2^j x - k) \qquad , k \in Z \qquad \text{(II.4)}$$

II.2.2.2 I.Daubechiers wavelet

Ingrid Daubechies, one of the brightest stars in the world of wavelets research, invented what are called compactly supported orthonormal wavelets — thus making discrete wavelet analysis practicable. The names of the Daubechies family wavelets are written dbN, where N is the order, and db the “surname” of the wavelet. The db1 wavelet, as mentioned above, is the same as Haar wavelet. Here are the wavelet functions psi of the next nine members of the family :



Fgure II. 2: Daubechies Wavelets : dbN

Except for db1, the wavelets of this family do not have an explicit expression. However, the square modulus of the transfer function of the associated filter h is explicit and relatively simple:

Let$P(y) = \sum_{K=0}^{N-1} C_{N-1+k}^{k} y^{k}$, where c_{N-1+k}^{k} are the binomial coefficients,

Then:$|m_0(\omega)|^2 = \left(\cos^2\left(\frac{\omega}{2}\right)\right)^N P\left(\sin^2\left(\frac{\omega}{2}\right)\right)$ where $m_0(\omega) = \frac{1}{\sqrt{2}} \sum_{K=0}^{2N-1} h_k e^{-ikw}$.

This family has the following properties:

− The ψ and φ support length is $2\ N - 1$. The number of zero moments of ψ is N.

− dbN wavelets are asymmetric (in particular for low values of N) except for the

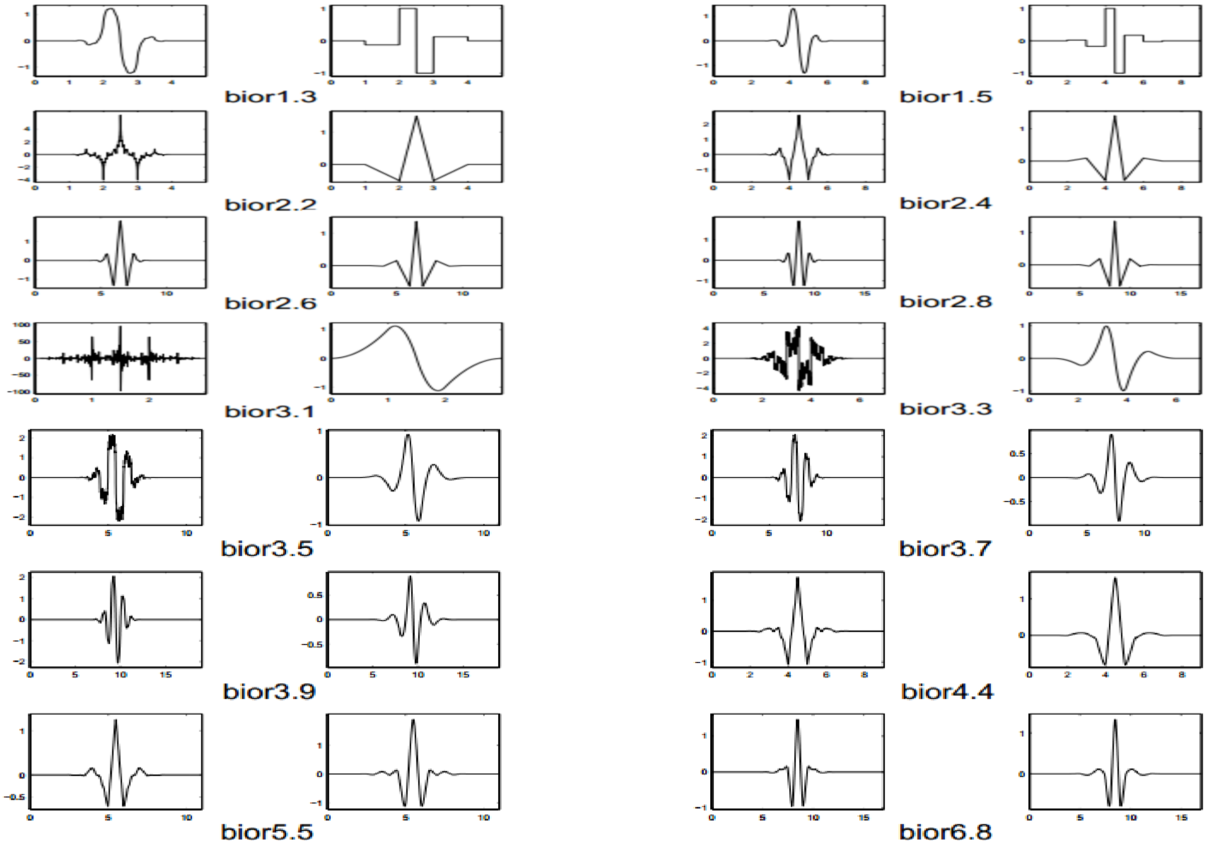
Haar wavelet.

− The regularity increases with order. When N becomes very large, ψ and ϕ belong to $C^{\mu N}$ where $\mu \approx 0.2$. This value μN is too pessimistic for relatively small orders, as it underestimates the regularity.

− The analysis is orthogonal. (15) (11)

II.2.2.3 Biorthogonal wavelet

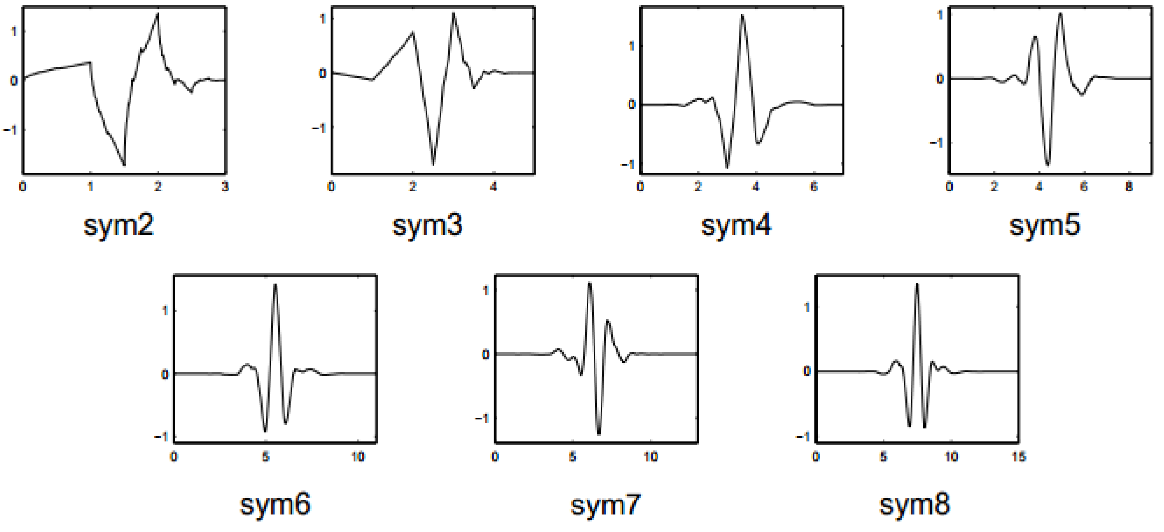
This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction. By using two wavelets, one for decomposition (on the left side) and the other for reconstruction (on the right side) instead of the same single one, interesting properties (15)



Fgure II. 3: Biorthogonal Wavelets Families

II.2.2.4 Symlet wavelet :

The symlets are nearly symmetrical wavelets proposed by Daubechies as modifications to the db family. The properties of the two wavelet families are similar. Here are the wavelet functions psi. (15)



Fgure II. 4: Symlet wavelets Families

II.2.3 Problem of the choice of the mother wavelet

Before carrying out a wavelet analysis, it is necessary to choose the analyzing function (The mother wavelet). The shape of the wavelet is important, but it is also important to choose its duration and bandwidth. These two parameters determine the resolutions of the transformed in the time and frequency domains. There are many families of wavelets which can be divided into two categories:

1) Filter wavelets which are associated with orthogonal multiresolution analyses (discrete wavelets) such as Daubechies wavelets (db1….db20), Symlet.

2) Unfiltered wavelets that are useful for the wavelet transform continue.

Which include the Gaussian wavelet, the Mexican hat, the Morlet wavelet, the complex Gaussian wavelet. These wavelets exhibit regularity properties infinite, of symmetry, of possible reconstruction and of explicit expression. The symmetry allows to ensure a better invariance by translation and thus provides a temporal localization More reliable.

In this study, we choosed the Haar wavelet because it's the simplest and filter wavelets Daubechies, symlet wavelets (12).

II.2.4 Types of wavelet transform

II.2.4.1 Continues wavelets transform :

The continuous wavelet transform is defined as :

$$X_{WT}(a,b) = \frac{1}{\sqrt{|a|}}\int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-b}{a}\right)\ dt\,. \qquad \text{(II.5)}$$

The transformed signal $X_{WT}(a,b)$is a function of the translation parameter b and the scale parameter a. The mother wavelet is denoted by ψ; the $*$ indicates that the complex conjugate is used in case of a complex wavelet. The signal energy is normalized at every scale by dividing the wavelet coefficients by$\frac{1}{\sqrt{|a|}}$. This ensures that the wavelets have the same energy at every scale. The mother wavelet is contracted and dilated by changing the scale parameter a. The variation in scale a changes not only the central frequency fc of the wavelet, but also the window length. Therefore the scale a is used instead of the frequency for representing the results of the wavelet analysis. The translation parameter b specifies the location of the wavelet in time, by changing b the wavelet can be shifted over the signal. For constant scale a and varying translation τ the rows of the time-scale plane are filled, varying the scale s and keeping the translation b constant fills the columns of the time-scale plane. The elements in $X_{WT}(a,b)$ are called wavelet coefficients; each wavelet coefficient is associated to a scale (frequency) and a point in the time domainSource spécifiée non valide..

The WT also has an inverse transformation, as was the case for the FT and the STFT. Let $\psi \in L_2(\mathbb{R})$ satisfyC_ψ,then for all $x \in (L_2 \cap L_\infty)(\mathbb{R})$, the inverse continuous wavelet transformation (ICWT) is defined by:

$$x(t) = \frac{1}{C_\psi}\int_0^\infty \int_{-\infty}^{+\infty} X_{WT}(a,b)\frac{1}{a^2}\psi\left(\frac{x-a}{b}\right)dbda \qquad (II.6)$$

Where $C_\psi$is the admissibility constant, and it is given by:

$$C_\psi = \int_0^\infty \frac{|\widehat{\psi(\omega)}|^2}{\omega}d\omega < +\infty \qquad (II.7)$$

The $\widehat{\psi}$ Is the Fourier transform ofψ (16) (16)

II.2.4.2 Discrete wavelet transform :

In the discrete case, we choose to restrict the values of the parameters a and b to a grid discreet. In this case we fix a dilation step $a_0 > 1$ and $b_0 > 0$ and by setting $a = a_0^m$ and $b = nb_0a_0^m$ withm and n $\in$ Z. The wavelet bases are defined by the functions :

$$\psi_{m,n}(x) = a_0^{-\frac{m}{2}}\psi(a_0^{-m}x - nb_0) \qquad (II.8)$$

As a result, the wavelet decomposition and the inverse transform are determined by the equations following:

$$W_f(m,n) = \langle f, \psi_{m,n}\rangle = \int_R f(x)\psi^*_{m,n}(x)dx \qquad (II.9)$$

$$f = \sum_{(m,n)\in Z^2} W_f(m,n)\psi_{m,n} \text{ cover in } L\text{^}2(R) \qquad (II.10)$$

Thus, the wavelet transform associates to the function $f \in L^2(R)$ a discrete set of Coefficient $W_f(m, n)$.

Yves Meyer constructed a dyadic wavelet transform for which $a_0 = 2$ and $b_0 = 1$, a shown that for particular functions y of $L^2(R)$, the family:

$$\psi_{m,n}(x) = 2^{-\frac{m}{2}}\psi(2^{-m}x - n) \qquad \forall(m,n) \in Z^2 \tag{II.11}$$

$\psi_{m,n}$ Constitutes an orthonormal basis of $L^2(R)$. (17)

II.2.5 First generation wavelet properties:

• P1: The wavelets form a Riesz basis of $L^2(R)$.

• P2: The wavelets are either orthogonal or their duals are known (in the case where they are biorthogonales).

• P3: The wavelets and their duals are localized in time and frequency. Frequency localization comes from their regularity and their moment's null polynomials.

• P4: The fast wavelet transformation, which allows obtaining the wavelet coefficients in one time linear. (18)

II.2.6 Multiresolution Analysis

A multiresolution analysis approximates a function $f(t)$ at various resolutions by orthogonal projection on a family of spaces $\{V\,j\}_{j\in Z}$.

The approximation of $f(t)$ at the 2^{-j} resolution is defined as its orthogonal projection on a space $V_j \subset L^2(\mathbb{R})$.

The multiresolution analysis is constructed using nested Vj subspaces in the others, such that the transition from one to the other is the result of a change of scale.

$$f(t) \in V_j \Leftrightarrow f\left(\frac{t}{2}\right) \in V_j \tag{II.12}$$

Generally: $\{0\} \subset \ldots \subset V_j \subset \ldots V_0 \subset \ldots V_{-j} \subset \ldots \subset L^2(\mathbb{R})$

With the following properties:

- $$\lim_{j\to-\infty} V_j = \bigcup_{j=-\infty}^{\infty} V_j = L^2(\mathbb{R})$$
- $$\lim_{j\to\infty} V_j = \bigcap_{j=-\infty}^{\infty} V_j = \{0\}$$
- $$f(t) \in V_0 \Leftrightarrow f(\frac{t}{2^j}) \in V_j$$

(II.13)

Also, there must exist a function $\phi(t)$ of $L^2(\mathbb{R})$ called a scaling function which by dilation and translation generates an orthonormal basis in Vj . This function will be noted

- $$\left\{\phi_{j,k}(t) = \frac{1}{\sqrt{2^j}}\phi\left(\frac{t-2^j k}{2^j}\right) \mid k = \ldots,-2,-1,0,1,2\ldots\right\}$$

(II.14)

In other terms, each element f (t) belonging to V_j (j is fixed) can be written as the following form: $f(t) = \sum_{k=-\infty}^{+\infty} C_{j,k}\phi_{j,k}(t)$ (II.15)

With: $C_{j,k} = < f(t), \phi_{j,k}(t) >$ (II.16)

The function $\phi(t)$ is called the scaling function.

Since $\phi(t) \in V_0 \subset V_{-1}, \phi(t)$ can be expressed in the subspace V_{-1} by:

$\phi(t) = \sqrt{2}\sum_{k=\_\infty}^{+\infty} h_k\phi(2t-k)$ (II.17)

With h_k given by: $h_k = < \phi(t), \sqrt{2}\phi(2t-k) >$ (II 18)

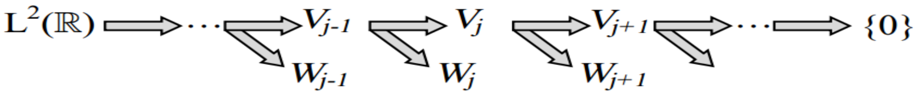
The sequence $H = \{h\_k \mid k = \cdots, -2, -1, 0, 1, 2, \ldots\}$ can be interpreted as a discrete filter, called ladder filter.

The purpose of the multiresolution decomposition of a signal is to construct a function wavelet to describe details in a signal. We are then interested in the error of approximation between the approximations of f(t) at scales j and $j-1$ which are represented by the projection of f(t) into V_j and V_{j-1}.

Like V_j is included inV_{j-1}, then there is an orthogonal complement of V_j in V_{j-1}such as:

$V_{j-1} = V_J \oplus W_J$ (II.19)

The previous writing means that each function in the subspace V_{j-1} can be expressed as the sum of two functions, one belonging to the subspace V_j and the other to the subspace W_j, which leads to the following sequence of nested spaces:



Fgure II. 5: Nested subspaces of a multiresolution analysis

Since V_j represents a rough approximation ofV_{j-1}, then Wj represents the details ofV_{j-1}. V_j is called approximation space and W_j is called detail space. The subspace W_j can be generated by the orthonormal basis formed by the translations of a function called wavelet, this function is noted:$\psi(t) \in L^2(\mathbb{R})$ The basic functions are built according to the relation:

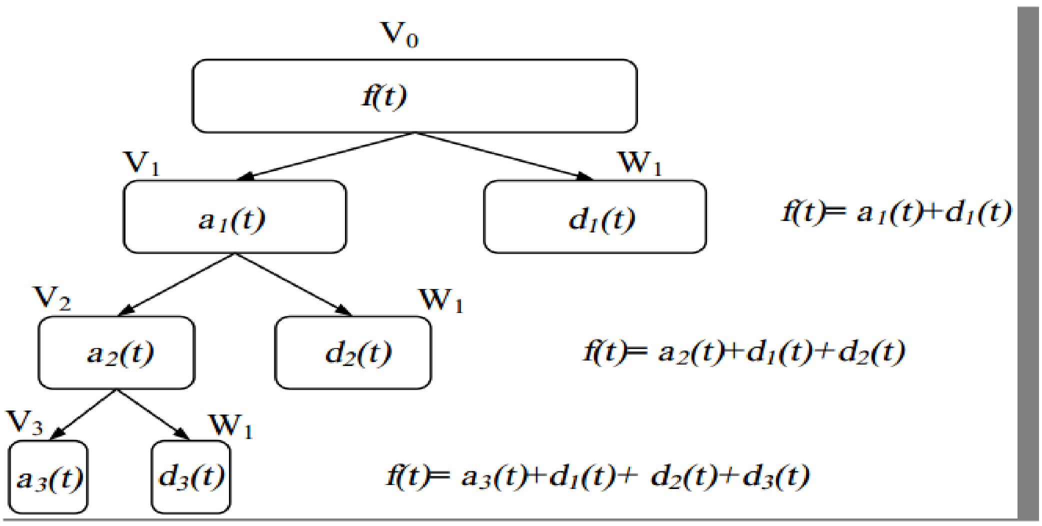
$$\psi_{j,k}(t) = \left\{\frac{1}{\sqrt{2}}\psi\left(\frac{t-2^j k}{2^j}\right) \backslash k = \cdots, -2, -1; 0,1,2, \ldots\right\} \quad \text{(II.20)}$$

With: $g_k = (-1)^{1-k} h_{1-k}$ (II.21)

Moreover all the translated and dilated versions of $\psi(t)$: $\{\psi_{j;k}(t) |\, j,k = \cdots, -2, -1, 0, 1, 2, \ldots\}$ form an orthonormal basis for $L^2(\mathbb{R})$; i.e. for any$f(t) \in L^2(\mathbb{R})$:

$$f(t) = \sum_{j=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} d_{j,k}\, \psi_{j,k}(t) \quad \text{(II.21)}$$

With: $d_{j,k=} < f(t), \psi_{j,k}(t) >$ called the wavelet coefficients. (19)



Fgure II. 6: Diagram of the multiresolution analysis of a function f (t).

II.2.4.7 Fast Wavelet Transform :

In practice, the signals we face are discrete, so it is necessary to have a discrete version of the wavelet transform. This version can easily be deduced from the scale equations. Consider the scale function $\phi(t)$. For a discrete signal$C = \{c_k |\, k = \cdots, -2, -1, 0, 1, 2, \ldots\}$, on associate C with a function f (t) inV_0:$f(t) = \sum_{k=-\infty}^{+\infty} c_k \phi(t-k)$

Mallat developed an algorithm called fast wavelet transform to express the signal f(t) in terms of the wavelet function ψ(t). The algorithm is defined as following:

$$c_{j,k} = \sum_{m=-\infty}^{+\infty} h_{m-2k} c_{j-1,m} \quad \text{(II.22)}$$

$$d_{j,k} = \sum_{m=-\infty}^{+\infty} g_{m-2k} c_{j-1,m} \quad \text{(II.23)}$$

According to the two previous equations:

$$\mathbf{C_j} = \{c_{j,k} \backslash k = \cdots, -2, -1, 0, 1, 2, \ldots\} \quad \text{(II.24)}$$

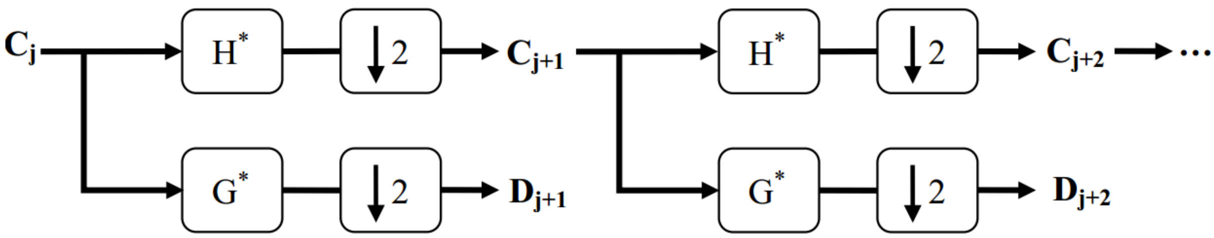
$$\mathbf{D_j} = \{d_{j,k} \backslash k = \cdots, -2, -1, 0, 1, 2, \ldots\} \quad \text{(II.25)}$$

Are respectively the result of the convolution of C_{j-1} with the filters $H^* = \{h_{-k} | k = \cdots, -2, -1, 0, 1, 2, \ldots\}$and $G^* = \{g_{-k} | k = \cdots, -2, -1, 0, 1, 2, \ldots\}$ followed by down-sampling of factor 2. We can write the two previous equations in a more compact way :

$$C_j = H * C_{j-1} \quad \text{(II.26)}$$

$$D_j = G * C_{j-1} \quad \text{(II.27)}$$

The decomposition process starts from $C_0 = C$ up to J levels of decomposition. After J levels of decomposition, the discrete signal C is transformed into a sequence of newly generated signals: $\{\mathbf{C_j};\ \mathbf{D_j}; \mathbf{D_{j-1}}; \ldots;\ \mathbf{D_1}\}$. (19)



Fgure II. 7: Discrete signal decomposition algorithm.

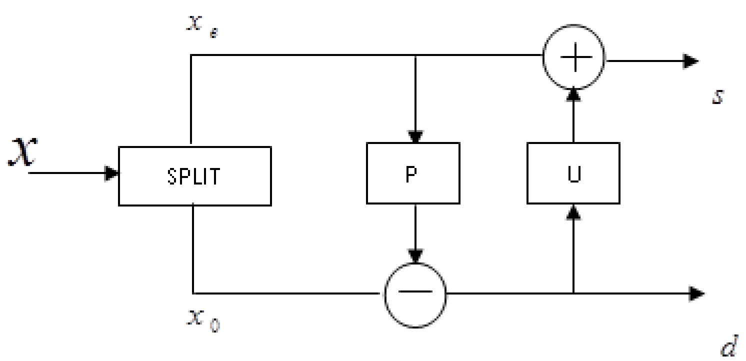
II.3 Lifting scheme

One of the limitations of the discrete wavelet transform (DWT) when applied to images is its reduced ability to decorrelate coefficients along contours. As correlation persists, large magnitude coefficients representing contours remain spatially collocated.

As a result, current research efforts are focusing on the design of schemes able to decorrelate these coefficients or to minimise their energy, and for that reason we are going to develop our study with the conception of 'lifting scheme', which represent an effective tool for the construction and implementation of wavelet transform, appears in the «second generation» wavelets, so we will describe the lifting as a general and flexible technic for designing wavelets and performing the discrete wavelet transform (DWT). In an implementation, it is often worthwhile to merge these steps and design the wavelet filters while performing the wavelet transform. This is then called the second wavelet generation. The technique was introduced by Wim Swelden. The lifting scheme factorizes any discrete wavelet transform with finite filters into a series of elementary convolution operators, so-called lifting steps, which reduces the number of arithmetic operations by nearly a factor two. Treatment of signal boundaries is also simplified. The discrete wavelet transform applies several filters separately to the same signal. In contrast to that, for the lifting scheme, the signal is divided like a zipper. Then a series of convolution _accumulate operations across the divided signals is applied.

II.3.1 The structure:

The structure of a simple lifting scheme shown in this figure (I.8) and it constructs from 3 operations which they are explained below: (20) (21) (22)



Fgure II. 8 : The simple lifting scheme

II.3.1.1 Polyphase transform

Its mechanism is simple. It takes an input signal and separates it into two polyphase components, even and odd. More precisely, the application of the polyphase transform to the original signal will partition it into two disjunct subsets: (all samples belonging to x of even index) and (all samples belonging to x of odd index). This separation is also called "Lazy Wavelet Transform"; Polyphase transform: $x \to (x_e, x_0)$

II.3.1.2 Prediction Operation

Most imaging signals have a local correlation structure in both the spatial and frequency domains. In other words, samples where the frequencies of the same neighbourhood will be strongly correlated while samples where frequencies not belonging to the same neighbourhood will have only a low correlation rate . If the signal χ has such a structure, then both sub-signals x_e and x_0 will be highly correlated. This results in the possibility of predicting x_0 from x_e. An operator P is applied to the subset x_e in order to predict x_0. The difference d between x_0 and its prediction constitutes the detail of the signal, in other words the wavelet coefficients.

$$d = x_0 - P(x_e) \qquad \text{(II.28)}$$

The operation of calculating the prediction and memorizing the detail constitutes a not dual lifting.

Thus, the second step of the lifting diagram brings us back to:

Step dual lifting : $(x_e, x_0) \rightarrow (x_e, d)$

II.3.1.3 UPDATE OPERATION:

THIS LAST STEP IS NECESSARY TO ACQUIRE A MORE ADEQUATE LOW-FREQUENCY REPRESENTATION OF THE SIGNAL, AND MAINTAIN SOME OF THE OVERALL PROPERTIES OF THE ORIGINAL SIGNAL. WE HAVE TWO SUBSETS: AND D. THE SET D OBTAINED BY AN OPERATION SIMILAR TO A FILTERING PASSES HIGH PLUS A SUBSAMPLING. THE SUBSET x_e IS

OBTAINED BY A SIMPLE SUBSAMPLING; THIS SIGNAL IS SPREAD OVER THE ENTIRE FREQUENCY BAND OF THE ORIGINAL SIGNAL. THE SHANNON CONDITIONS ARE NOT MET, HENCE THE NEED FOR A THIRD STEP IN ORDER TO ACQUIRE AN ADEQUATE REPRESENTATION OF THE LOW FREQUENCY SIGNAL: THE APPLICATION OF AN UPDATE OPERATOR ON THE DETAIL SIGNAL.

$$s = x_e + U(d) \qquad \text{(II.29)}$$

THIS SECOND STEP IS CALLED A LIFTING STEP.

LIFTING STEP: $(x_e, d) \rightarrow (s, d)$ (II.30)

II.3.2 Proprieties of lifting scheme:

- **Fast calculation:**
  The lifting allows you to perform the calculations on site. In other words, the output signal is encoded on the same memory location as the input signal. This method allows a significant saving in memory.
- **Efficiency:**
  In most cases the number of operations is reduced compared to independent filtering of the two sub-bands. Indeed, the facelift simultaneously calculates high frequency low bands.

- **Reversibility:**

  The realization of the reverse lifting scheme is trivial. It is obtained by reversing the order of operations and undoing them.

- **Generality:**

  The transform is implemented without any reference to the Fourier domain. This makes it extremely easy to extend the scheme to other application frameworks.

- **Increasing vanishing moments, stability, and regularity:**

  A lifting modifies biorthogonal filters in order to increase the number of vanishing moments of the resulting biorthogonal wavelets, and hopefully their stability and regularity. Increasing the number of vanishing moments decreases the amplitude of wavelet coefficients in regions where the signal is regular, which produces a more sparse representation. However, increasing the number of vanishing moments with a lifting also increases the wavelet support, which is an adverse effect that increases the number of large coefficients produced by isolated singularities. Each lifting step maintains the filter biorthogonality but provides no control on the Riesz bounds and thus on the stability of the resulting wavelet biorthogonal basis. When a basis is orthogonal then the dual basis is equal to the original basis. Having a dual basis that is similar to the original basis is, therefore, an indication of stability. As a result, stability is generally improved when dual wavelets have as much vanishing moments as original wavelets and a support of similar size. This is why a lifting procedure also increases the number of vanishing moments of dual wavelets. It can also improve the regularity of the dual wavelet. A lifting design is computed by adjusting the number of vanishing moments. The stability and regularity of the resulting biorthogonal wavelets are measured a posteriori, hoping for the best. This is the main weakness of this wavelet design procedure.

- **Non-linearites:**

  The convolution operations can be replaced by any other operation. For perfect reconstruction only the invertibility of the addition operation is relevant. This way rounding errors in convolution can be tolerated and bit-exact reconstruction is possible. However, the numeric stability may be reduced by the non-linearities. This must be respected if the transformed signal is processed like in lossy compression. Although every reconstructable filter bank can be expressed in terms of lifting steps, a general description of the lifting steps is not obvious from a description of a wavelet family. (22) (23)

II.3.3 Advantages of lifting

1. The lifting allows a transformation "in place", making implementation easier for fast wavelets. That said no need to allocate an auxiliary storage space.
2. The use of the lifting makes it particularly possible to nonlinear wavelets: for example, one can make the interentiers transforms. This is important for hardware implementations and image encoding.
3. Any transformation done with the lifting is immediately reversible and has exactly the same cost in complexity, as the she herself.
4. The lifting allows the transform to be adapted into a wavelet.
5. The lifting allows the construction of the wavelets without making use of the transform Fourier. This means that it can be used to build the wavelets which are not necessarily translated or dilated by a function (on talk about second generation wavelets).
6. The lifting is more flexible for non-purist mathematicians, as does not appeal to the notions of the Fourier transform, and can be in this way easily introduced, only by its arguments in the space field.
7. Finally, the lifting exposes the inherent parallelism of the transform into wavelets. All The operations of a lifting step can be done completely in parallel. (22) (23)

II.3.4 Applications of lifting:

The lifting has a large area of uses and applications like:

- Wavelet transforms that map integers to integers
- Fourier transform with bit-exact reconstruction
- Construction of wavelets with a required number of smoothness factors and vanishing moments
- Construction of wavelets matched to a given pattern
- Implementation of the discrete wavelet transform, image JPEG 2000
- Data-driven transforms, e.g., edge-avoiding wavelets

- Wavelet transforms on non-separable lattices, e.g., red-black wavelets on the quincunx lattice. (22) (23)

II.4 NONLINEAR WAVELET: (20)

One of the main reasons for creating these second-generation wavelets is, as we have already pointed out, to be able to adapt to particular constraints, in terms of starting space, or finishing space - this is the case of whole wavelets or wavelets over an interval - either to obtain fundamentally non-linear wavelets, in the very design of the update and prediction operators. (21)

II.4.1 Whole wavelets:

One of the difficulties of wavelets is the difficulty of creating wavelets that can retain the entire nature of the data, especially for lossless compression.

One of the advantages of lifting is that there is an entire version of the diagram. The Integer version uses a rounding operator to project an actual variable over a set of samples defined in the domain of relative integers $\mathbf{Z}$. The lifting scheme on the integers is defined by the set of equations :

$$\begin{cases} s^{(l)} = s^{(l-1)} - \left[U^l * d^{(l-1)} + \dfrac{1}{2} \right] \\ d^{(l)} = d^{(l-1)} - \left[P^l * s^l + \dfrac{1}{2} \right] \end{cases} \qquad \text{(II.31)}$$

For l from 1 to L .

With

L : indicates the number of no lifts.

One of the main interests of this operation is that the scheme remains reversible.

Therefore, the whole facelift scheme is a perfect reconstruction transform, which processes integers and returns integers. This type of transform offers a notable interest within a lossless compression chain, and filtering by lifting on integers is non-linear, even if the filters $P\ et\ U$ are.

The lifting is not the only transform used to develop an inverted transformation defined on the integers. (13) (21) (24)

II.4.2 Correspondence between lifting scheme and transformed into first generation wavelets:

Daubechies et Sweldens ont établi la correspondance entre la transformée en ondelettes et le schéma lifting.

The first step is the application of the polyphase transform of the two bands to the two synthesis filters and .The polyphase representation of $\tilde{h}$ is given by :

$$\tilde{h}(z) = \tilde{h}_e(z^2) + z^{-1}\tilde{h}_0(z^2)$$

With:

$$\tilde{h}_e(z) = \sum_{k=k1}^{k2} \tilde{h}_{2k}(z).z^{-k} \quad \text{And} \qquad \tilde{h}_0(z) = \sum_{k=l1}^{l2} \tilde{h}_{2k+1}(z).z^{-k}$$

k1, k2, l1, l2 being the maximum and minimum degrees of $\tilde{h}_e$ and $\tilde{h}_0$ given by $\tilde{h}$.

The same applies to $\tilde{g}$ we have :

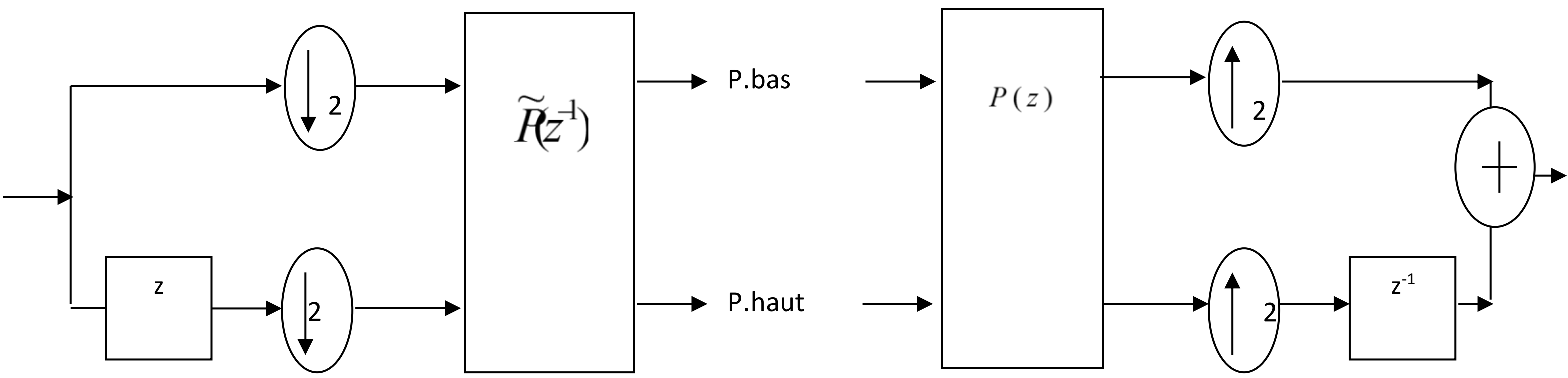
$$\tilde{g}(z) = \tilde{g}_e(z^2) + z^{-1}\tilde{g}_0(z^2)$$

With : $\tilde{g}_e(z) = \sum_{k=k1}^{k2} \tilde{g}_{2k}(z).z^{-k}$ And $\tilde{g}_0(z) = \sum_{k=l1}^{l2} \tilde{g}_{2k+1}(z).z^{-k}$

The polyphase representations of $\tilde{h}$ and $\tilde{g}$ allow us to write the polyphase matrix $\tilde{p}$ as :

$$\tilde{P}(z) = \begin{bmatrix} \tilde{h}_e\,(z) & g_e(z) \\ \tilde{h}_0\,(z) & \tilde{g}_0(z) \end{bmatrix}$$

The polyphase representation (Figure 1.5) of the wavelet transform is obtained by subsampling into even and odd index elements, then by applying the polyphase matrix.



For the reconstruction, we apply the polyphase matrix $\tilde{p}$ then, we gather the elements of even and odd indices. The perfect reconstruction property then becomes:

$$p(z)\tilde{p}^T(z^{-1}) = I \qquad \text{(II.32)}$$

Either

$$h_e(z) = \tilde{g}_0(z^{-1}) \qquad h_0(z) = -\tilde{g}_e(z^{-1}) \qquad \text{(II.33)}$$

$$g_e(z) = -\tilde{h}_0(z^{-1}) \qquad g_0(z) = \tilde{h}_e(z^{-1}) \qquad \text{(II.34)}$$

The factoring of the polyphase matrix $\tilde{p}^T$ leads to a lifting scheme implementation of filters h and g . z transforms into polyphase filters are Laurent polynomials and do not belong to an Euclidean space. The polyphase matrix is factored by the application on $\tilde{h}_e$ and $\tilde{h}_0$ a «Euclid» algorithm extended to the Laurent polynomials.

$$\begin{pmatrix} \tilde{h}_e(z) \\ \tilde{h}_0(z) \end{pmatrix} = \prod_{i=0}^{M-1} \begin{pmatrix} q_i(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} K \\ 0 \end{pmatrix} = \prod_{i=1}^{L-1} \begin{pmatrix} 1 & q_{2i-1}(z) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ q_{2i}(z) & 1 \end{pmatrix} \begin{pmatrix} K \\ 0 \end{pmatrix} \qquad \text{(II.35)}$$

The Polynomials q_i , for all $i \in \{1,......., M-1\}$, are the successive quotients of the division $(M-1)$, the number of divisions obtained, $L-1 = \left[\frac{M-1}{2}\right]$ *et* K and the last remains non-zero. Based on the decomposition of h , The additional filter g^0 checking : $\tilde{p}^0(z) = \prod_{i=1}^{L-1} \begin{pmatrix} 1 & q_{2i-1}(z) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ q_{2i}(z) & 1 \end{pmatrix} \times \begin{pmatrix} K & 0 \\ 0 & 1/K \end{pmatrix}$

So a filter exists which verified :

$$\tilde{p}(z) = \tilde{p}^0(z) \times \begin{pmatrix} 1 & s(z) \\ 0 & 1 \end{pmatrix} \qquad \text{(II.36)}$$

The identification of the terms of equality (1.26) makes it possible to find the function s, which corresponds to an additional step facelift. The upper triangular matrices represent the "lifting" steps, while the lower triangular matrices represent the dual "lifting" steps.

Let's put : (13) (21) (24)

$$u^{(i)}(z) = q_{2i-1}(z) \quad p^{(i)}(z) = q_{2i}(z) \quad \forall i \in \{1,.......,L-1\} \tag{II.37}$$

$$u^{(L)}(z) = K^2 s(z) \quad p^{(L)}(z) = 0 \tag{II.38}$$

The reverse lifting pattern is deduced from the equation (1.27) and is determined by:

$$\tilde{p}(z) = \prod_{i=1}^{L} \begin{pmatrix} 1 & u^{(i)}(z) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ p^{(i)}(z) & 1 \end{pmatrix} \times \begin{pmatrix} K & 0 \\ 0 & 1/K \end{pmatrix} \tag{II.39}$$

The lifting scheme is given by transposing the matrix of :

$$p^T(z^{-1}) = \begin{pmatrix} 1/K & 0 \\ 0 & K \end{pmatrix} \times \prod_{i=0}^{L-1} \begin{pmatrix} 1 & 0 \\ -p^{(L-i)}(z) & 1 \end{pmatrix} \begin{pmatrix} 1 & -u^{(L-1)}(z) \\ 0 & 1 \end{pmatrix} \tag{II.40}$$

II.5 Second wavelet generation:

(24) (21)

II.5.1 Overview:

First generation wavelets are a powerful tool and it has are many areas of application. Nevertheless, they are formulated in the mould of a multiresolution analysis, well adapted in a theoretical framework of functional analysis (unconditional bases of functional spaces, characterization of functional spaces based on detail coefficients, etc.) but too restrictive in some practical applications. These limitations are mainly due to :

- Translation invariance (uniformity), which makes it difficult or impossible to create wavelets on domains other than the real line, such as the sphere, or even simply an interval of;

- The scale change invariance, which is the use of the same filters at all levels (stationary).

The second-generation wavelet transform (SGWT) is a wavelet transform where the filters (or even the represented wavelets) are not designed explicitly, but the transform consists of the application of the lifting scheme. Actually, the sequence of lifting steps could be converted to a regular discrete wavelet transform, but this is unnecessary because both design and application is made via the lifting scheme. This means that they are not designed in the frequency domain, as they are usually in the *classical* (so to speak *first generation*) transforms such as the DWT and CWT**).** The concept of second generation wavelets has been introduced to overcome these problems and extend the field of application of multi-resolution techniques. There is no precise definition of what second-generation wavelets are, but the fundamental characteristic is to no longer assume that the scale functions are derived from a single parent function.

The idea of moving away from the Fourier domain was introduced independently by David Donoho and Harten in the early 1990s.

II.5.2 Advantages of second generation of wavelet:

The SGWT has a number of advantages over the classical wavelet transform in that it is quicker to compute (by a factor of 2) and it can be used to generate a multiresolution analysis that does not fit a uniform grid. Using a priori information the grid can be designed to allow the best analysis of the signal to be made. The transform can be modified locally while preserving invertibility; it can even adapt to some extent to the transformed signal.

II.5.3 Applications of second generation wavelet:

There are many applications using second-generation wavelets, such as compression, denoising, and mathematical analysis. The first recognized application of second-generation wavelets (geometric type), is in the construction and progressive transmission of large virtual scenes. Thus, the Wavier application allows the visualization of virtual terrains, by scalable transmission mode.

Gortler et al. examined the effect of light reflection on virtual objects. This phenomenon is called radiosity (direct translation of the English term "Radiosity"). Otherwise known as illumination, the goal is to obtain a diffuse reflection of light and an icy effect that brings realism to the stage for virtual objects. Conventional light transport methods approximated the integral equation of radiosity, projecting the unknown function into a base composed of a set of n linear equations and limited support. This operation generates n linear units where n is the number of discrete elements of the scene. These methods generally require the resolution of n2 interaction coefficients. In order to reduce this number of coefficients, efforts have led to increase the order of the functions of the analysis base, to use the meshes as well as a hierarchical model. Finally, the number of interaction coefficients to be calculated becomes an equivalent in O(n). The authors propose to project this radiosity function into a base of wavelet functions, and also introduce the base of "flatlet" wavelets.

II.6 Conclusions:

The transformation into wavelets is a tool which is capable of giving a multi-resolution representation of a monodimensional (1D) signal; it has been a major success in different fields of application such as image and signal processing, data compression and transmission, numerical solutions of differential and integral equations, etc. The reasons for the successful transformation into wavelets are that for a class of functions, most of the information contained in the function is concentrated in a small number of wavelet coefficients.

In the case of multidimensional signals such as images, it is possible to build bases of wavelets separable by tensorial product. However, the latter have an indeformable square support and are isotropic: for these reasons, they cannot optimally represent the regions of an image with local contours or singularities. To compensate for this disadvantage, many bases of anisotropic wavelets (Curvelets, Contourlets, Bandelet, wavelets orientates...) were then proposed to allow a more economical representation of the images.

At the same time, the discovery of the lifting structure made it possible to simply construct multiresolution transforms, always inversible and allowing the implementation of non-linear operators capable of capturing the singularities of a signal. In addition, the lifting scheme makes it possible to implement the transform in wavelets more efficiently, by reducing the number of arithmetic operations like the convolution. The lifting scheme is used to implement direct and reverse transforms without going through the Fourier domain.

Chapter III: analysis ECG signals by wavelet

III.1 Introduction

In this chapter we will make processing to the ECG signals for selected pathology cases from a preserved database of diagnostics taken from **physionet database**, our study in this part considered on discrete wavelet transform (multiresolutional analysis), which is based on ECG decomposition into levels to address the best way for denoising the ECG signals and also determine the best wavelet to analyse and process this kind of signals as a tool for filtering, our work will be on **MATLAB** interface using **WAVELET TOOLBOX**, this chapter will be devise into 2 parts, the first will be theoretical takes ECG cases in explaining but no far as medical analysis, the second will be practical with a MATLAB program which decompose ECG by wavelets(symlet, haar, daubauchies, Mexican) , finished by results about what will be the best choice (wavelet) to analysis and denoising ECG signals.

III.2 Theoretical part:

III.2.1 The first case (myocardial infraction)

III.2.1.1 Definition and mechanism

A myocardial infarction (commonly called a heart attack) is an extremely dangerous condition caused by a lack of blood flow to your heart muscle. The lack of blood flow can occur because of many different factors but is usually related to a blockage in one or more of your heart's arteries. Without blood flow, the affected heart muscle will begin to die. If blood flow isn't restored quickly, a heart attack can cause permanent heart damage and death. When a heart attack happens, blood flow to a part of your heart stops or is far below normal, which causes that part of your heart muscle to die. When a part of your heart can't pump because it's dying from lack of blood flow, it can disrupt the pumping sequence for the entire heart. That reduces or even stops blood flow to the rest of your body, which can be deadly if it isn't corrected quickly. And this is figure shows how it is the heart attack. (25) (2) (26)

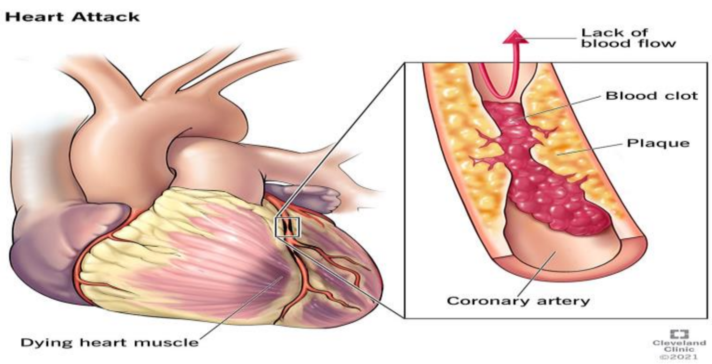


Figure III. 1: myocardial infraction patient's heart

III.2.1.2 The causes and Symptoms

The vast majority of heart attacks occur because of a blockage in one of the blood vessels that supply your heart. This most often happens because of plaque, a sticky substance that can build up on the insides of your arteries (similar to how pouring grease down your kitchen sink can clog your home plumbing). That build-up is called atherosclerosis.

Sometimes, plaque deposits inside the coronary (heart) arteries can break open or rupture and a blood clot can get stuck where the rupture happened. If the clot blocks the artery, this can deprive the heart muscle of blood and cause a heart attack.

Heart attacks are possible without a blockage, but this is rare and only accounts for about 5% of all heart attacks. This kind of heart attack can occur for the following reasons : (27)

- **Spasm of the artery**: Your blood vessels have a muscle lining that allows them to become wider or narrower as needed. Those muscles can sometimes twitch or spasm, cutting off blood flow to heart muscle.
- **Rare medical conditions**: An example of this would be any disease that causes unusual narrowing of blood vessels.
- **Trauma:** This includes tears or ruptures in the coronary arteries.
- **Obstruction that came from elsewhere in the body**: A blood clot or air bubble (embolism) that gets trapped in a coronary artery.
- **Electrolyte imbalances**: Having too much or too little of key minerals like potassium in your blood can cause a heart attack.
- **Eating disorders:** Over time, an eating disorder can cause damage to your heart and ultimately result in a heart attack. (27) (26) (28)

And the Symptoms most often described by people having a heart attack:

- Chest pain (angina). This symptom can be mild and feel like discomfort or heaviness, or it can be severe and feel like crushing pain. It may start in your chest and spread (or radiate) to other areas like your left arm (or both arms), shoulder, neck, jaw, back or down toward your waist.
- Shortness of breath or trouble breathing.
- Nausea or stomach discomfort. Heart attacks can often be mistaken for indigestion.
- Heart palpitations.
- Anxiety or a feeling of “impending doom.”

- Sweating.
- Feeling lightheaded, dizzy or passing out.

III.2.2 The second case (Dysrhythmia)

III.2.2.1 definition and mechanism

Cardiac dysrhythmia (or arrhythmia) is a disturbance in the rate of cardiac muscle contractions, or any variation from the normal rhythm or rate of heart beat. The term encompasses abnormal regular and irregular rhythms as well as loss of rhythm.

Cardiac dysrhythmias are found in a vast range of conditions and may be defined in a number of ways, including by site of origin (e.g., supraventricular, ventricular, atrial), mechanism of disturbance (e.g., fibrillation, automaticity, re-entry or triggered activity), rate of disturbance (e.g., tachycardia, bradycardia) and electrocardiogram appearance (e.g., long QT syndrome). Dysrhythmias may be acute or chronic, and some (especially ventricular arrhythmias) may be life-threatening. Sudden cardiac death is the most severe manifestation of ventricular arrhythmias (e.g., ventricular fibrillation) (2) (26) (28)

III.2.2.2The causes and Symptoms

The following causes can lead to dysrhythmia development (27) (2) (25):

- A high-fat diet
- Certain OTC and prescription drugs or supplements
- Coronary artery disease (blockage in the arteries)
- Diabetes
- Drug abuse
- Excessive use of alcohol (more than two drinks per day)
- High blood pressure
- High cholesterol
- Obesity
- Sleep apnea
- Smoking
- Stress

Dysrhythmia symptoms can vary from silent to severe. That’s why regular check-ups are so important. Symptoms like these may be noticed on a regular basis or every once in a while:

- Chest pain or tightness
- Dizziness or light-headedness
- Fainting
- Palpitations – a feeling of skipped heartbeats or fluttering
- Pounding in the chest
- Shortness of breath
- Weakness or fatigue

III.2.3 The third case (Cardiomyopathy)

III.2.3.1definition and mechanism

Cardiomyopathy refers to conditions that affect the myocardium (heart muscle). Cardiomyopathy can make your heart stiffen, enlarged or thickened and can cause scar tissue. As a result, your heart can't pump blood effectively to the rest of your body. The main types of cardiomyopathy include dilated, hypertrophic and restrictive cardiomyopathy. Treatment which might include medications, surgically implanted devices, heart surgery or, in severe cases, a heart transplant depends on the type of cardiomyopathy and how serious it is.

III.2.3.2 The causes and Symptoms

Healthcare professionals may categorize cardiomyopathy based on the general cause. These two categories are:

- Ischemic cardiomyopathy**,** caused by heart attacks or Coronary artery disease (CAD).
- Non-ischemic cardiomyopathy, types unrelated to CAD.

- Autoimmune diseases, connective tissue diseases.
- Conditions that damage the heart, such as high cholesterol diseases, hemochromatosis or sacoidosis.
- Endocrine conditions, such as diabetes or thyroid disease.
- Family history of heart failure, cardiomyopathy or sudden cardiac arrest
- Previous heart attacks.
- Pregnancy.

There might be no signs or symptoms in the early stages of cardiomyopathy. But as the condition advances, signs and symptoms usually appear, including:

- Breathlessness with activity or even at rest

- Swelling of the legs, ankles and feet
- Bloating of the abdomen due to fluid build-up
- Cough while lying down
- Difficulty lying flat to sleep
- Fatigue
- Heartbeats that feel rapid, pounding or fluttering
- Chest discomfort or pressure
- Dizziness, light headedness and fainting

III.3 partical part

In this part we will analyse the three cases by three wavelet (haar, db, symlet) and compare them to obtain results for determine the best wavelet between them. This process goes through two steps :

- ❖ Analyse the signal by the wavelet for filtering the high frequency
- ❖ Transform this last in the frequency domain by FFT to make it easier to read the statement.

III.3.1 The normal case

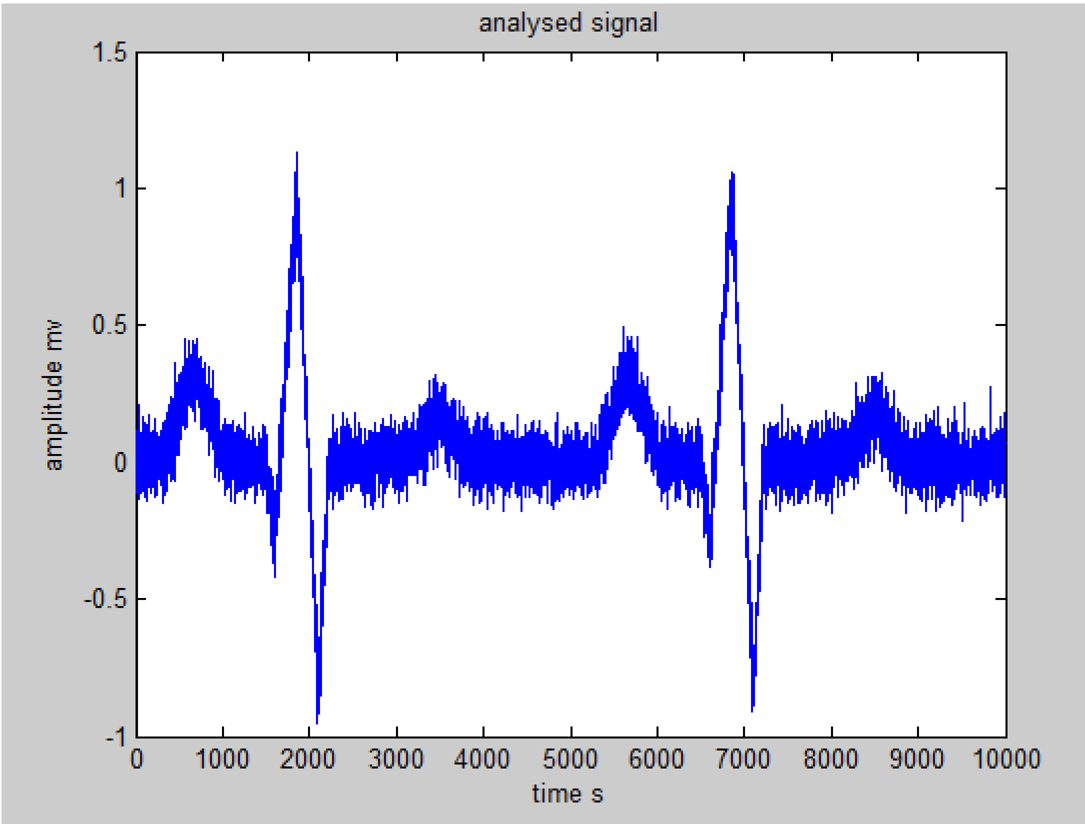


Figure III. 2: The analyse of normal signal

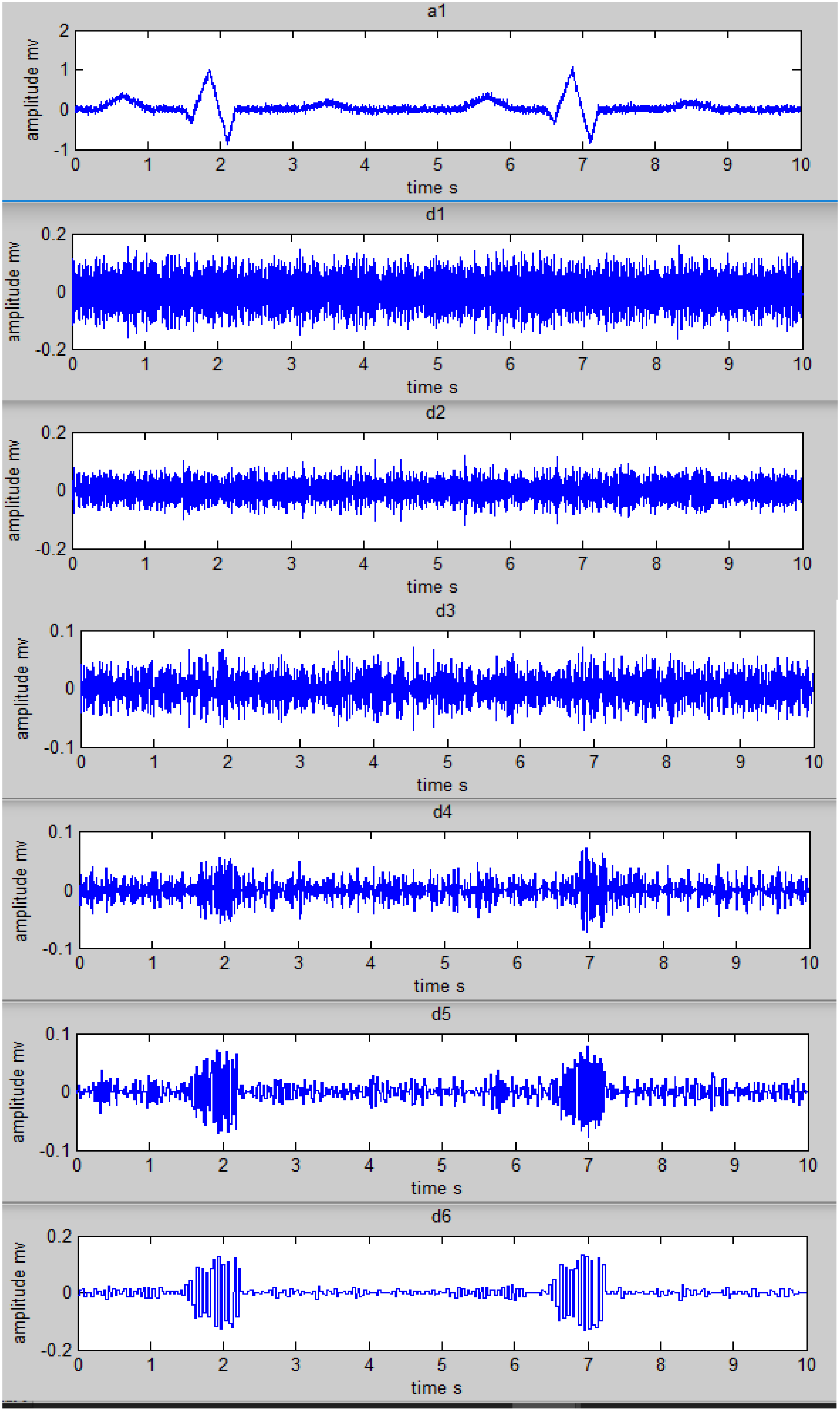


Figure III. 3: The approximation and detais of the signal using haar

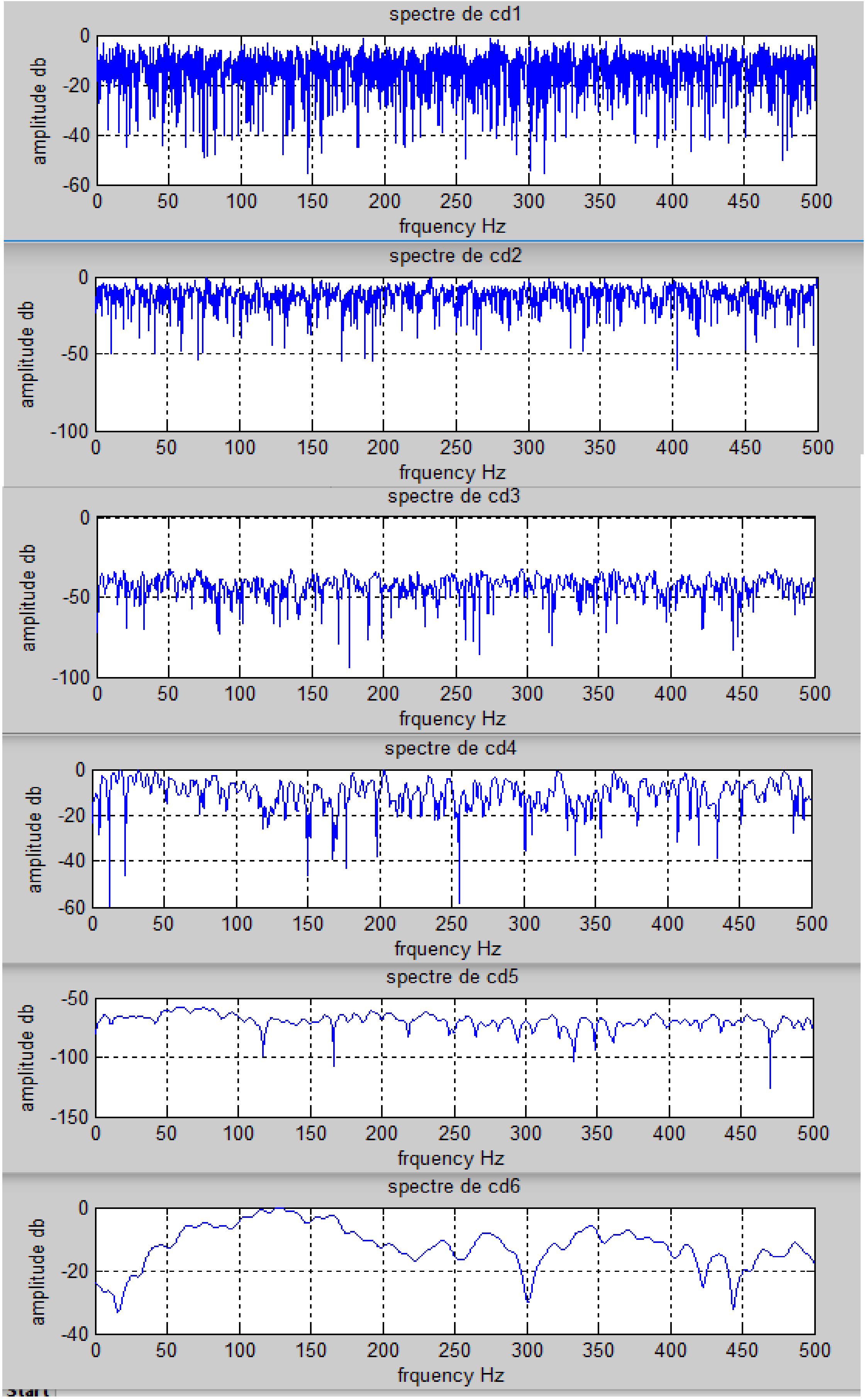


Figure III. 4: FFTs of details using haar

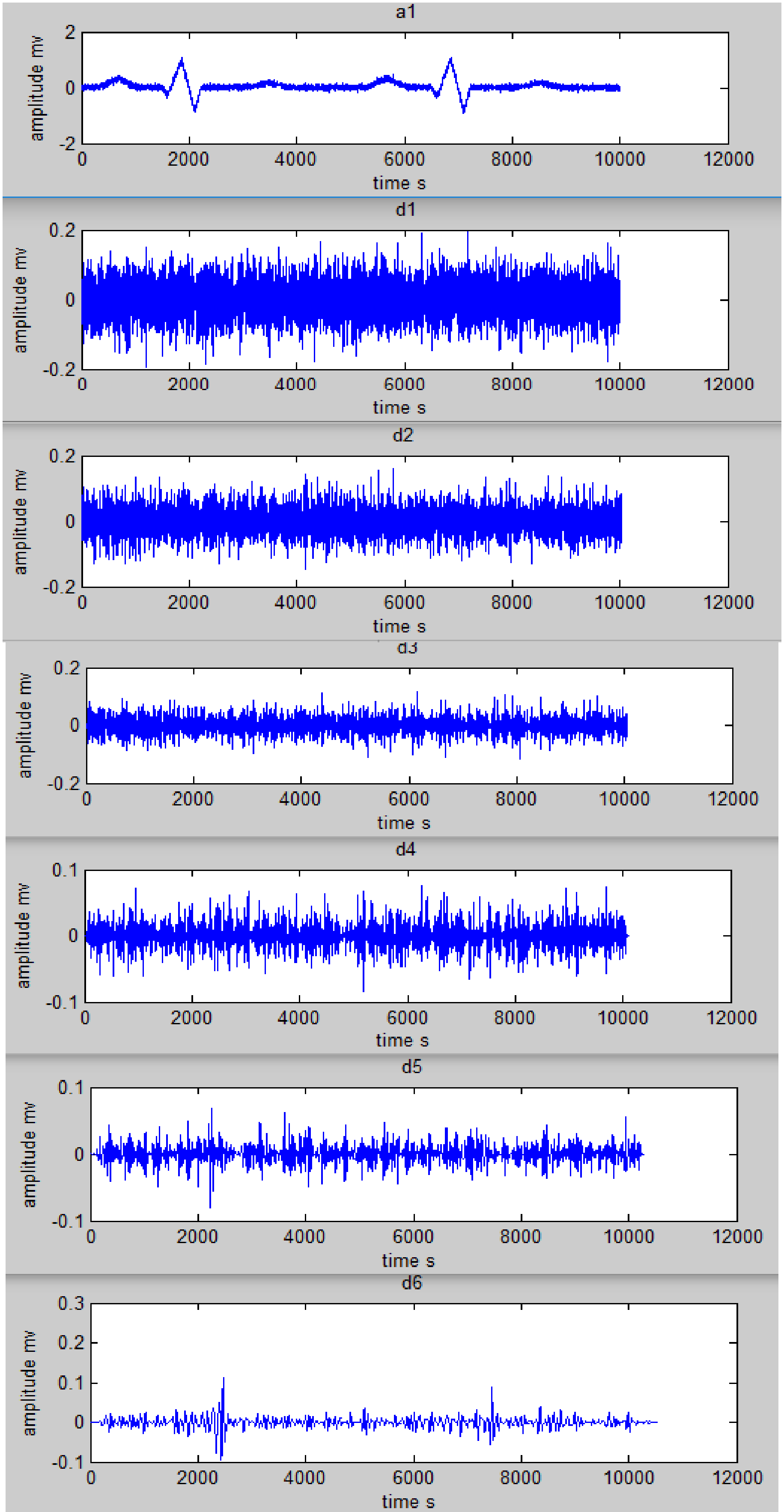


Figure III. 5: The approximation and detais of the signal using db

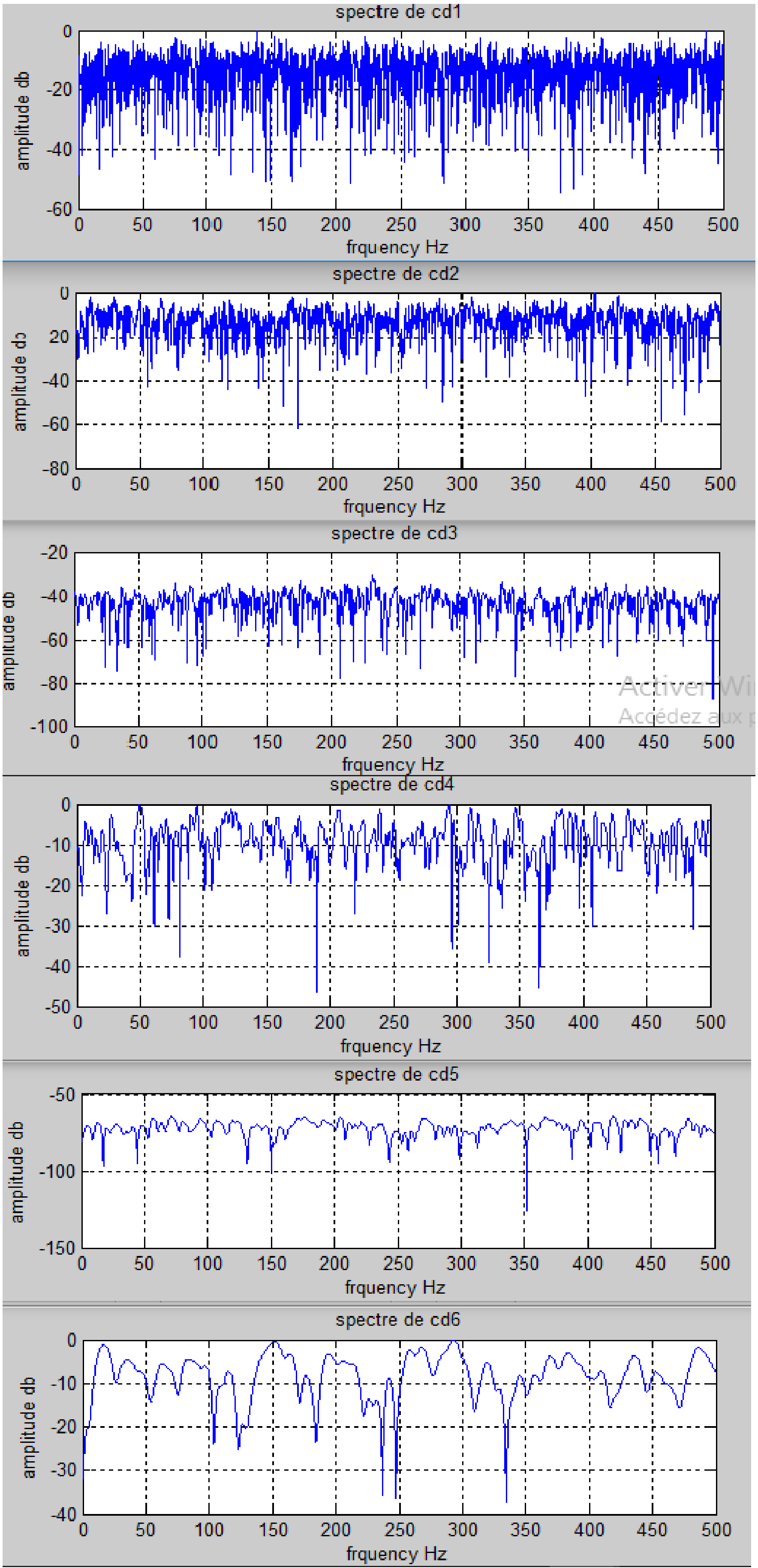


Figure III. 6: The FFTs of the details using db

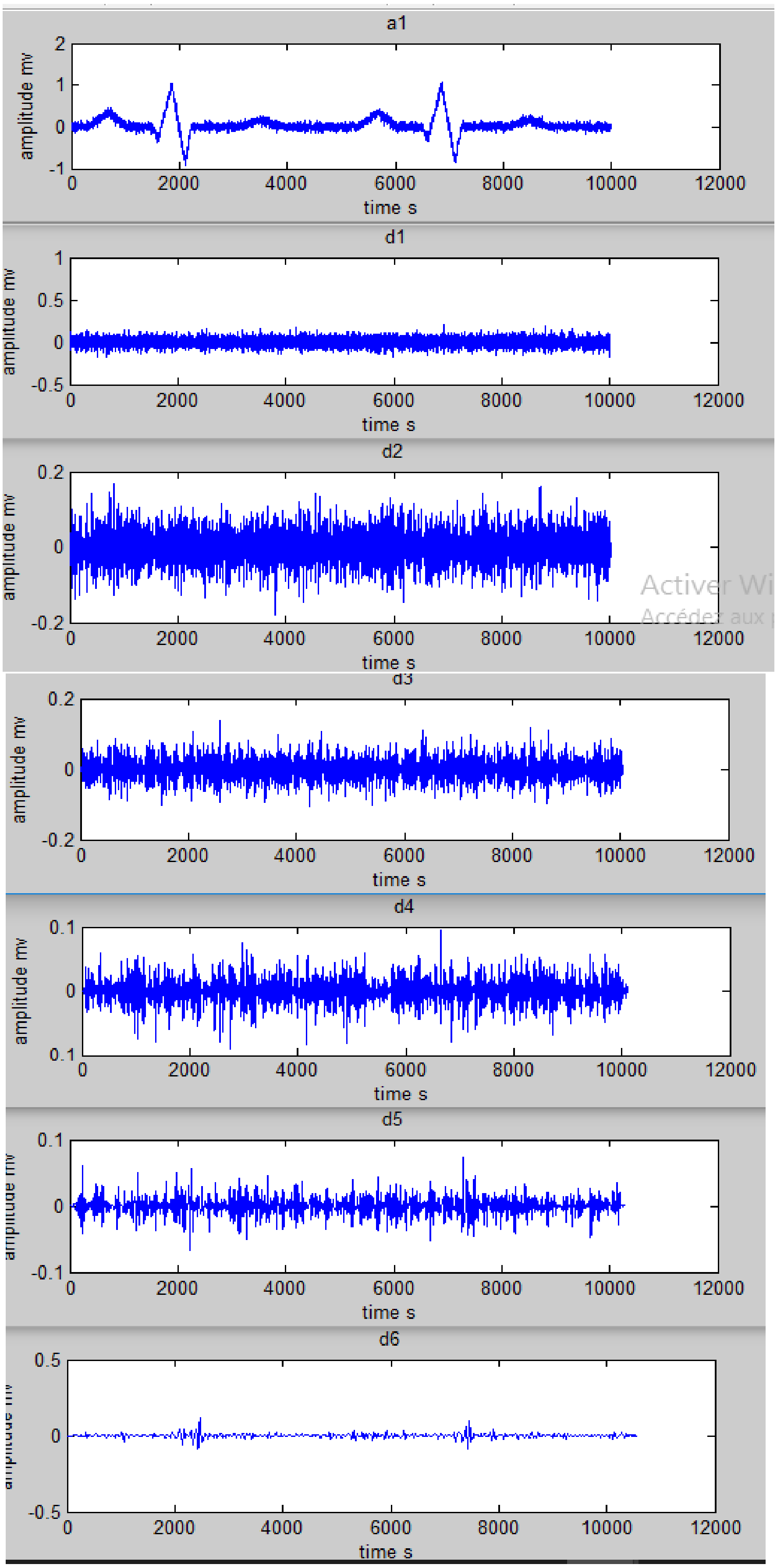


Figure III. 7: The approximation and detais of the signal using symlet

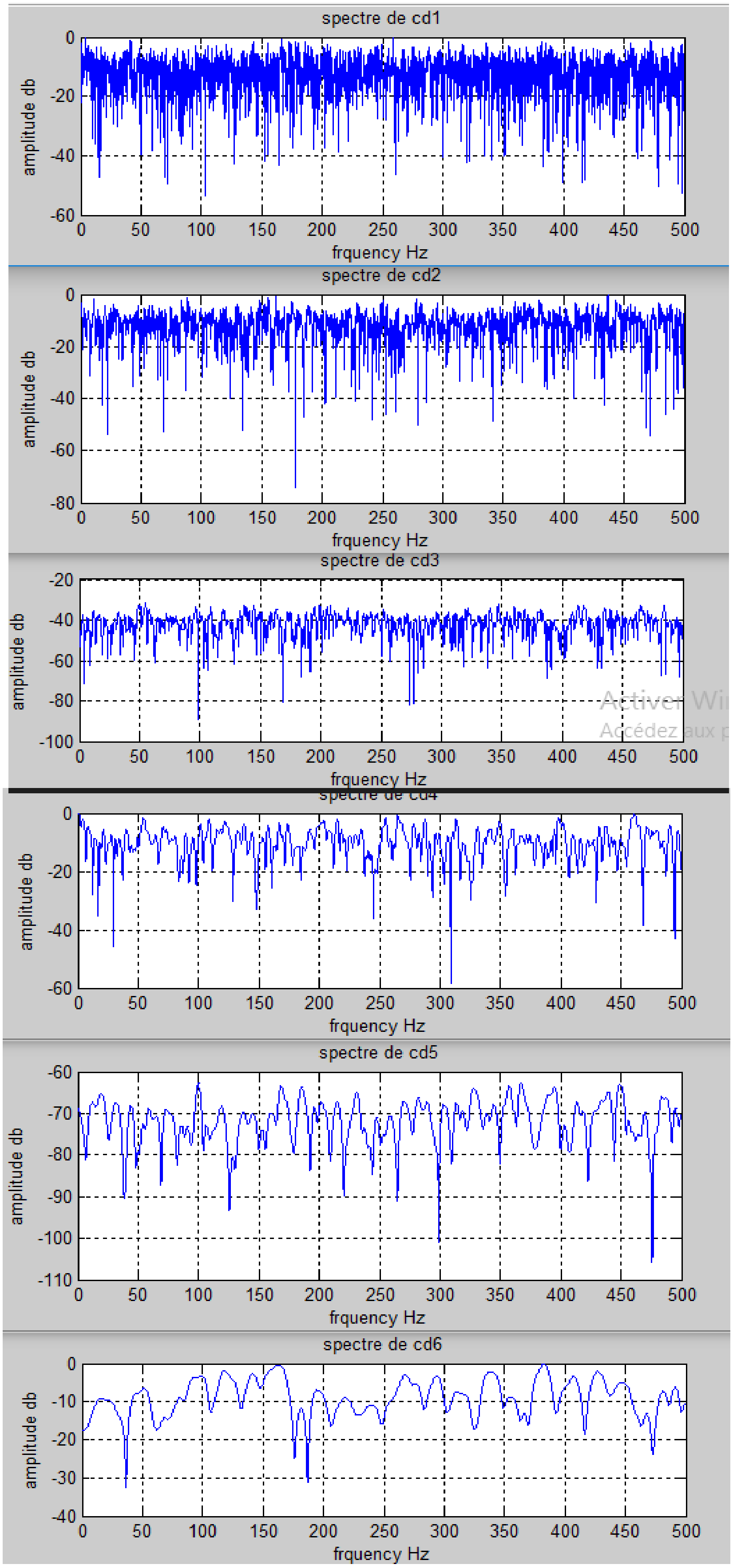


Figure III. 8: The FFTs of the details using symlet

III.3.2 Cardiomyopathy case

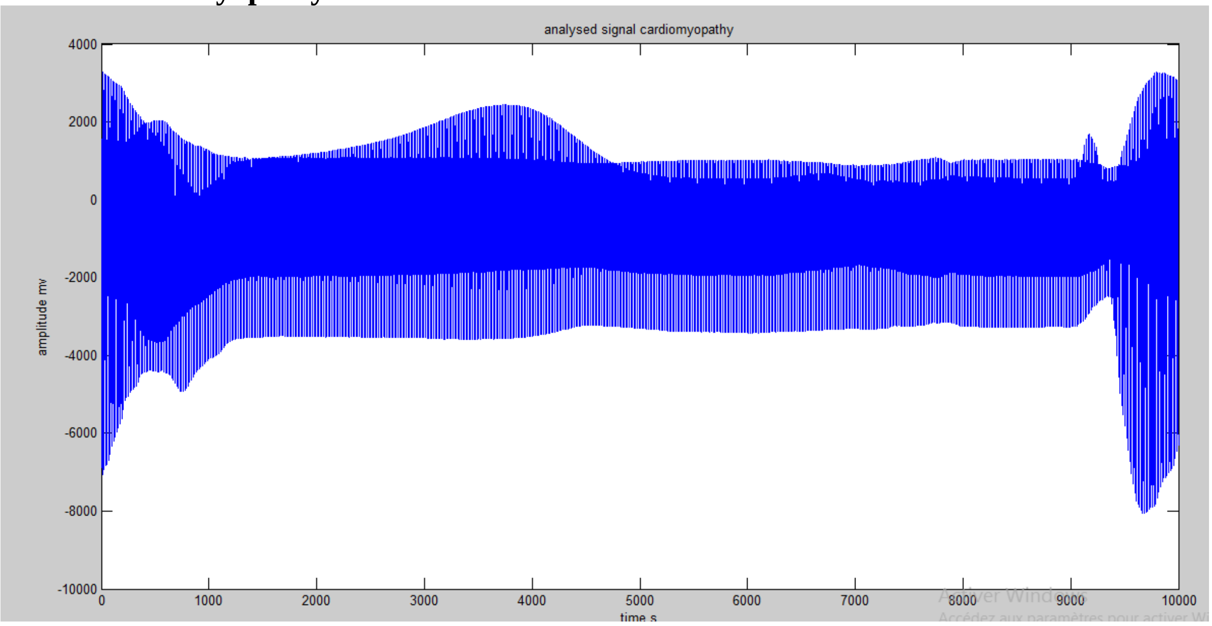


Figure III. 9: The analysed of Cardiomyopathy signal

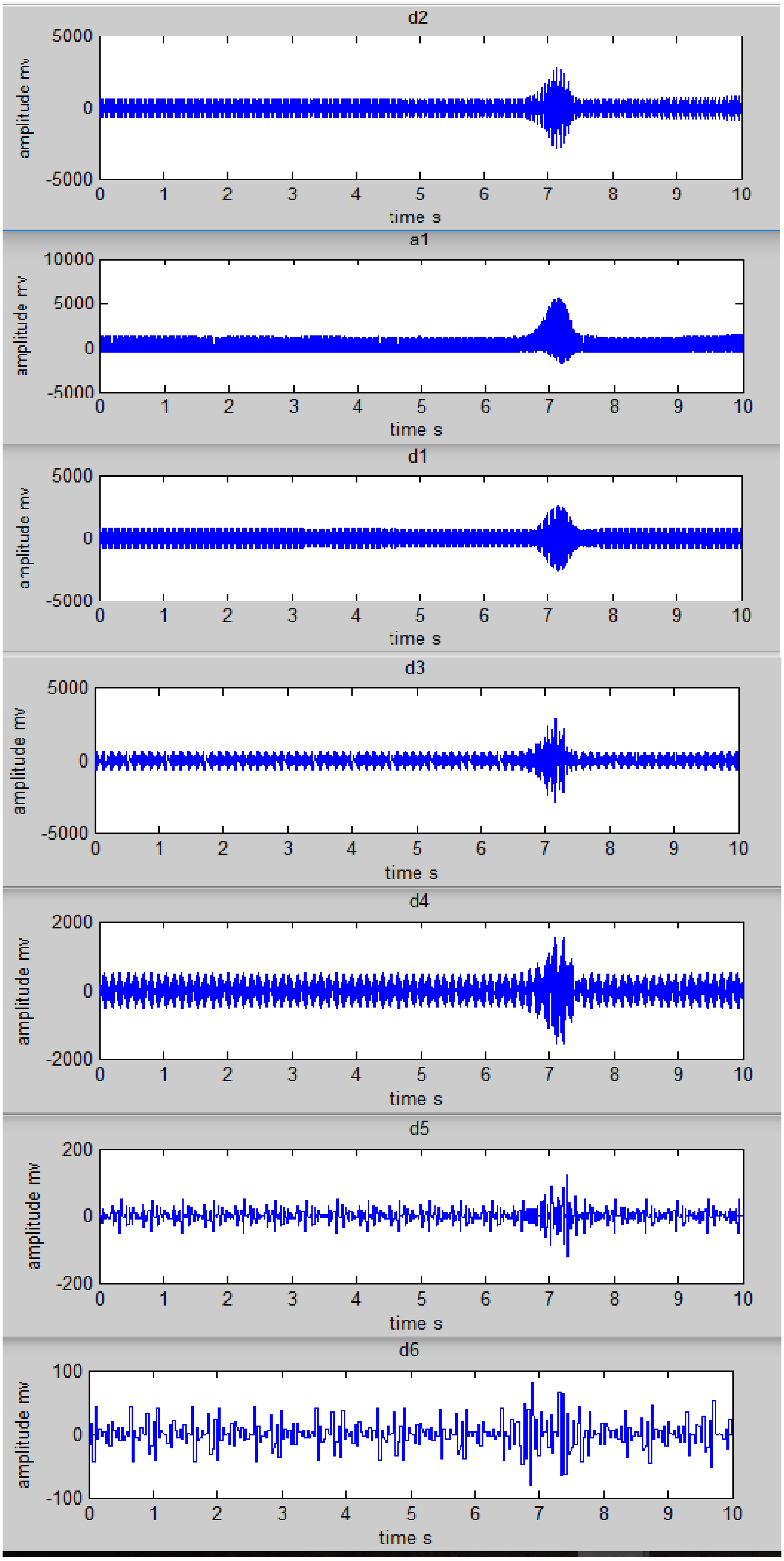


Figure III. 10: The approximation and detais of the signal using haar

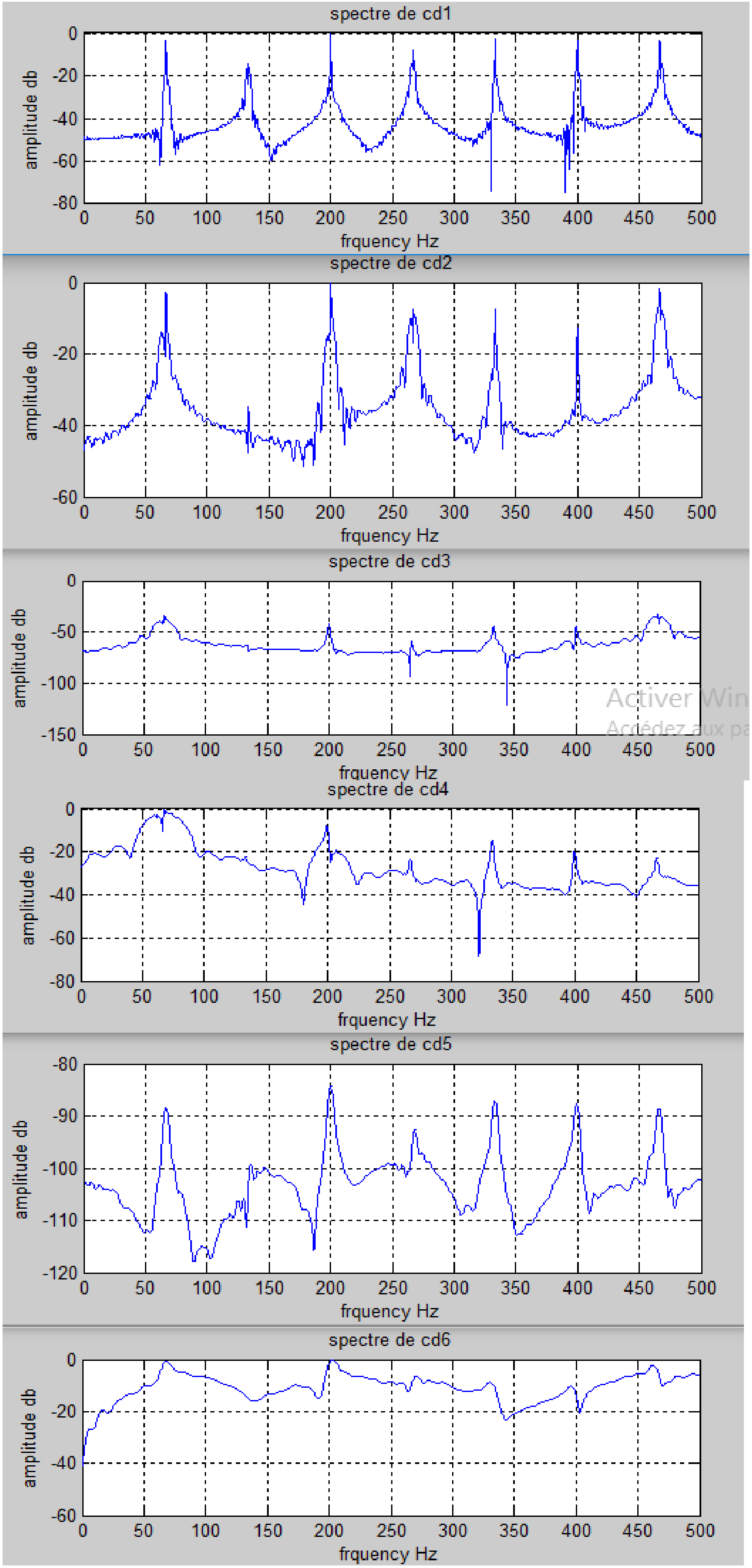


Figure III. 11: The FFTs of the details using haar

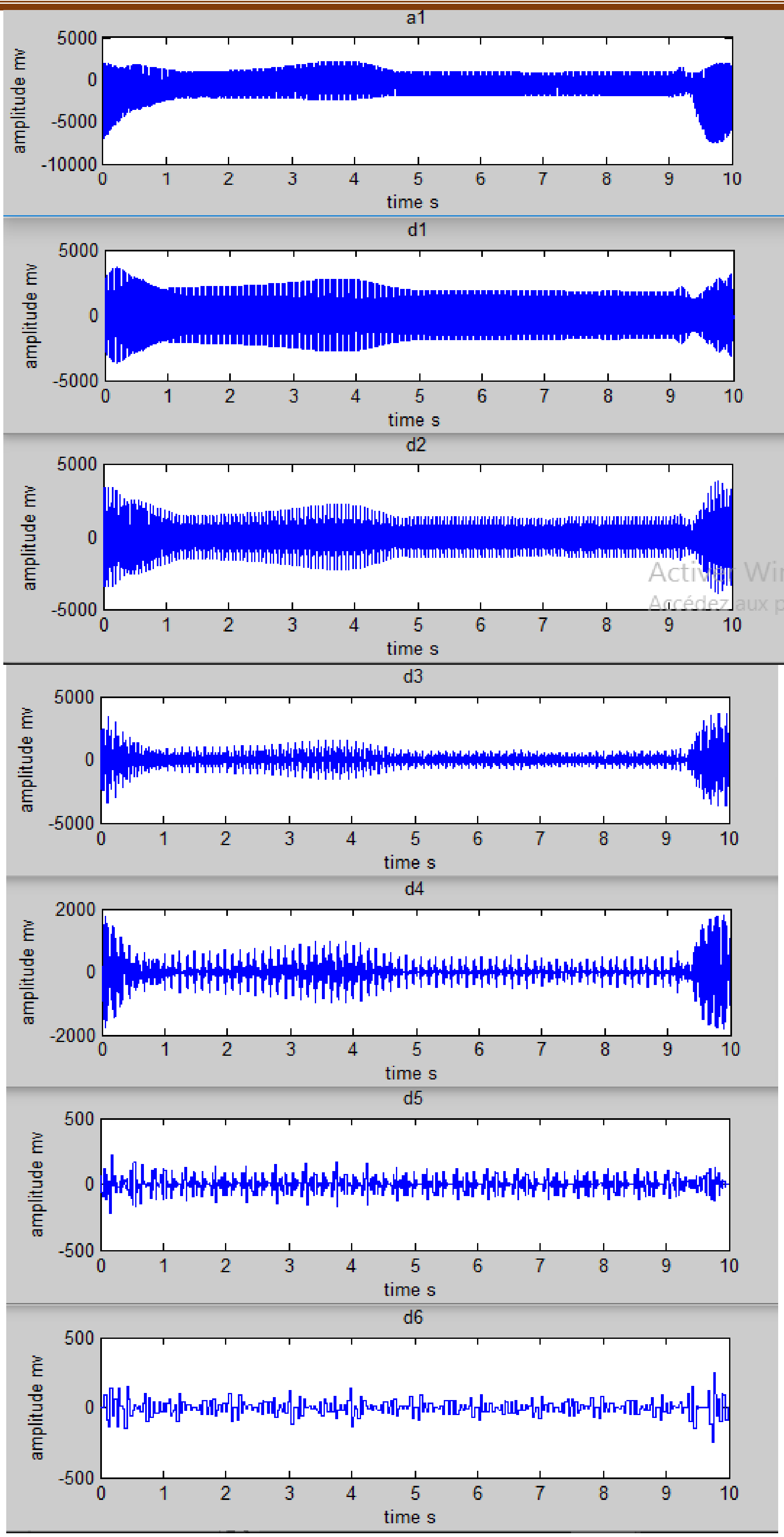


Figure III. 12: The approximation and detais of the signal using db

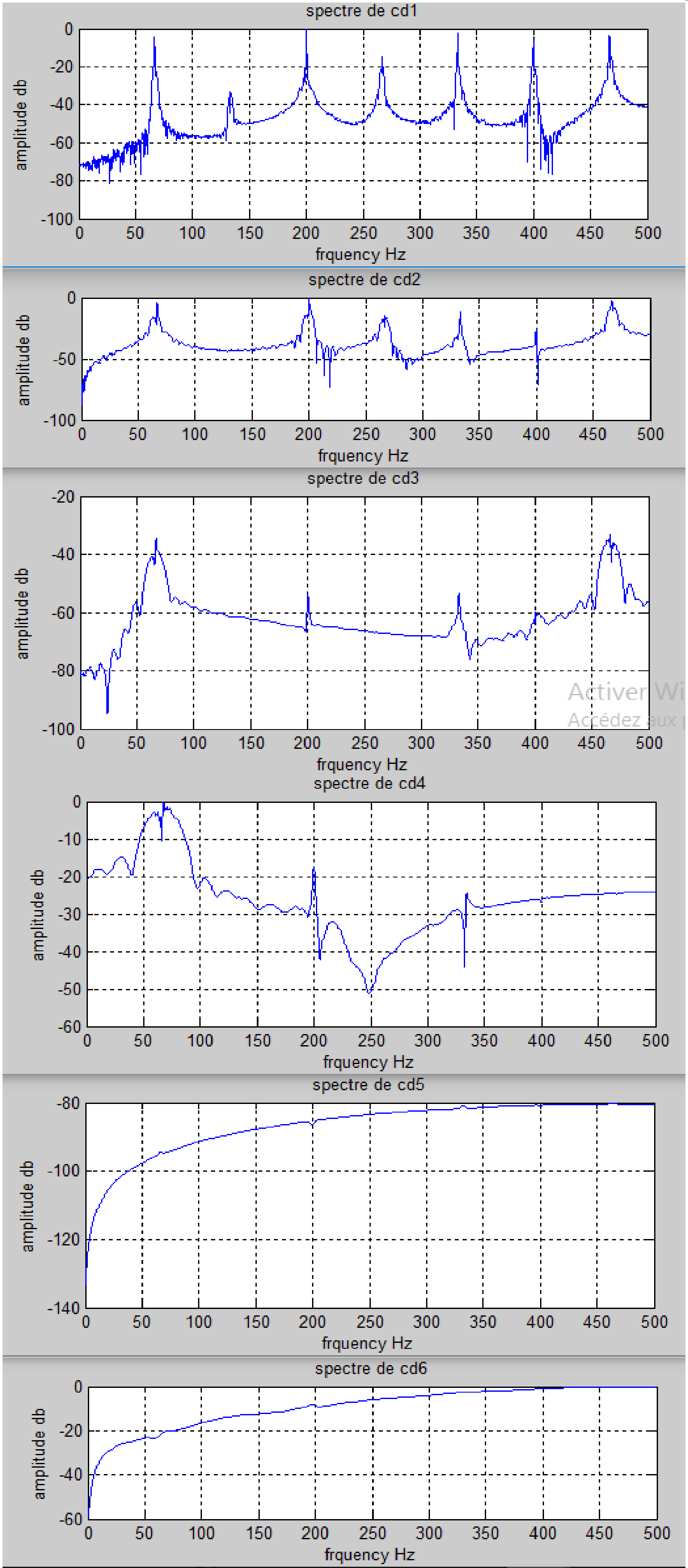


Figure III. 13: The FFTs of the details using db

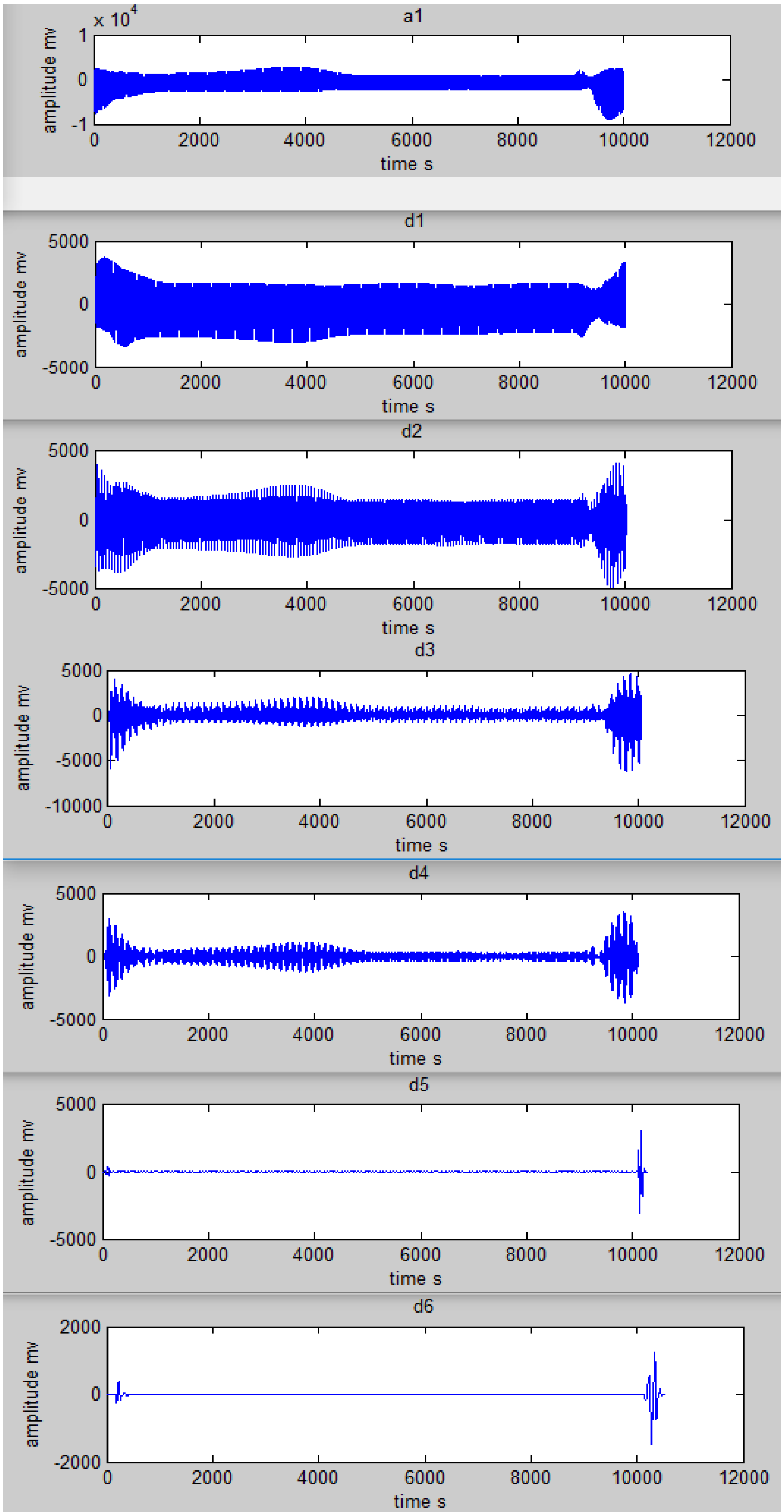


Figure III. 14: The approximation and detais of the signal using symlet

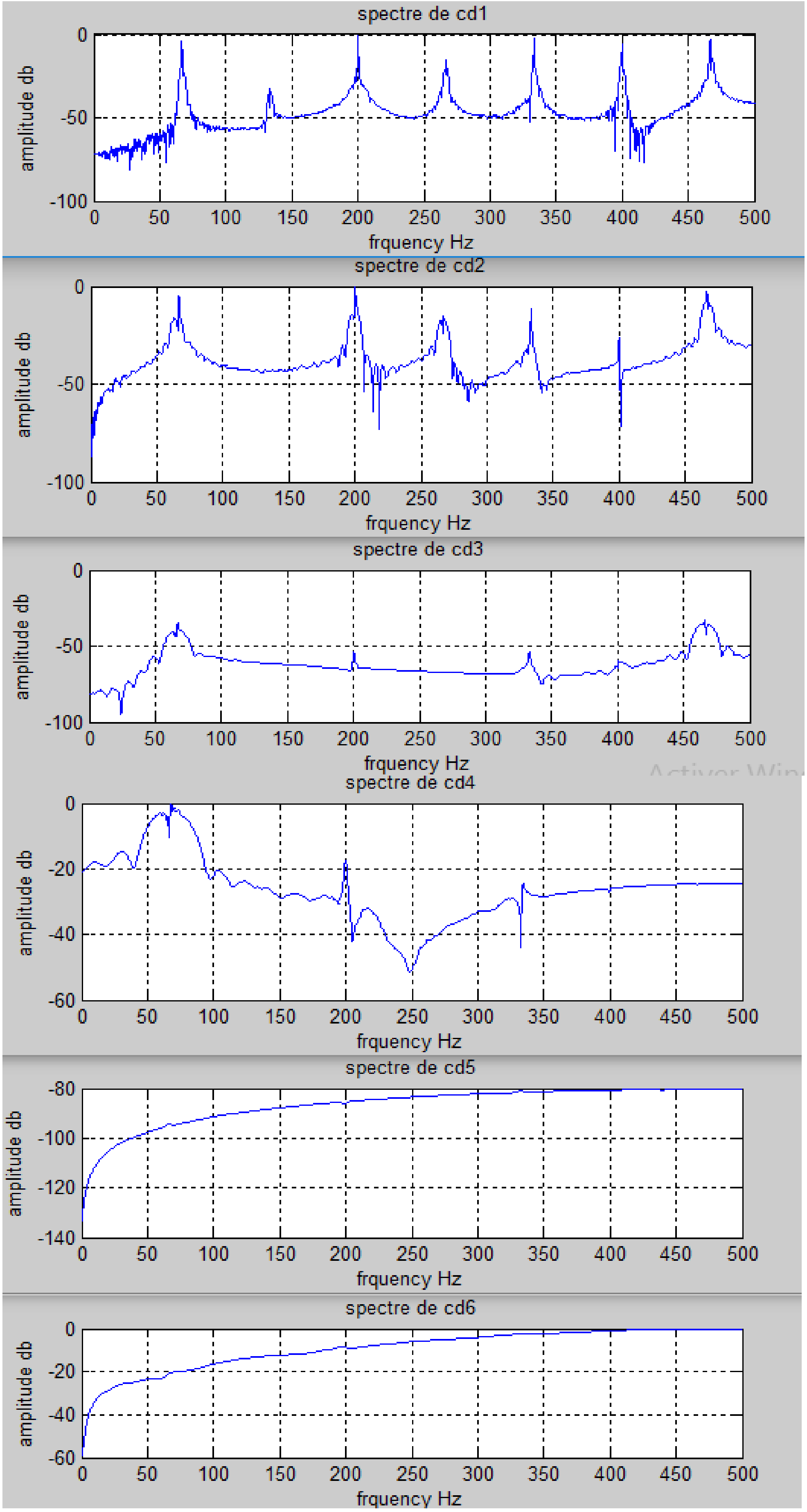


Figure III. 15: The FFTs of the details using symlet

III.3.3 Myocardial infraction case

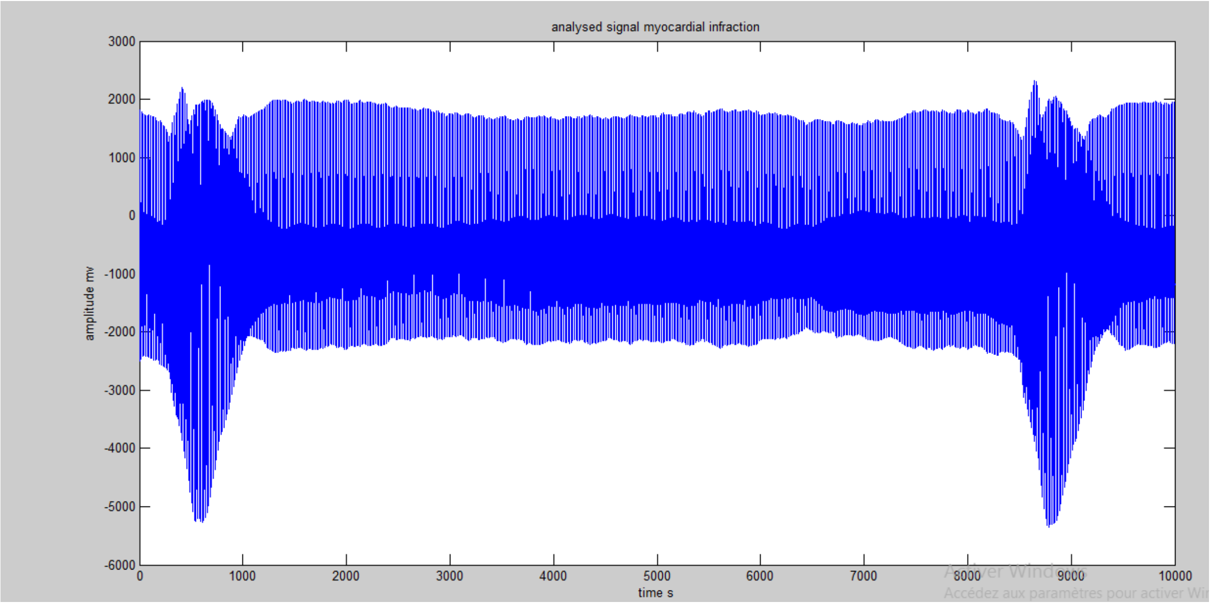


Figure III. 16: The analysed of Myocardial infraction signal

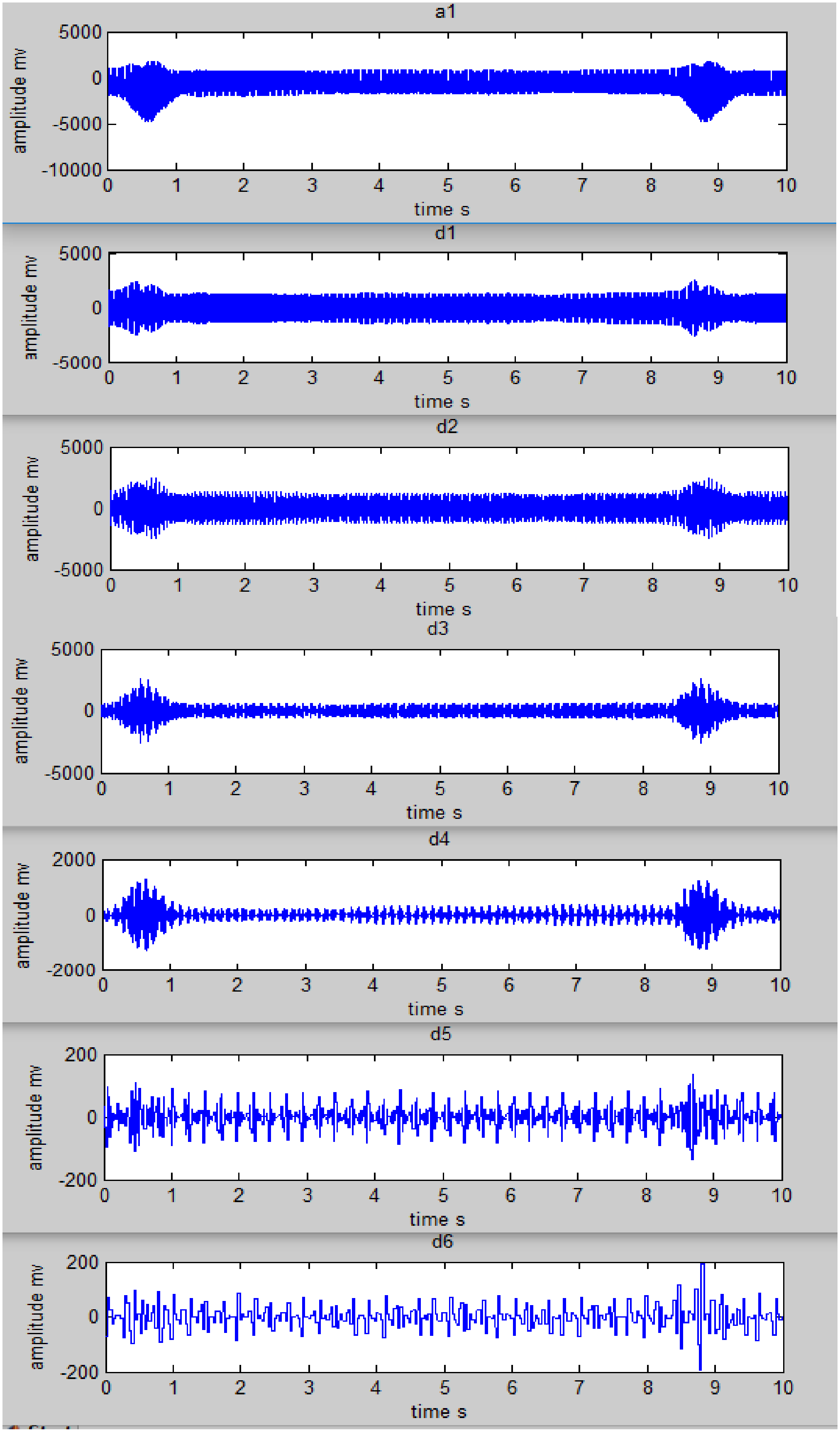


Figure III. 17: The approximation and detais of the signal using haar

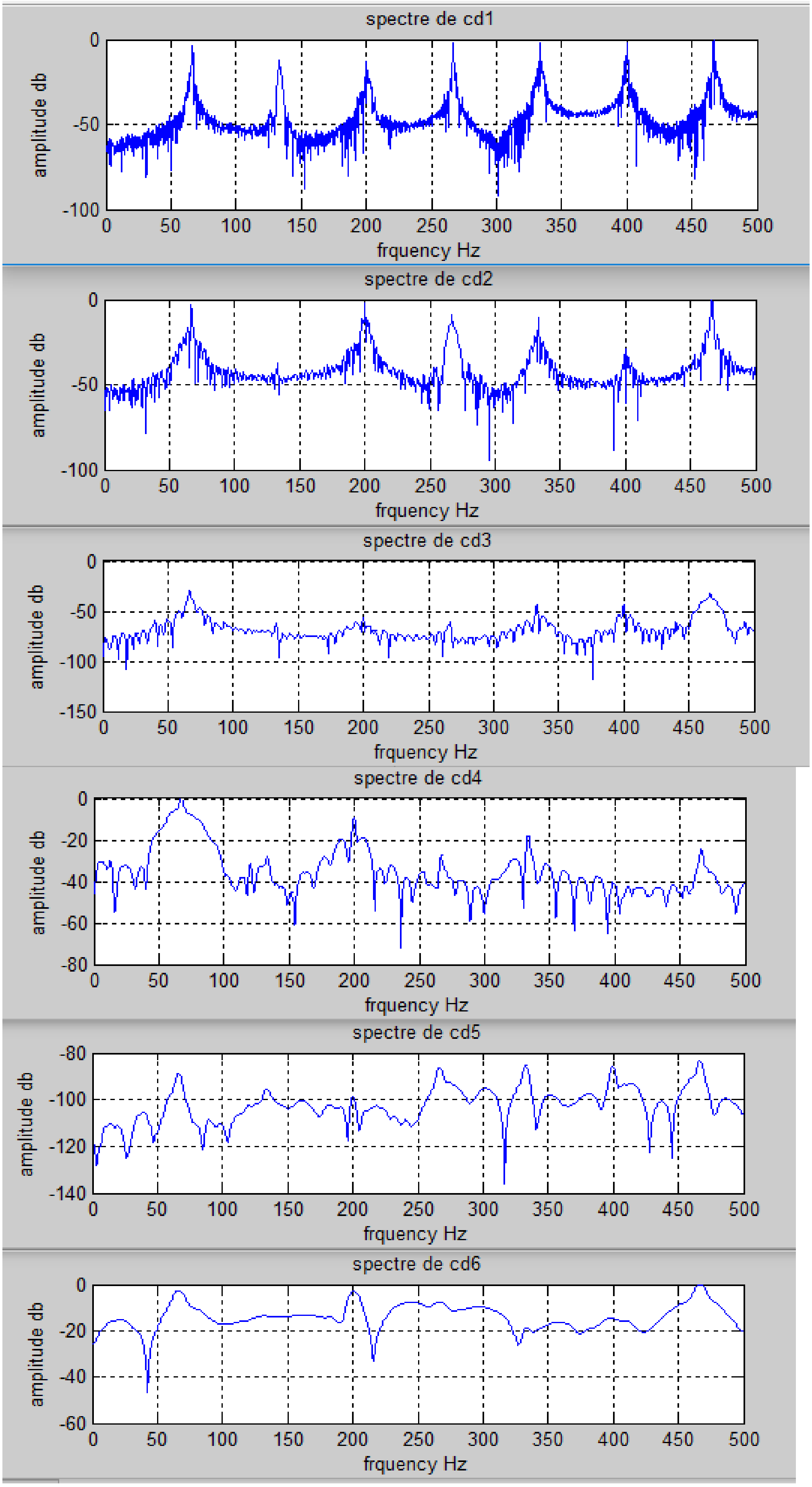


Figure III. 18: the FFTs of details using haar

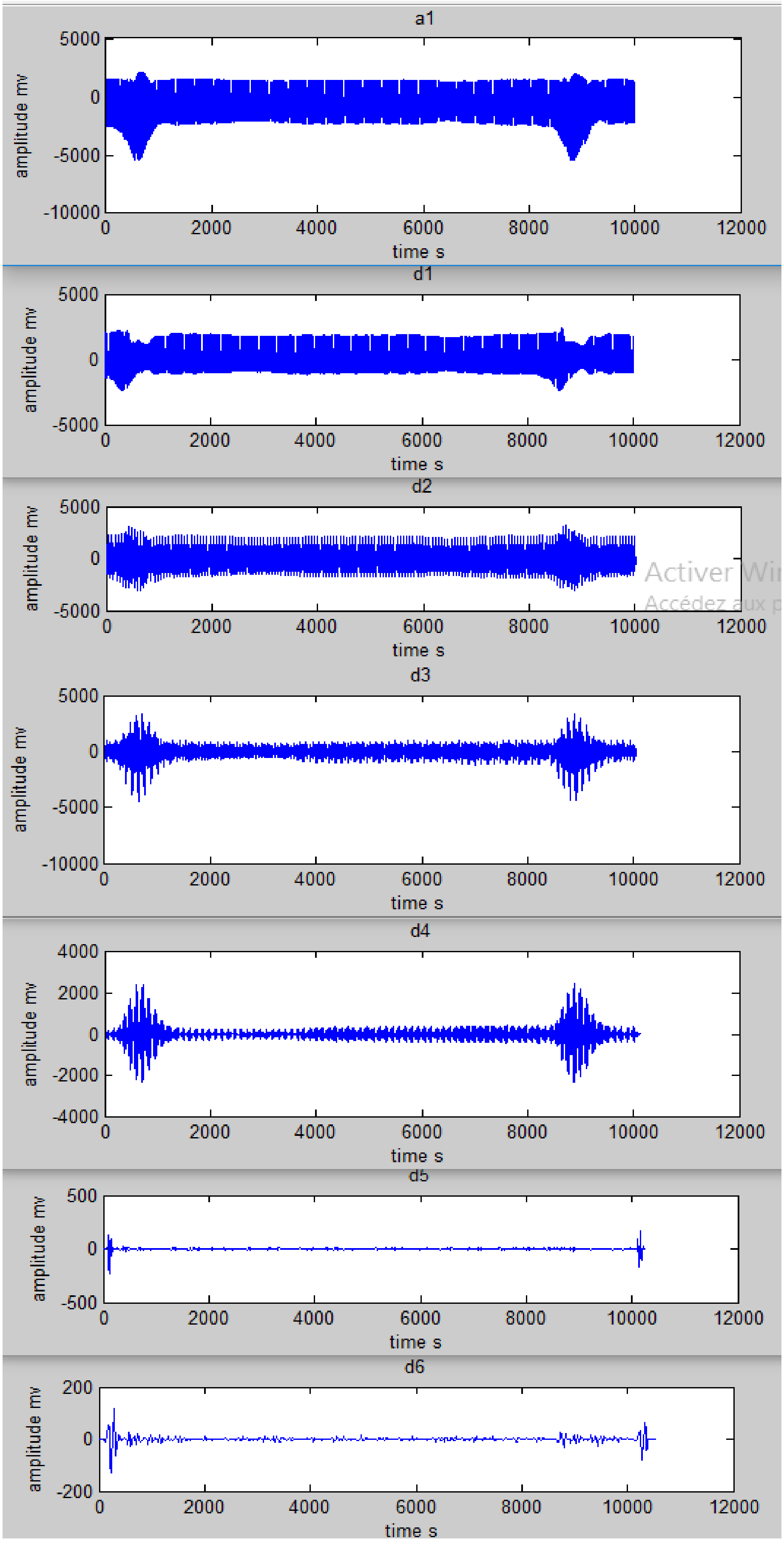


Figure III. 19: The approximation and details using db

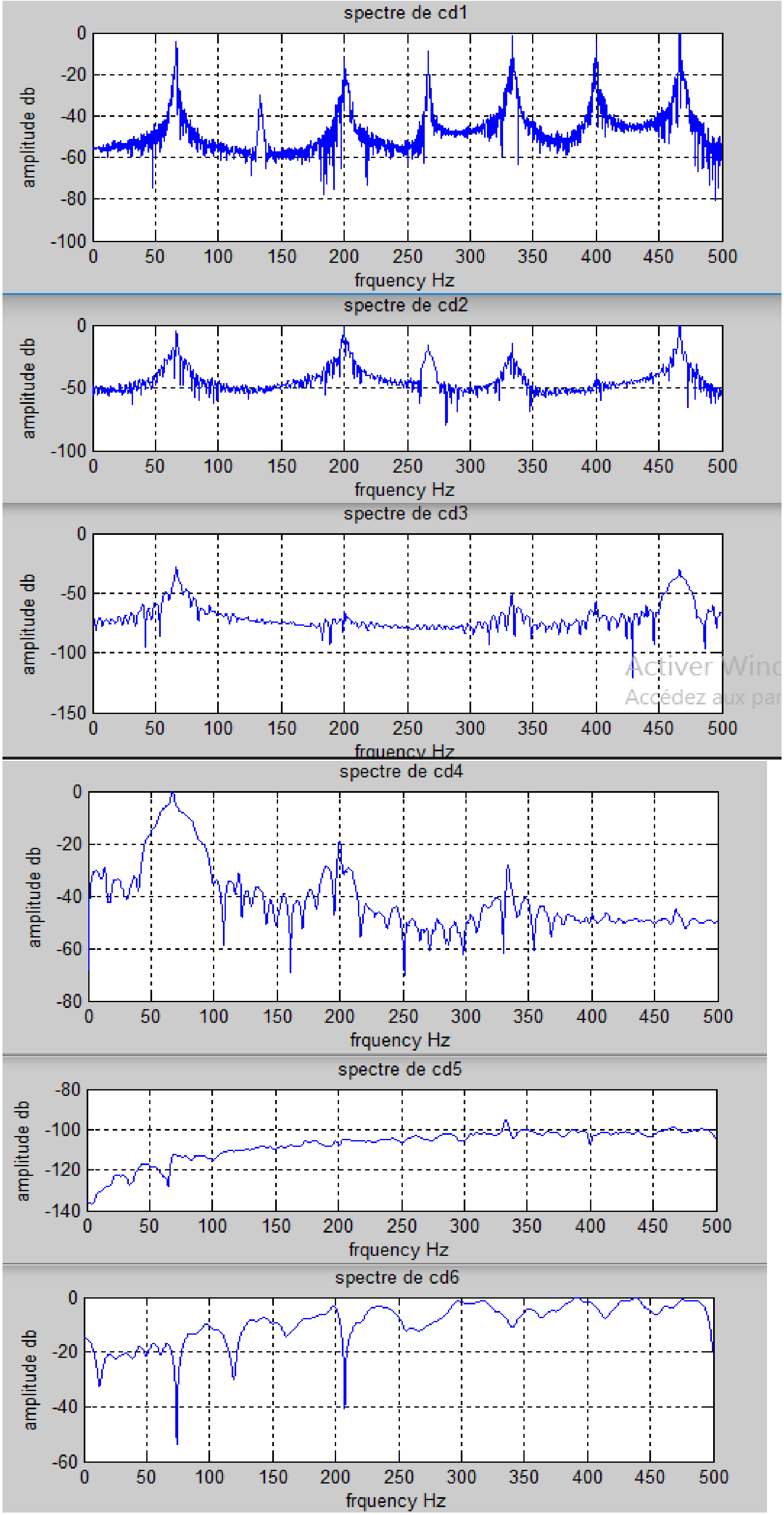


Figure III. 20: The FFTs of detail using db

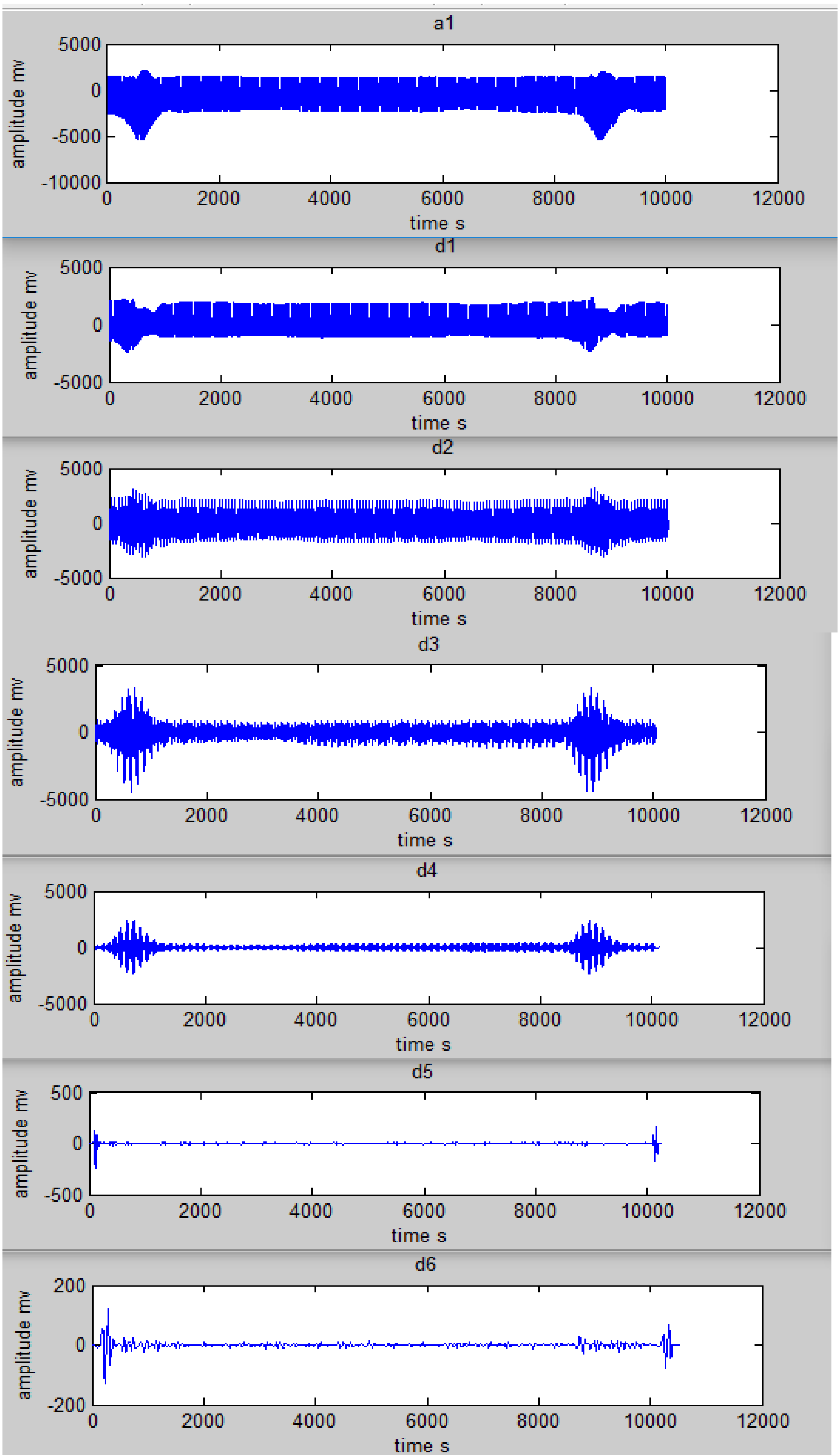


Figure III. 21: The approximation and details using symlet

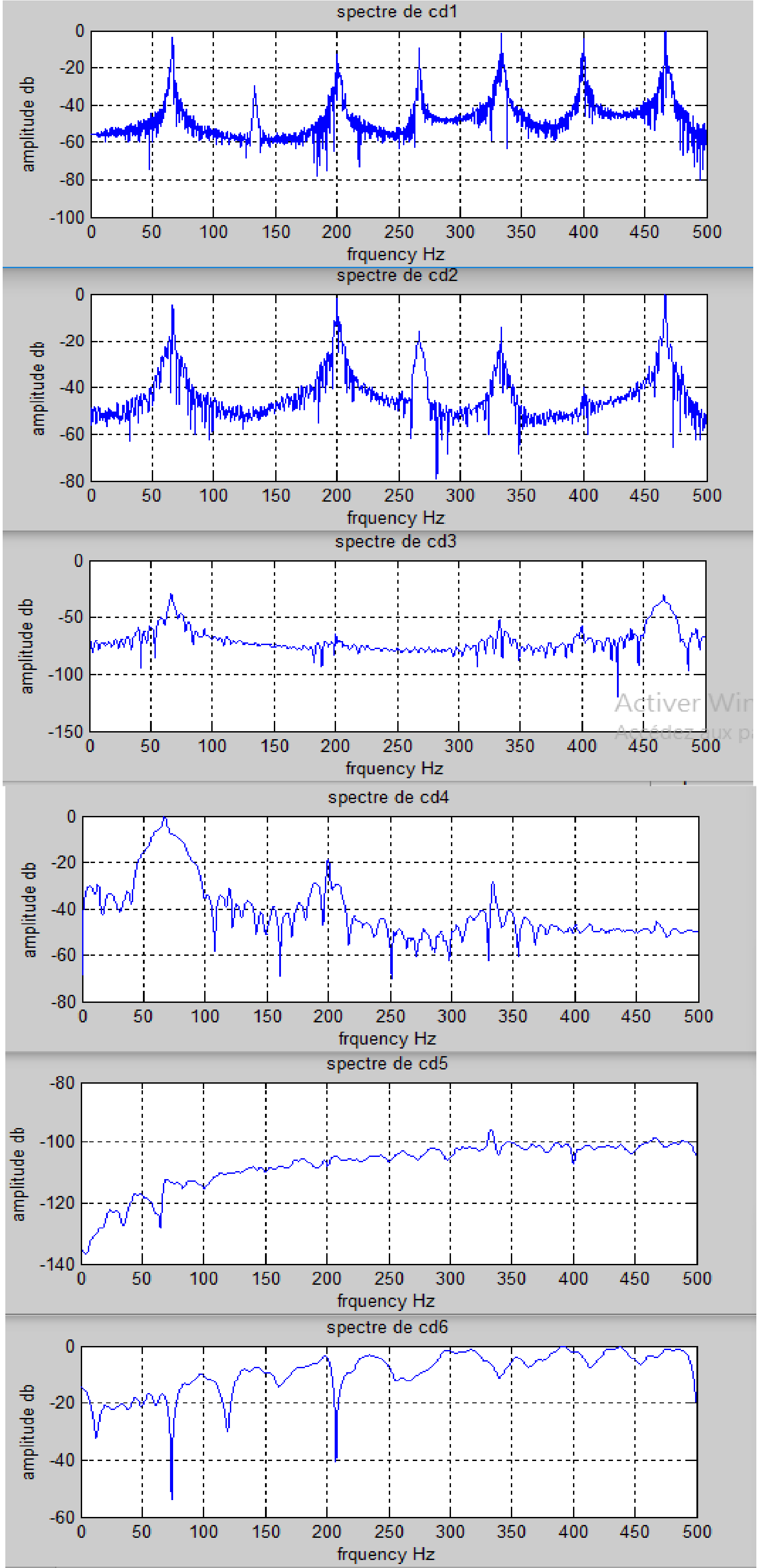


Figure III. 22: The FFTs of detail using symlet

I.3.4 Dysrhythmia

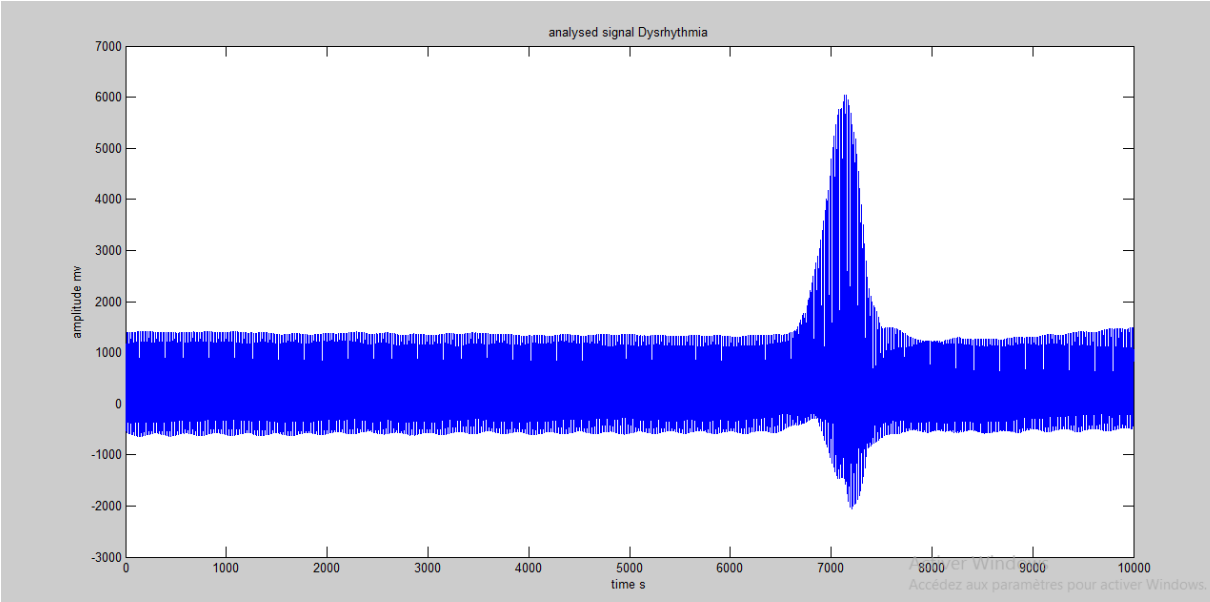


Figure III. 23: The analydes of Dysrhythmia signal

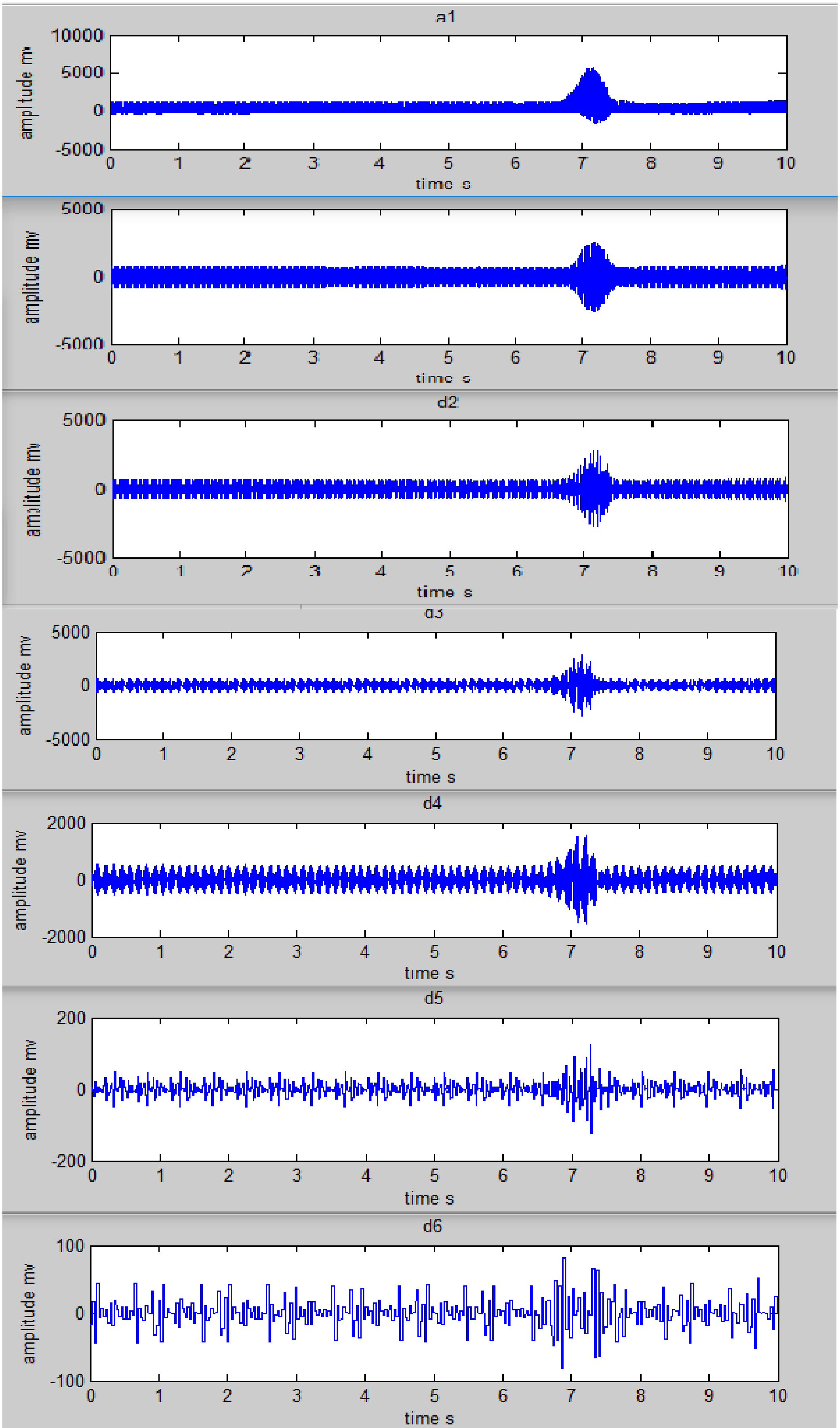


Figure III. 24: The approximation and details using haar

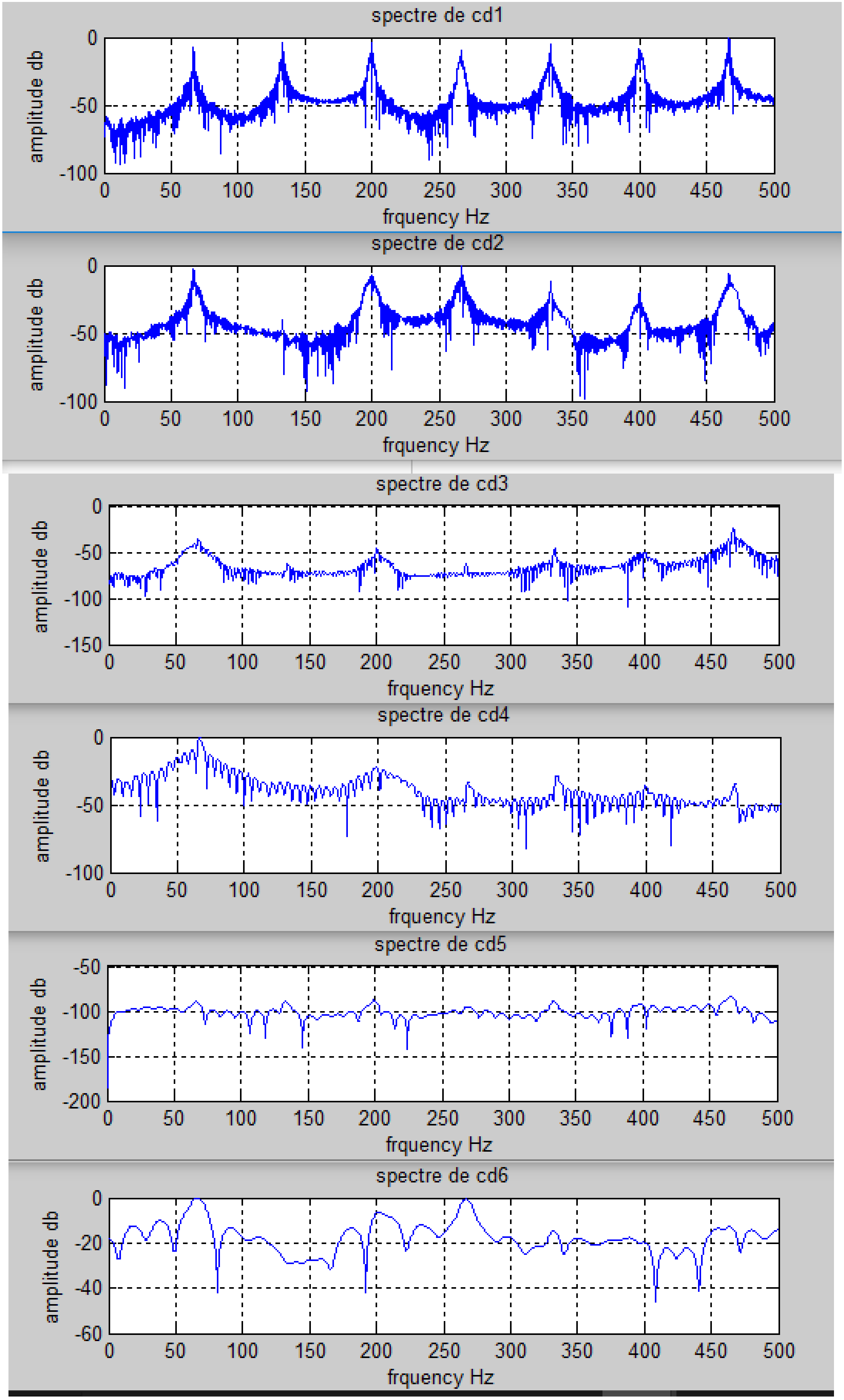


Figure III. 25: The FFTs of detail using haar

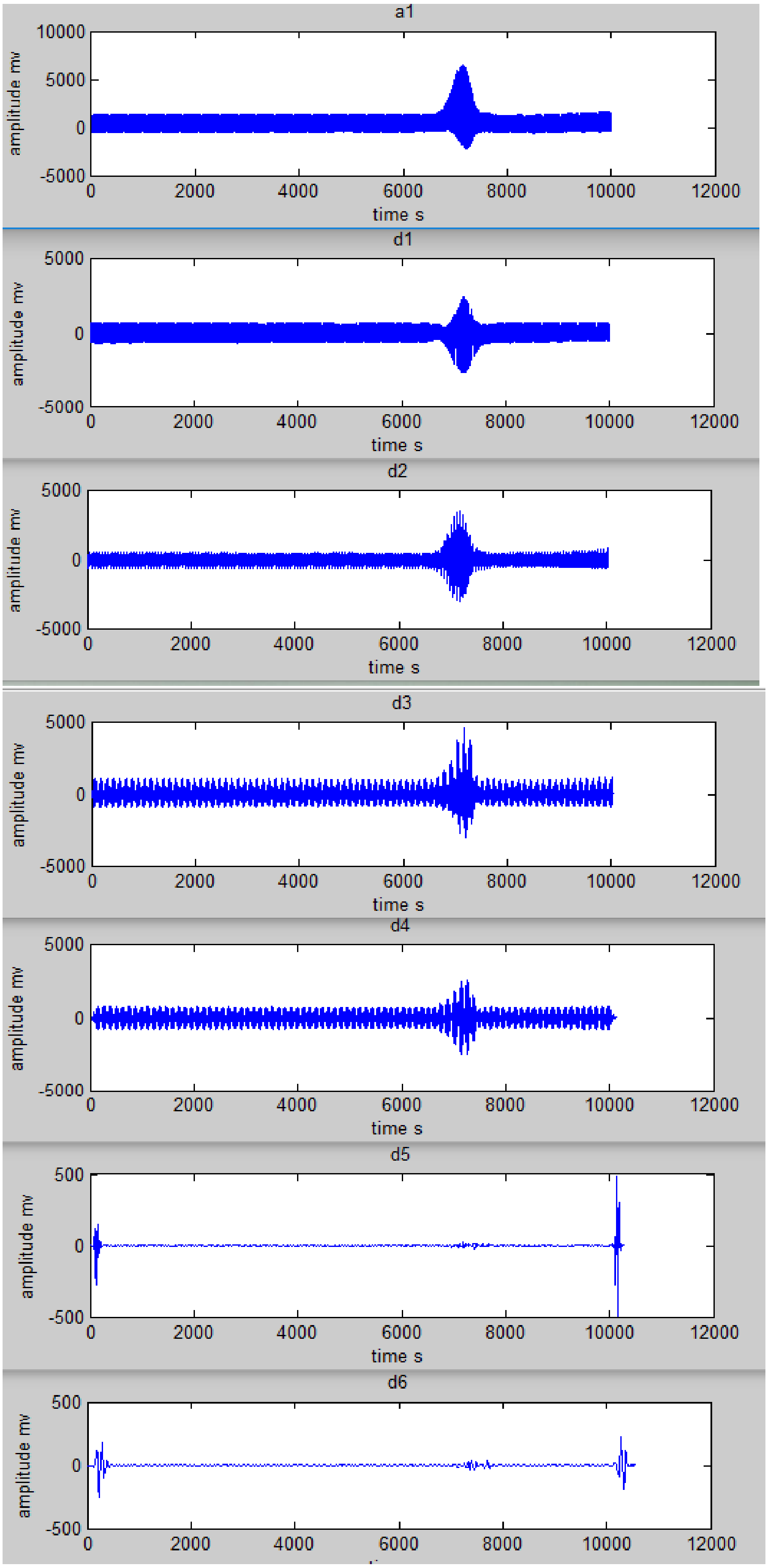


Figure III. 26: The approximation and details using db

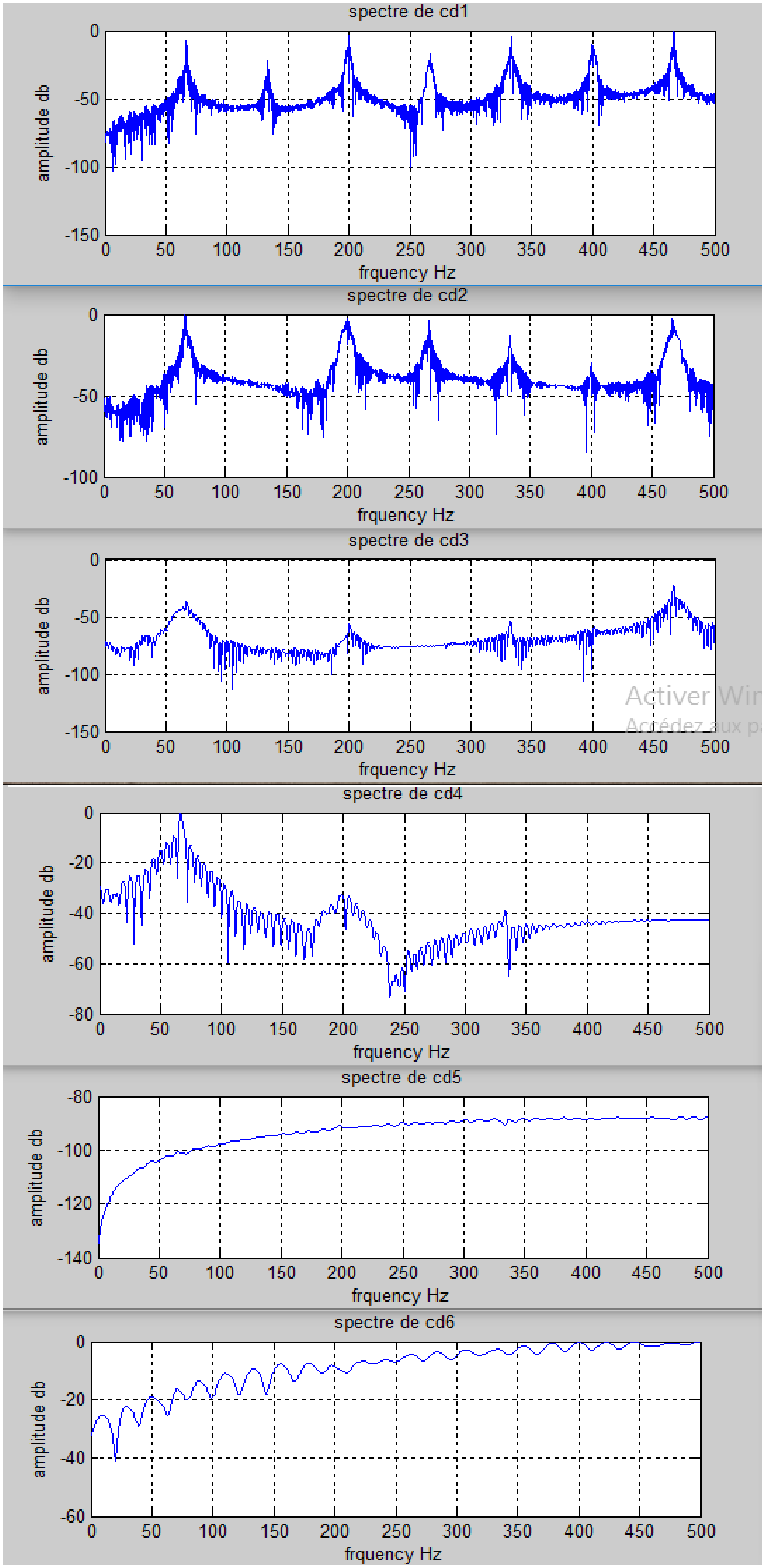


Figure III. 27: The FFTs of details using db

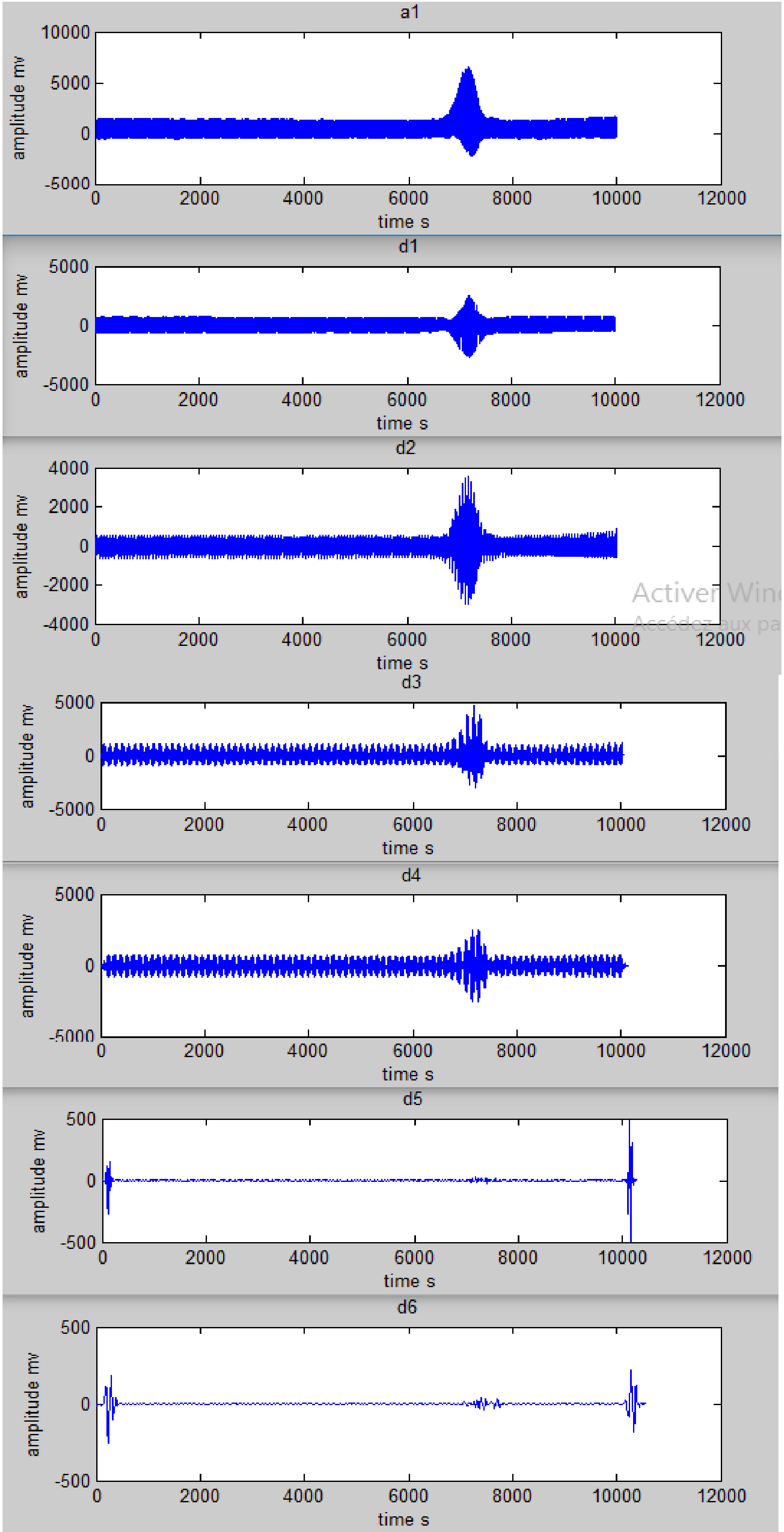


Figure III. 28: The approximation and details using symlet

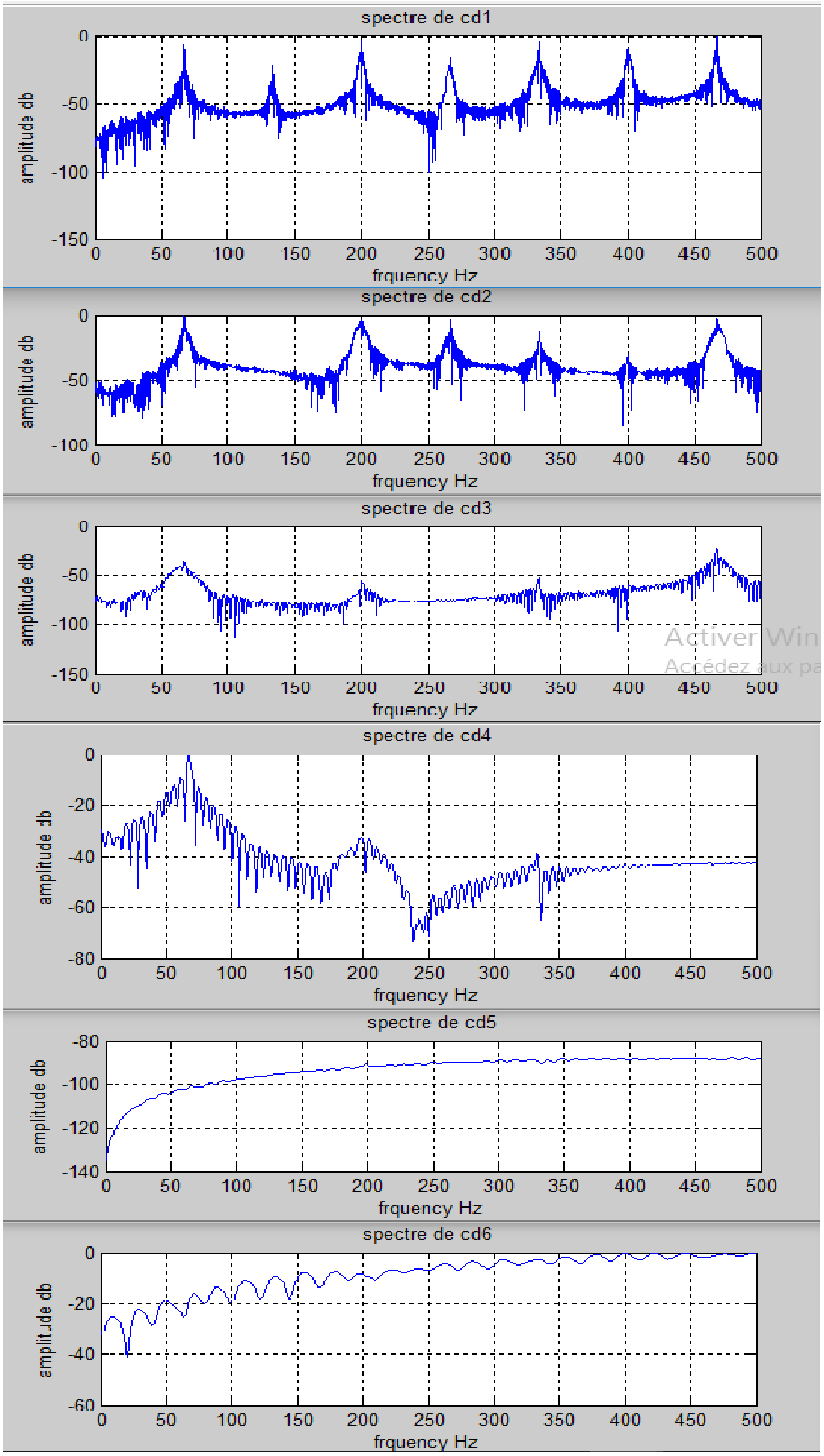


Figure III. 29: The FFTs of details using symlet

III.4 Conclusion and discussion

In this chapter, we got knowledge about the cases of ECG signals, we analyse by DWT (wavelet transform), and also we made the FFT of the DWT's details, and after comparing we concluded the pathologies comparing to the normal case, we found that:

A. We notice that the two waves (db3 and symlet3) they have the same statements so they process the same operation(work)

B. In the beginning, all the cases have the same fft comparing by the fft of normal case, when the level increases (after the 3 level), the pathology be more remarkabale by the variations on the frequencies details.

So according to these notes we can say that the DWT is limited and bounded, slow, and that made us ask a question about if we can develop a new method or technique which is supposed to be capable to surpasses the problem of DWT .

Chapter IV: Analyse signals with Adaptive lifting

IV.1 Introduction

In this chapter, we introduce the concept of adaptive lifting. Then, we can make a comparison between analysis ECG signals with classical wavelet transform and analysis ECG signals by adaptive lifting schemes. This chapter is divided into 2 parts, theoretical part that introduce adaptive lifting definition and its basic conceptions and a practical part which will a simulation based on MATLAB. The obtained results of analyzing ECG signals (the results in the case of a healthy patient and certain chronic diseases are compared) are reported.

IV.2 Theoretical part

IV .2 .1 Adaptive wavelets state

The decompositions into adaptive and non-linear wavelets capable of understanding the geometric and directional nature of signals or images. These transforms are based on lifting structures where the update operator is modified at each sample, according to a decision based on a local gradient calculated on the input signal. We are particularly interested in computing decisions in determining the necessary and sufficient conditions for these decisions to be reconstructed during the synthesis, thus allowing the perfect reconstruction of the original signal. **Source spécifiée non valide.Source spécifiée non valide.**

The works of Piella, Heijmans and Pesquet-Popescu (30) (22) (20) (31)describes a lifting structure where the update operator is modified at each sample according to a decision made on the input signal. These decisions are taken by thresholding a semi standard norm calculated on the input signal gradient, thus leading to a choice between two update filters. The authors then show the existence of necessary and sufficient conditions linking decision-making and update filters, and allowing perfect reconstruction. Although very attractive, this adaptive transform is not flexible enough because it only allows a binary decision criterion. In a 2D geometric context, we can only discriminate between two geometric events as a contour and a homogeneous region. In order to be able to take into account the richness and variety of images, it is desirable to be able to use several criteria, thus leaving a multiple choice between several update filters.

IV.2.2 Adaption possible types

The adaptive lifting scheme is a modified version of the classic lifting scheme, the adaptation consists in choosing between several filters, according to the local information of the signal, and there are two adaptive facelift structures: we start with the prediction then the update, either the update operator is applied and the prediction is shown in figure 4.1.

The initial idea would be to write :

$$h = x_0 - P_{adap}(x_e)\ (2.1) \qquad (4.1)$$

$$l = x_e + U(h) \qquad (4.2)$$

Or :

$$P_{adap}(x_e)(n) = \sum x_e(k) F_n(n-k) \qquad (4.3)$$

The order of *Fn* depends on the regularity x_e of n.

Achieving this in the conventional no prediction then no update framework is feasible, but it has at least two drawbacks:

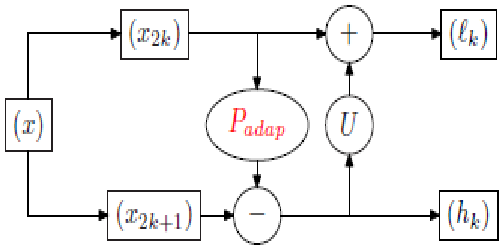


Figure IV. 1 : Block diagram of the lifting, with non-linear prediction step before updating

Calyppole (32)shows that the update structure before prediction figure IV.2 is better adapted to avoid problems of stability and synchronization

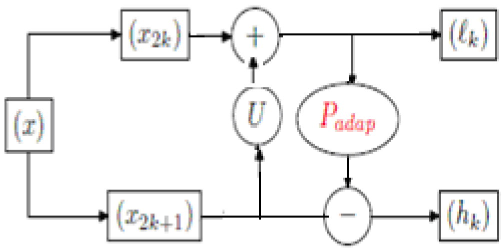


Figure IV. 2: Schematic of the lifting, with first updating step, then with a non-linear prediction step

IV.2.3 Decomposition with adaptive update

According to the work of Piella, Heijmans and Pesquet-Popescu transformed it into adaptive wavelets in lifting form, reversible and based on an adaptive update step and a fixed prediction step presented in Figure IV.3 the adaptability of the scheme is based on the choice of different update filters, depending on the local information provided by the input sub-bands. Let us first describe the general structure of the decomposition. (20) (31)

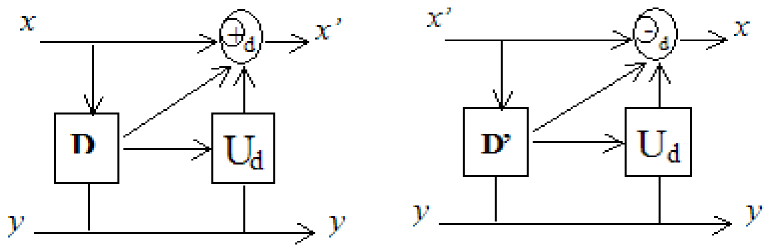


Figure IV. 3: Structure of the lifting diagram with adaptive update

IV.2.4 The General structure

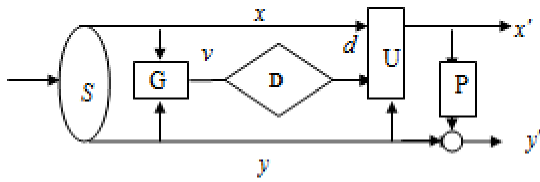


Figure IV. 4: Lifting diagram with adaptive update

From this figure we can see that the lifting diagram with adaptive update contains the following different operations:

- Operation of sharing S of the input signal into different bands (we assume that we have only one approximation band x and the detail y :{ y can be broken down into several details $y^{(1)},..........y^{(k)}$; however the illustration in the figure shows only one detail band.

-Operation to estimate the D decision map based on a grading vector.

- An adaptive update step, where adaptability appears from U dependency in the output of d decision card D.

-A prediction stepP is fixed.

In this case the input signal $x^n : Z^d \rightarrow R$ is split into two x, y signals, where y may have more than one subset: $y^{(1)},..........y^{(k)}$ with the decomposition is inversible and thus it is possible to reconstruct from its components (x and y).

- **The gradient vector:**

Either:

$y_j(n) = y^{p_j}(n + l_j)$ With j=1... N , $p_j \in \{1,....,k\}$ and $l_j \in L$ $N \leq k.|L|$ (4 .4)

Here:

$L \in Z^d$: Is a window centred on the origin

N: the number of elements in the L window.

It may be noted that the p_j are not necessarily different.

The grading vector v to the components v_j is given by the equation:

$$v_j(n) = x(n) - y_j(n) \qquad (4\ .5)$$

Withj=1 ... N

Another notation is introduced

$$v(n) = G(x, y)(n) \qquad (4\ .6)$$

- **The decision card:**

It is considered that

$$D\ (v) = [p\ (v) > T] \quad \text{with} \quad D(v) \in \{0,1\} \qquad (4\ .7)$$

p : means a semi standard.

In this case the expression $[P\ ]$ equal to one if the predicate P is true is zero if necessary.

- **The update stage:**

The decision value taken by D in position n is represented by

$$d_n = D(v(n)) \in \{0,1\} \qquad (4\ .8)$$

x'(n) is updated according to the relationship:

$x'(n) = \alpha_{d_n} x(n) + \sum \beta_{d_n,j} y_j(n)$ And $\beta_{d_n,j} = \alpha_d \lambda_{d_n,j}$

(4 .9)

Indeed it can be written by:

$$x'(n) = x(n) \oplus_{dn} U_{dn}(y)(n) = x(n) \oplus_{dn} U_{dn}(y^{(1)},....,y^{(k)})(n)$$

(4 .10)

We assume that $\alpha_d \neq 0$ For both decision values d = {0, 1} and $\mu_{0,j} \neq \mu_{1,j}$ for all $j \in \{1,.....,N\}$.

And can be inverted x'(n) in the direction of reconstructing x if x', y, d are given

$$x(n) = \frac{1}{\alpha_{d_n}}(x'(n) - \sum \beta_{d_n,j} y_j(n)) = x'(n) \Theta_{dn} U_{dn}(y)(n) \quad (4\ .11)$$

- **The prediction step:**

In the same way the prediction uses x to modify y :

$$y'(n) = p(y/x)(n) \quad (4\ .12)$$

P : refers to the prediction operator.

The prediction P is reversible, and we can recover y from x , y'

$$y(n) = p^{-1}(y'/x)(n) \quad (4\ .13)$$

4.2.5 Three-stage adaptive nonlinear lifting scheme (Principe in 1D case as an example)

We consider the lifting diagram illustrated by fig IV.5

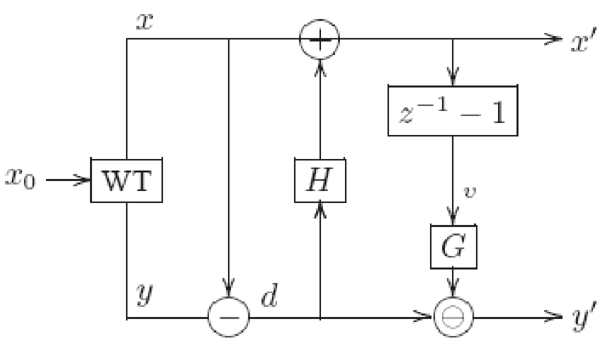


Figure IV. 5: The three steps of the Lifting Diagram (Analysis)

Here:

WT: represents lazy wavelet transform, $x(n) = x_0(2n), y(n) = x_0(2n - 1)$, and H, G are thresholding operators defined on everything $u \in R$ by:

$$H(u) = \begin{cases} 1/2\ u & si|u|\langle T \\ \alpha T / 2sign(u) & si\ non \end{cases}$$

(4 .14)

$$G(u) = \begin{cases} u & si|u|\rangle T' \\ \alpha' T' & \text{si}\,non \end{cases}$$

(4 .15)

With

T, T' : Are thresholds with positive values.

$\alpha, \alpha' \in \{0,1\}$: Two constants that determine the type of thresholding.

The signal approximation x' is given by :

$$x'(n) = x(n) + H(d(n)) \quad (4\ .16)$$

Where

$$d(n) = y(n) - x(n) \quad (4\ .17)$$

The reasoning behind this procedure is as follows. In regions where the signal x_0 is locally smooth, signal d (which can be considered as a local gradient of) will be low amplitudes and signal x will be calculated as a linear combination of the polyphase components x and y. On the contrary, near discontinuities signal d takes large values and from which x is slightly modified to prevent smoothing edges.

As shown in Fig. 2.7 the detail signal is obtained by the non-additive prediction step of lifting. Note that the subtraction standard has been replaced by a non-linear operator defined as

$$t \Theta u = \frac{t}{\beta + |u|}, \beta \geq 1 \quad (4\ .18)$$

For $t, u \in R$, the detail signal y' is given by :

$$y'(n) = \frac{d(n)}{\beta + |G(v(n))|} \tag{4 .19}$$

Where

$$.\, v(n) = x'(n-1) - x'(n) \tag{4 .20}$$

Here, $v(n)$ can be considered as an estimate of d (n).

IV.3 Practical part

In this part we will decompose our signal according the adaptive lifting desired in previous section and we will compare it with the results of the previous chapter our comparison considers on the way of reconstitution and decomposition between DWT and AL, so we propose 6 levels of decomposition and reconstitution of ECG signal.

IV.3.1 Results of processing by adaptive lifting

We took an ECG signal in case of myocardial infraction and we applied the technique of adaptive lifting in it, this is the original signal which is showing below :

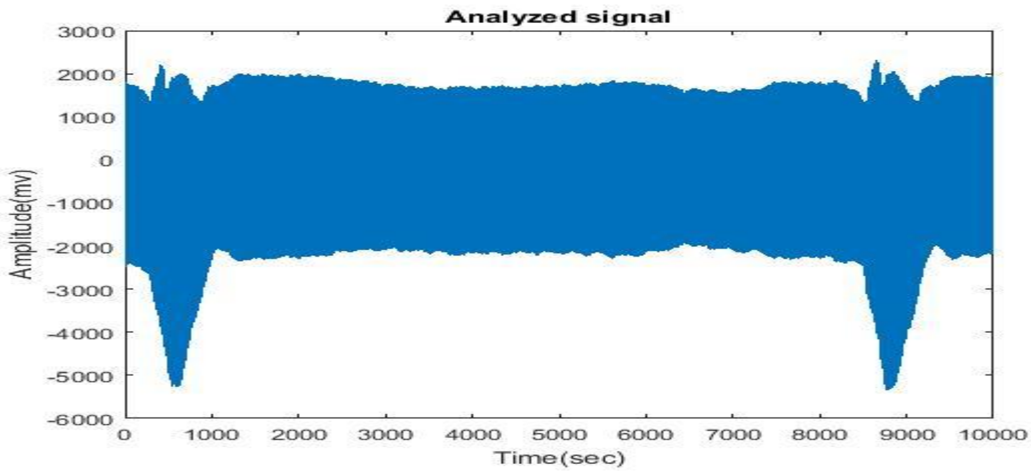


Figure IV. 6: the original signal

We will fixe the parameters of lifting in this case on :

T=1, T1=0,2, β=0,5 , α=α1=0,2,fs=1000 Hz ,N= 10000

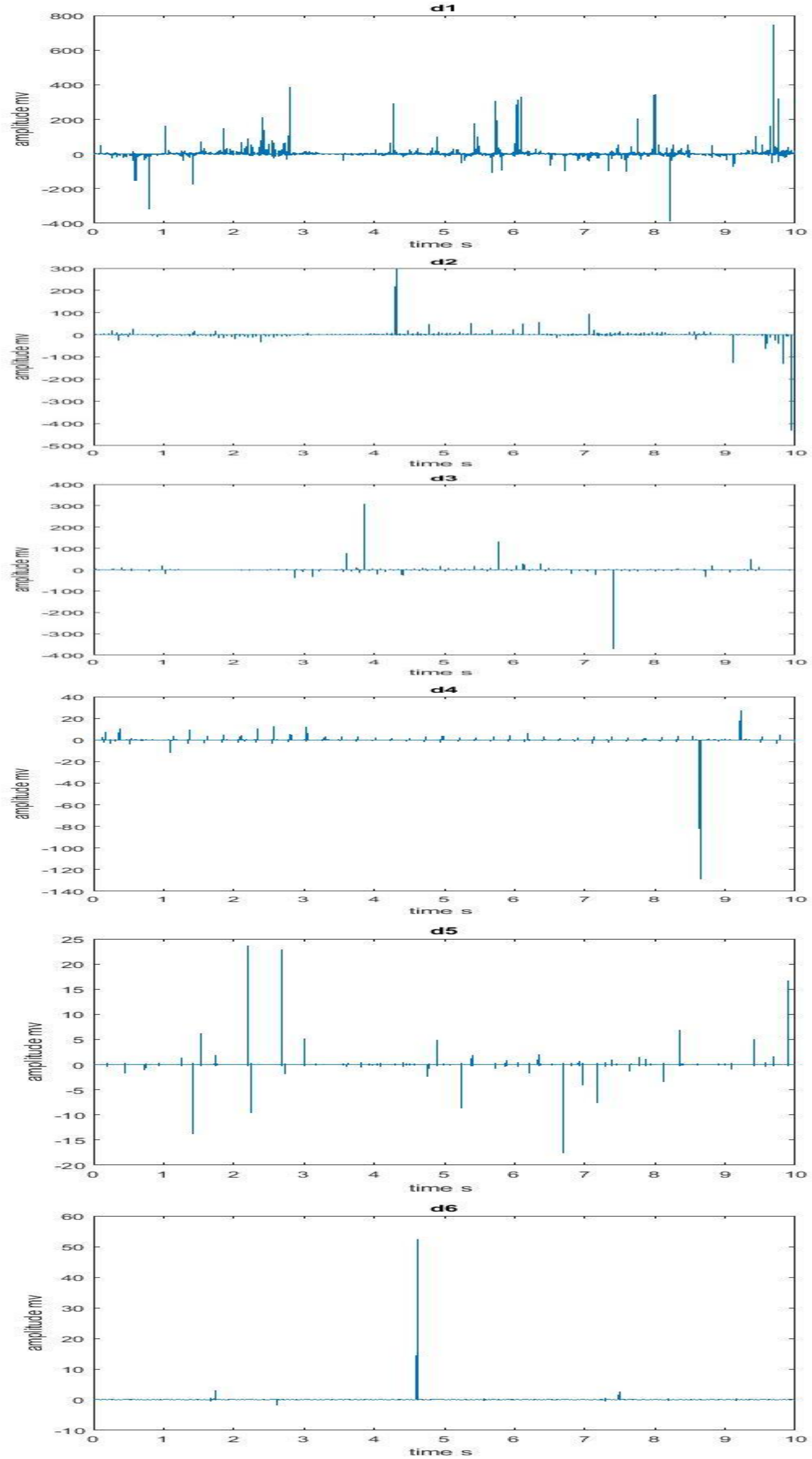


Figure IV. 7: Details of decomposition by adaptive lifting (6levels)

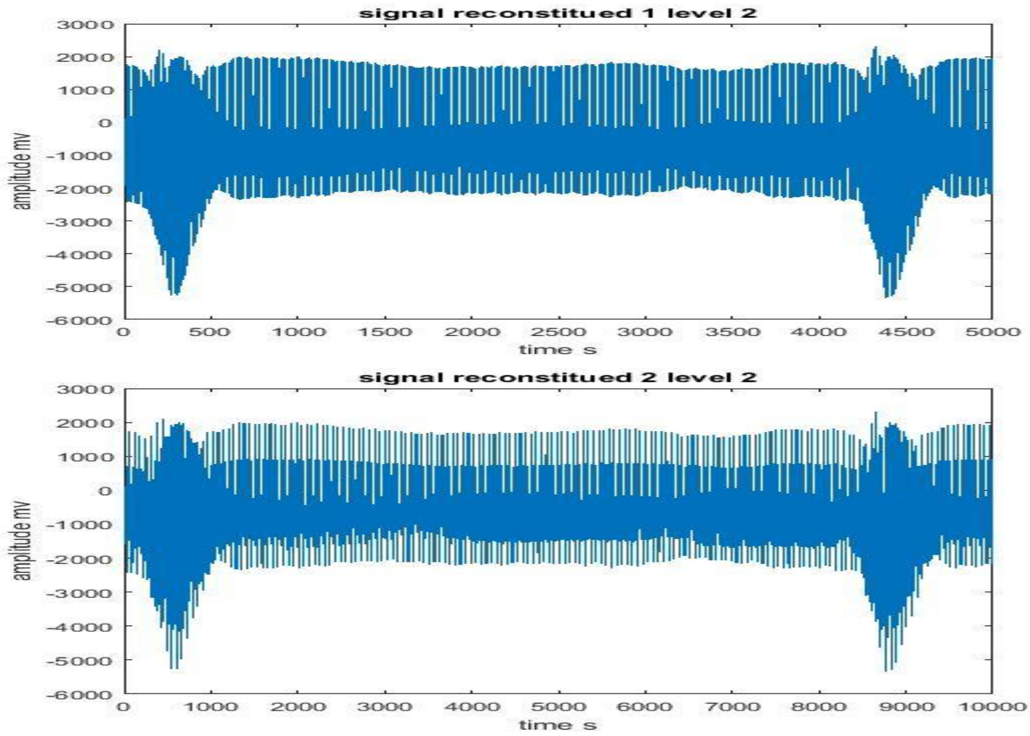


Figure IV. 8: Reconstituted signal after 2 phases (phase 1, phase 2)

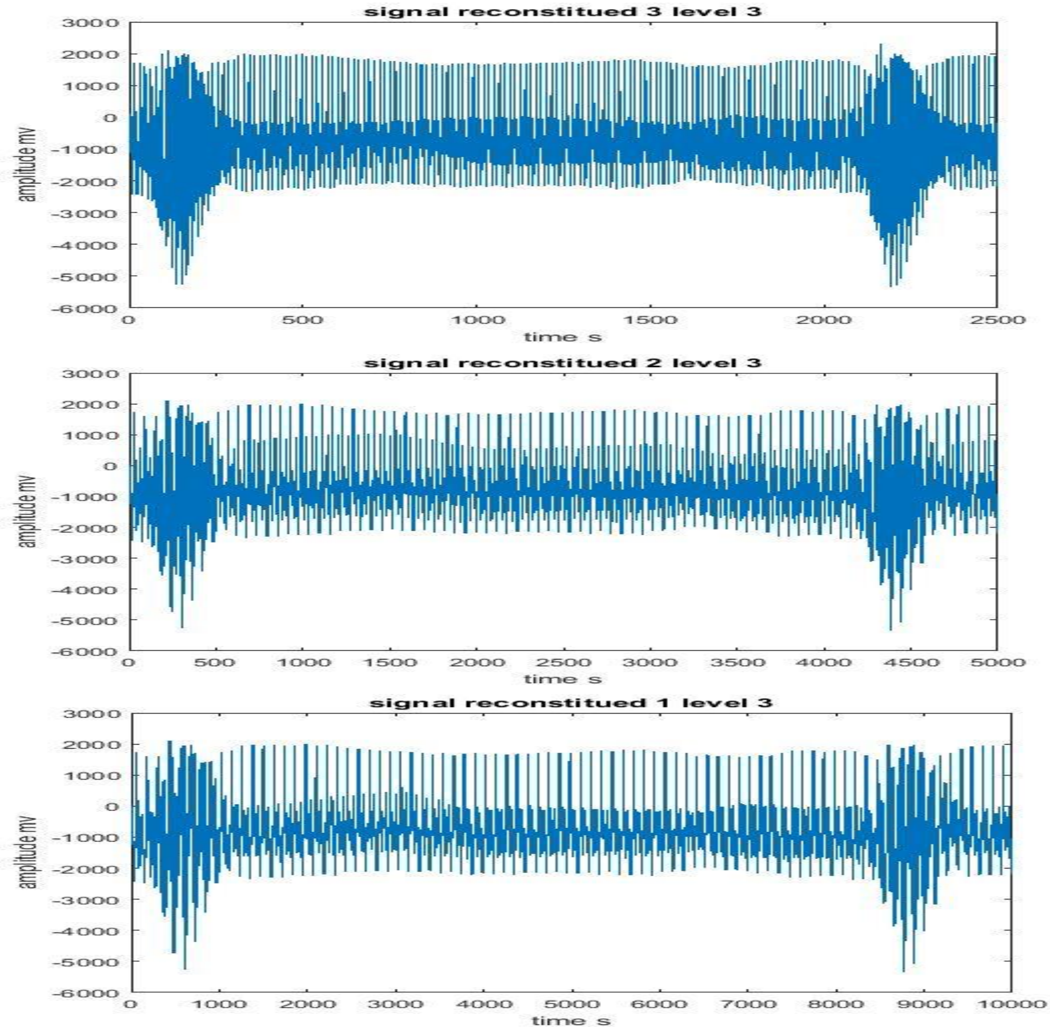
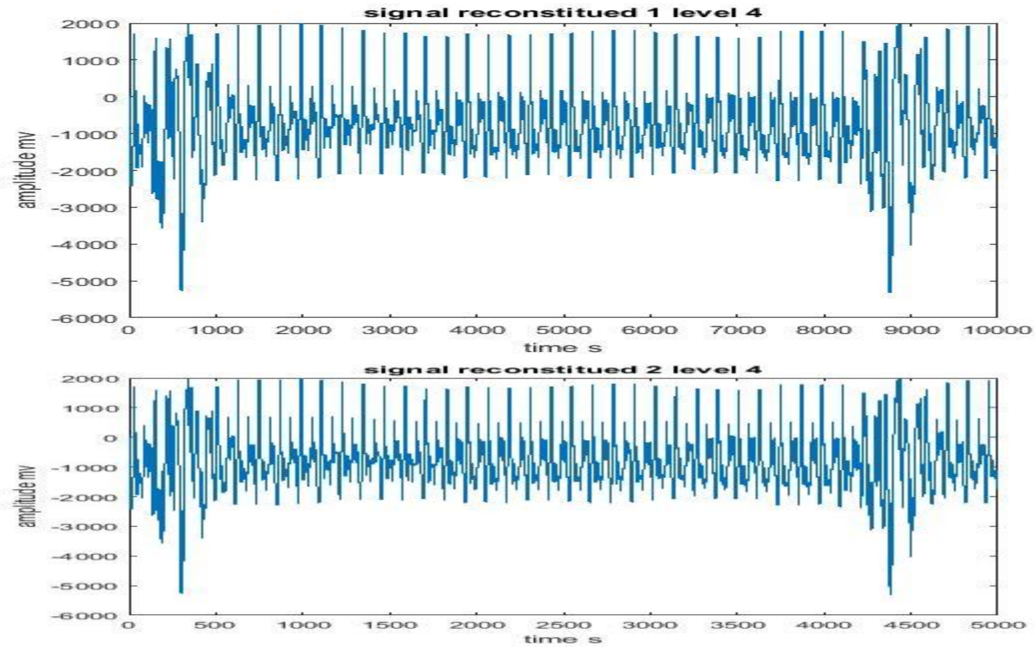


Figure IV. 9: Reconstituted signal after 3 phases (phase 3, phase 2,and phase 1)



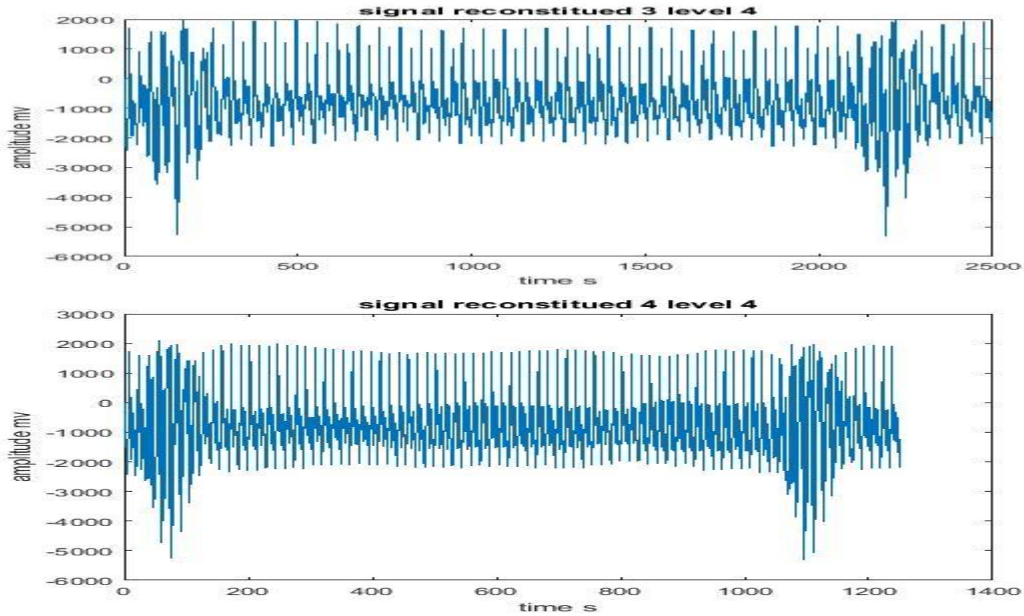
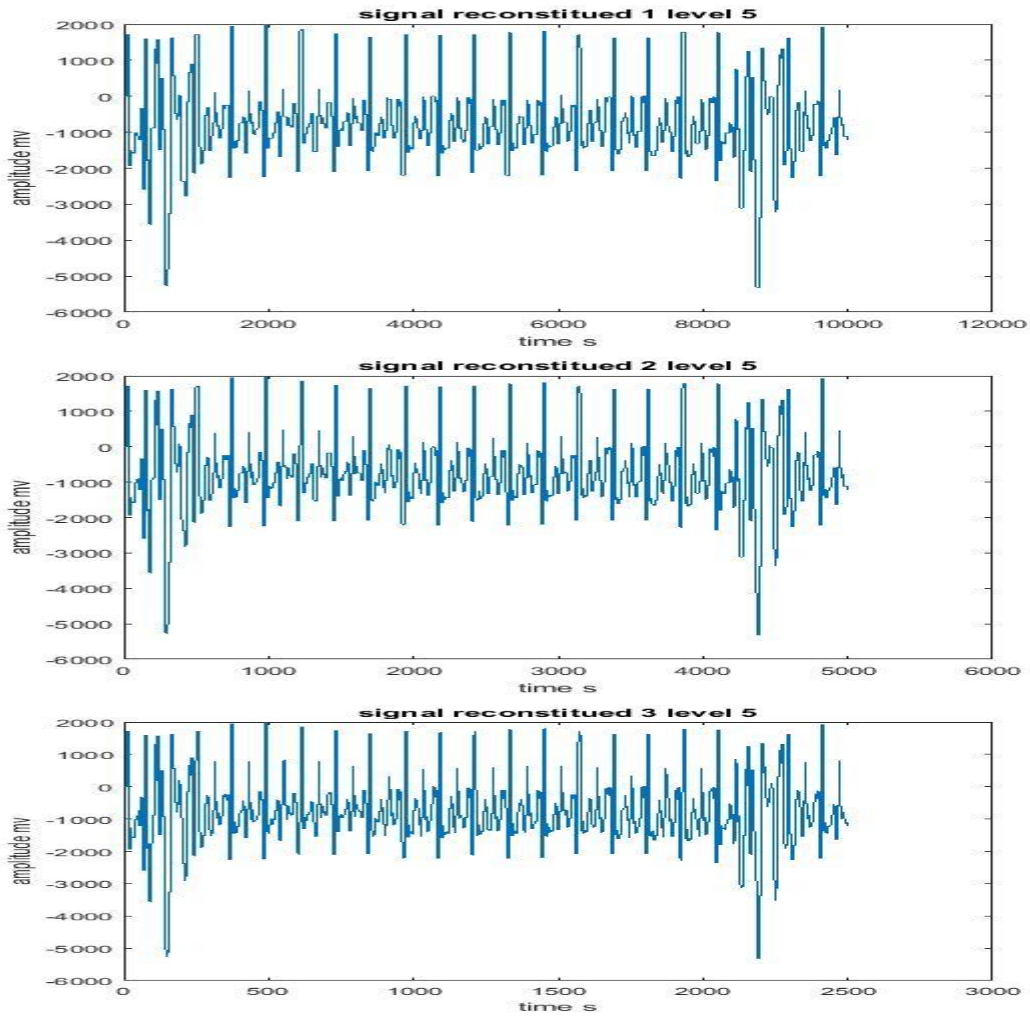


Figure IV. 10: Reconstructed signal after 4 phases (phase 4, phase 3, phase 2, phase 1)



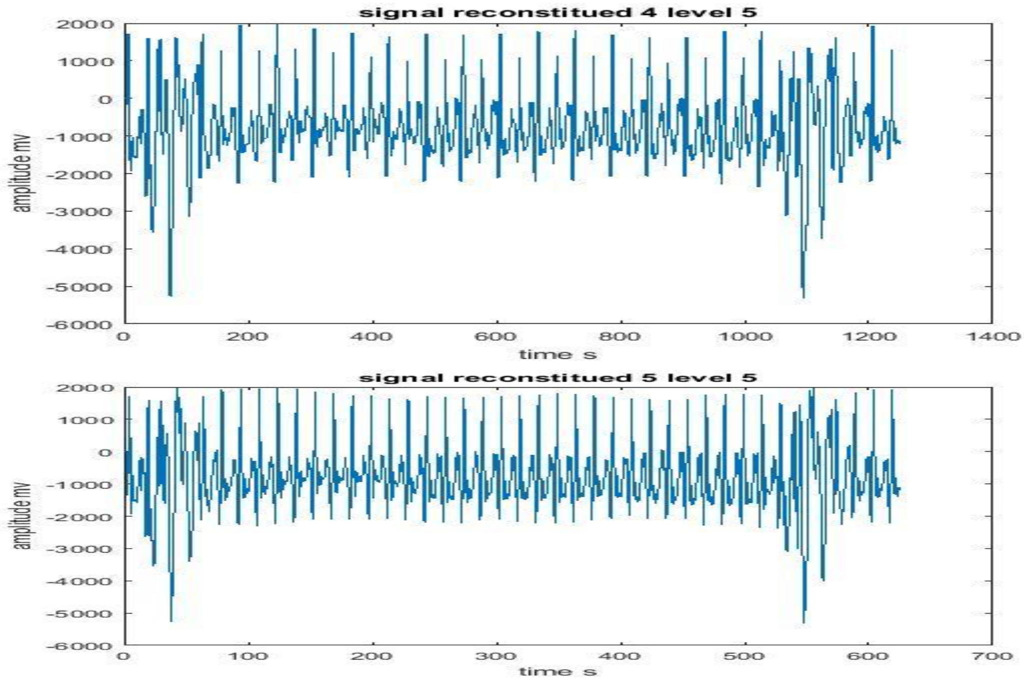
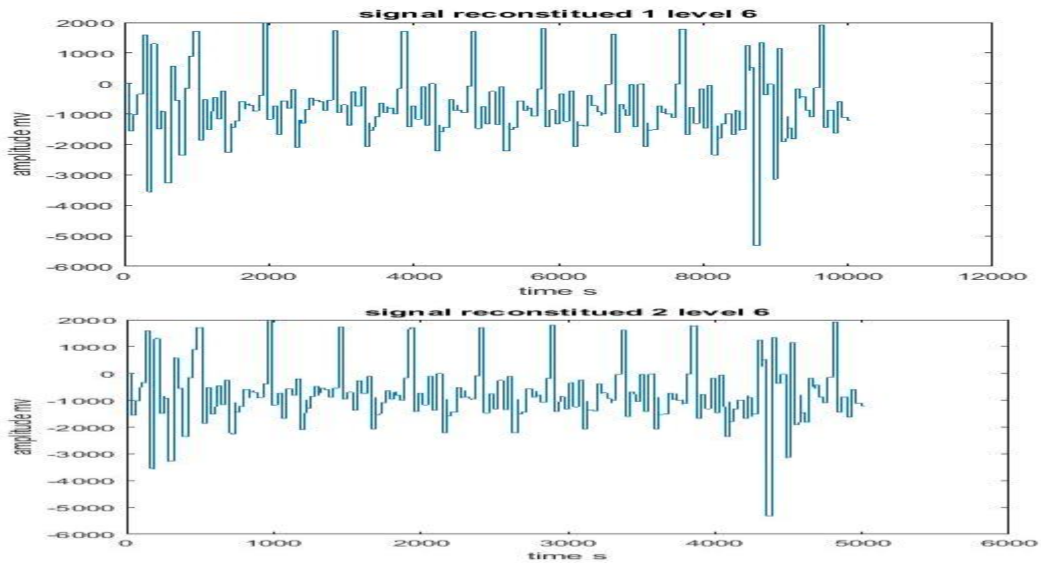


Figure IV. 11: Reconstituted signal after 5 phases (phase 5, phase 4, phase 3, phase 2, phase 1)



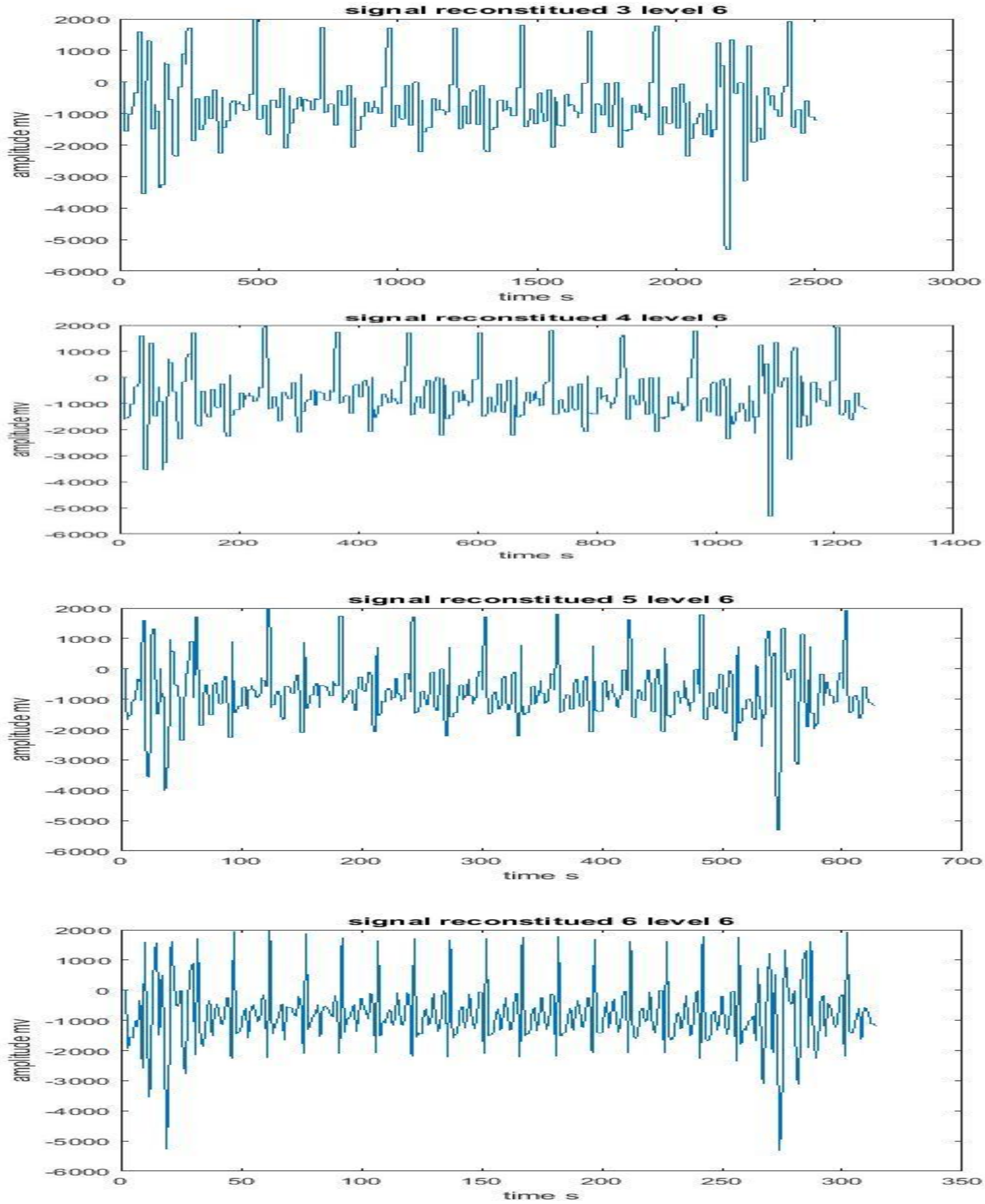


Figure IV. 12: Reconstituted signal after 6 phases (phases 6,5,4,2,1)

IV.3.1.1 Discussion and analyse the results

After applying AL decomposition on ECG signal by 6 levels of decomposition which is representing the process of signal analysis, after that we inversed the process which we can call it the synthesis of signal.

The decomposition, decomposes signal into 2 parts the first called the approximation and the second called the detail, by evaluating levels the process consider on completing the mechanism in the next approximation from 1 to n level (n=6 in our work), and also we remark the this process reduces the frequency into the half after every decomposition, whenever the level of decomposition goes higher, the frequency reduces into the half in every time so:

$$f(n+1) = \frac{f(n)}{2} \qquad (2.21)$$

n: number of level

And that is appearing in the figures demonstrated previously.

In the approximations we remark that after the continuous decompositions the filtering goes to be more efficacy in clearing the signal and this last become more real and be more similar to the original signal .

The detail goes to minimise continuously and this is belong to the advancing of the approximation (and also the vice-versa), because the decomposition process consider on the approximations.

The reconstitution of signal (synthesis),goes to be more complicated when the decomposition levels evaluate, and after the fourth level 4 the signal ECG goes to get the perfect reconstitution because of evaluating levels causes the evaluating of filtering quality and that appeared in the level 6, but we should know that after every decomposition n , the process of reconstruction increases the signals reconstructed phases and for example 1 level we need only one phase to reconstruct signal , in case of 4 levels we need to do 4 phases continuously for generation for the decompositions to address the reconstructed final signal, and that is appear in the figures previously, and in those phases to signal goes to be more denoise.

we remarked that in the last 2 levels our reconstituted signal goes to be more denoised and more identical to the original signal (ECG in case of myocardial infraction).

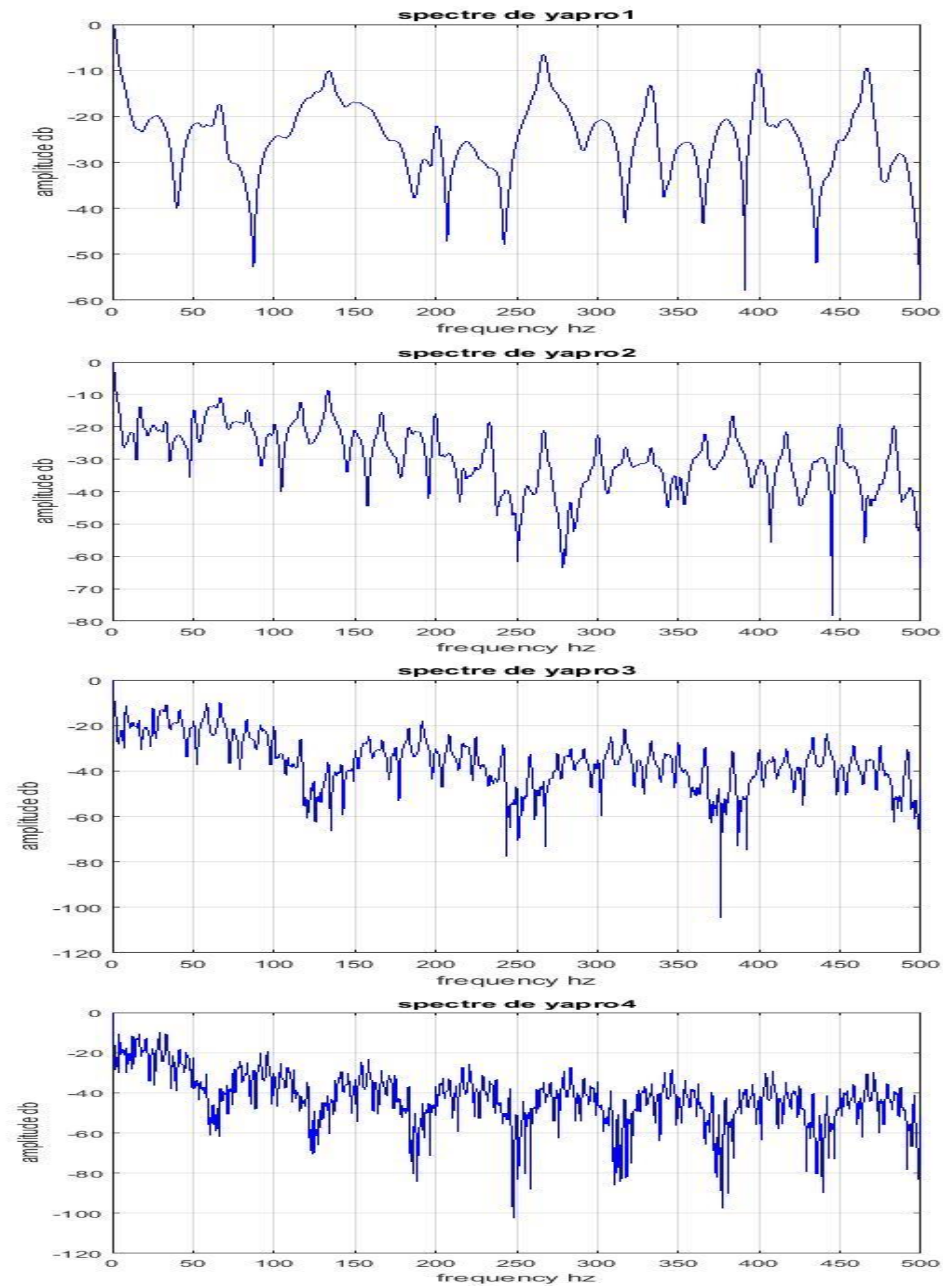
This is a schedule shows the relation between the details and the approximations and also N :

Table IV. 1 : Relation between a and d and N in 6 levels of decomposition

Levels	**approximations**	**details**	**N**
1	a1	d1	5000
2	a2	d2	2500
3	a3	d3	1250
4	a4	d4	625
5	a5	d5	312.5
6	a6	d6	156.25

4.3.2 Results of the reconstructed signal in the levels by FFT:

This comparison addresses the difference between frequencies between the phases of the reconstructed signal and we will take the level 6 as an example for this process :



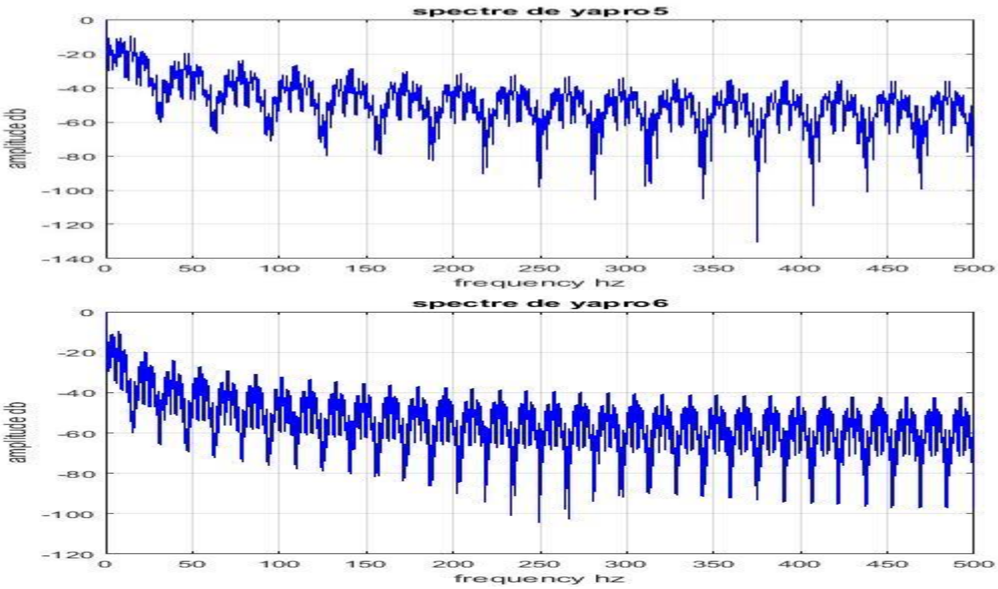


Figure IV. 13: FFT of reconstituted signal by 6 phases (phase 1, 2, 3, 4, 5, 6)

we remark in these figures that the process of reconstitution of the signal goes to be develop and optimise phase after phase and also the filtering of low pass band gets rid of high frequencies continuously quickly (speed up), so at the last level we will see a perfect filtering (high band frequencies get rid), and also we remark that the factor N increase after each level by multiplying in double.

we see that the filtering of signal influences by the level of decomposition, evaluating level makes the filtering more efficacy and helps to get a perfect reconstitution of signal, so if we decompose only in 1 or 2 levels, we don’t wait for a good filtering and reconstitution by comparing if we do the same process with 5 or 6 levels, so we can say the evaluating of decomposition levels causes the perfect signal synthesis (reconstitution of the signal).

IV.4 Comparison between the detail’s FFT of ECG signal cases

We took samples of ECG signal cases (cardiompathy, dysrhythmia, myocardial infraction), and after that, we made the FFT of their details from the decomposition which is based on adaptive lifting, and we made a comparison analytic for trying to see how much is possible to detect the case just from this FFT, and how much we can make this method large, and efficacy and what is the relation with number of decomposition and the power, efficacy of this technique, and that will be appear in these figures which they took from level 1 to 6:

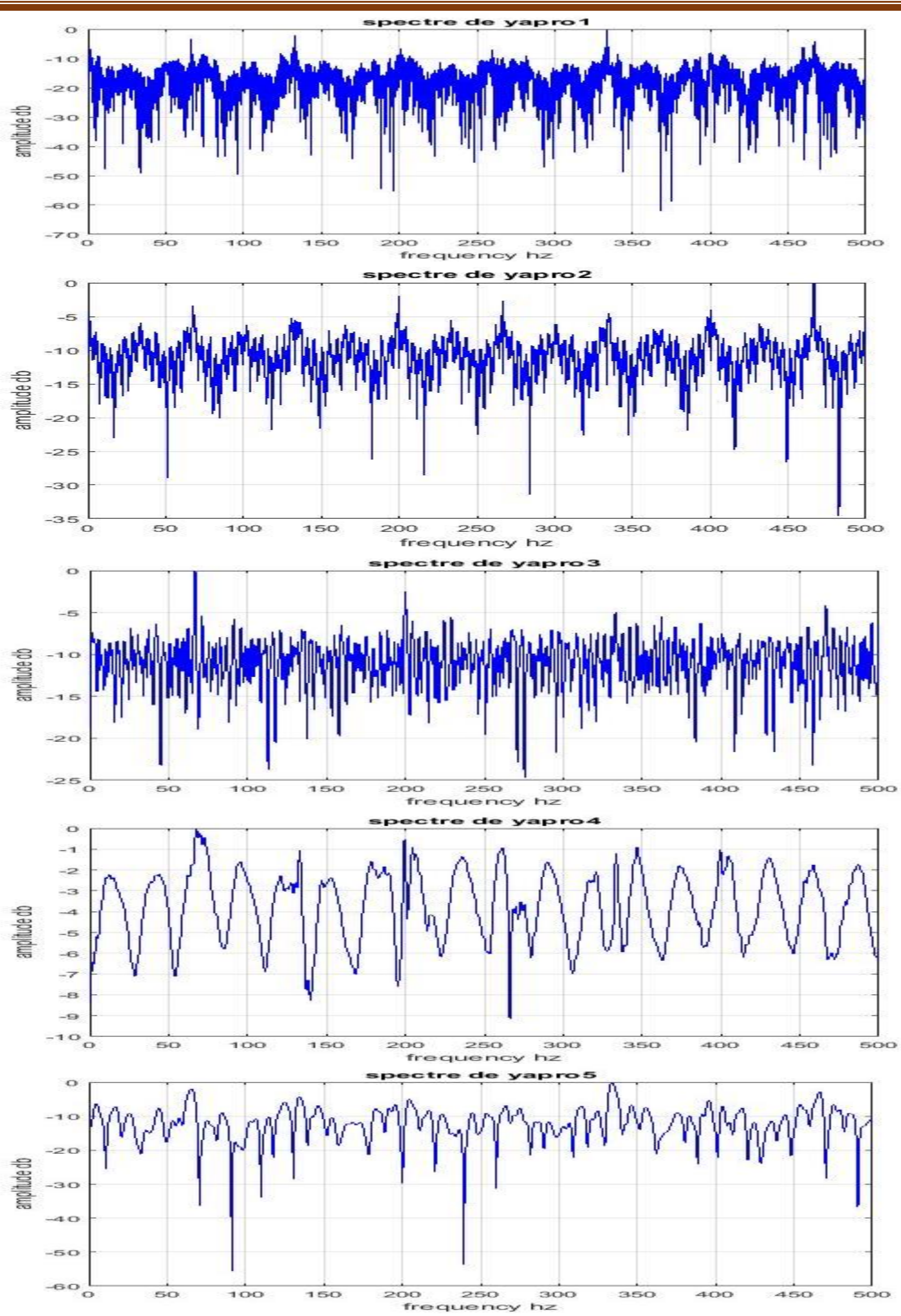


Figure IV. 14: FFT of detail AL of myocardial infraction

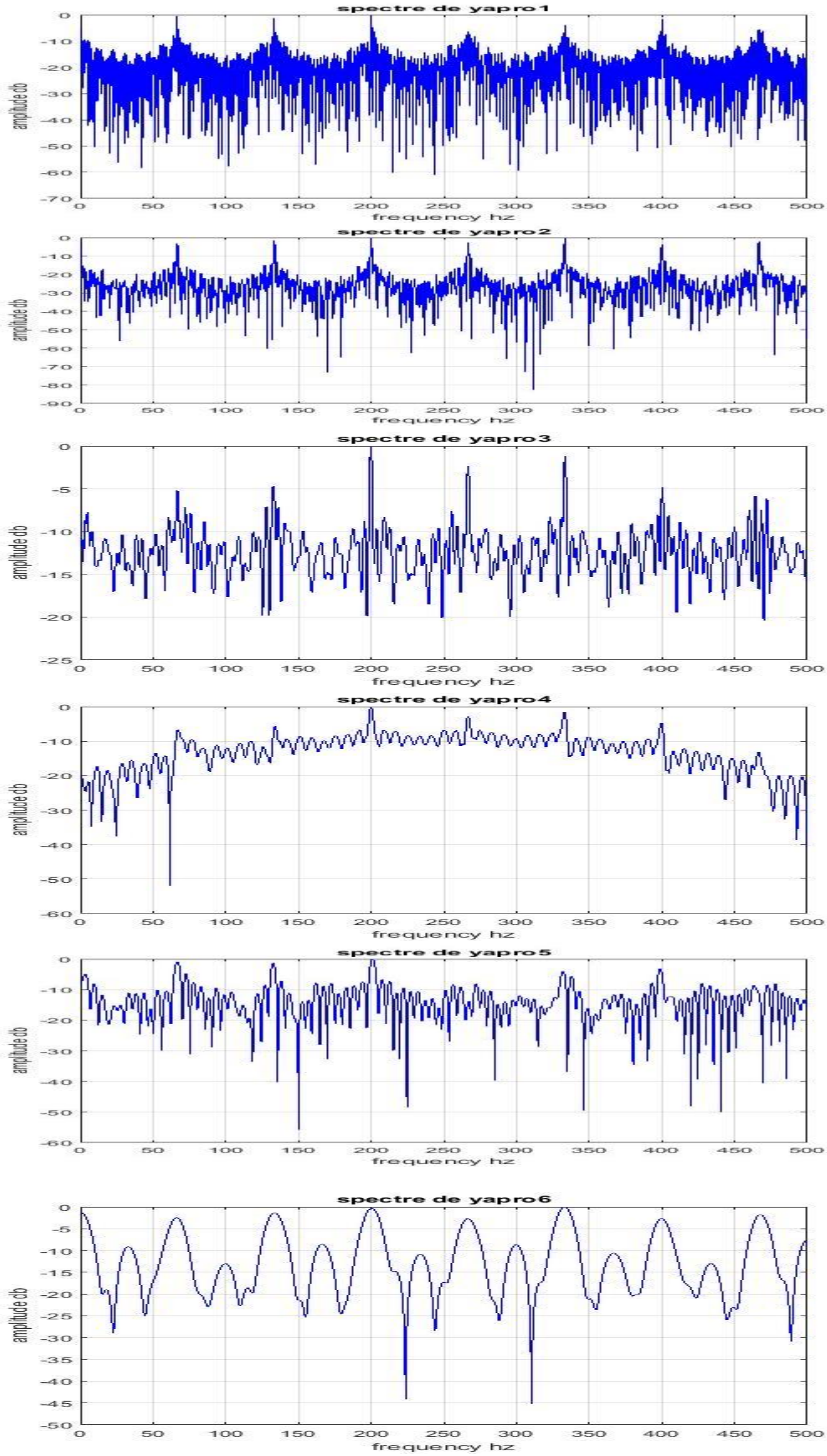

spectre de yapro1
amplitude db
frequency hz
spectre de yapro2
amplitude db
frequency hz
spectre de yapro3
amplitude db
frequency hz
spectre de yapro4
amplitude db
frequency hz
spectre de yapro5
amplitude db
frequency hz
spectre de yapro6
amplitude db
frequency hz

Figure IV. 15: FFT of detail AL of cardiompathy

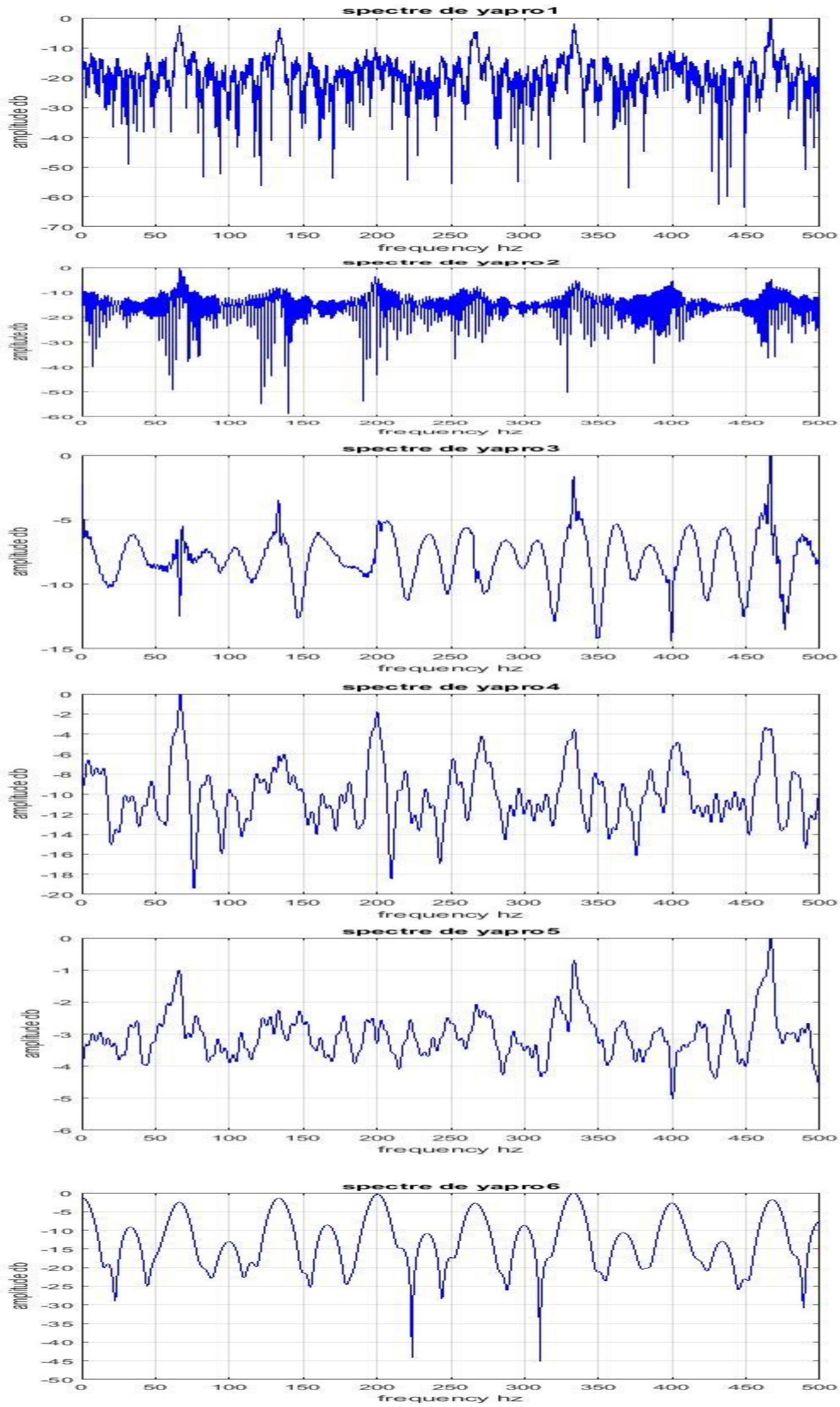


Figure IV. 16: FFT of detail AL of dysrhythmia

IV.5 Comparison between FFT of details in case of AL and DWT

After comparing between FFT of details in case of AL and DWT, we remarked that is possible and efficacy to detect and determine the ECG pathologies, this is based on identic frequencies and their amplitudes, but what is important than determine it is how much possible we can continue by this rhythm, in case of DWT we see that in level 3 of decomposition by db3 after that we can determine the pathology, and that was appear when the FFT of DWT cases even the pathologies signals are different but the spectrograms stay similar even we add new levels, to be more clear if we suppose that we will going to detect 50 pathology for example, the problem is that the determining of this grand number of cases needs many levels to go deeper inside frequencies variations so according of our experiment, the DWT is not capable to do that,in addition the slow makes that more complicated so what we can do it with 3 levels of AL surpasses what we can do it by DWT, oppositely the AL is more capable because whenever we go far in decomposition levels, the FFT details in this case stay different so that what will make our capability to determine more cases possible, but we should know that we have to be more specific in our studies so we need to choose all of our parameters carefully and exact.

IV.6 Conclusion:

The adaptive lifting diagram is a modified version of the classic lifting diagram; the adaptation consists in choosing between several filters based on the local information of the signal; we have taken this case into consideration because research shows that the update structure before the prediction is better suited to avoid problems with stability and timing.

This technique guarantees the best reconstruction and design ways for researchers, the AL is efficacy in reconstruction signals (synthesis), so we can get our information without loss information, from the other hand the denoising of AL is so fast (speed up) without forgetting how much it makes the calculations simplified, and also we can say in our experiment it was so practical and efficacy in the continuity of determining ECG pathologies in a large way.

Final conclusion

In this study we had given a general conception about the heart functioning and the ECG signals which was the axe of our study, starting from the anatomic structure of the heart, finishing with explained its conductional system, with showing the normal ECG waveform's construction (QRS complex, ST segment, P wave, T wave…etc.), this was the content of the first chapter.

In the second chapter we went far into the theory of wavelets and consider on the conception of wavelet transform, which is a capable tool for giving a multi-resolution representation of a monodimensional (1D) signal; it has been a major success in different fields of application such as image and signal processing, data compression and transmission, numerical solutions of differential and integral equations, etc. The reasons for the successful transformation into wavelets are that for a class of functions, most of the information contained in the function is concentrated in a small number of wavelet coefficients.

In the third chapter we made our practical part of this study which is analysing ECG signals of cases (myocardial infraction, dysrhythmia, cardiompathy), of course the comparison bases on analyse these signals by discrete wavelets (Haar, db, symlet), by decomposition into levels and getting approximations and detail, also trying to blend this technique with FFT algorithm to determine cases according this blended process, and verify if this tool (DWT), is capable to be more efficacy in showing the ECG pathologies, and how much this method work on case where we will have many pathologies, after the experiments with MATLAB tool, we saw that this technique works but at the same time is bounded and limited.

so for this reason we defined in the fourth chapter a tool which the adaptive lifting diagram which is a modified version of the classic lifting diagram; the adaptation consists in choosing between several filters based on the local information of the signal; we have taken this case into consideration because research shows that the update structure before the prediction is better suited to avoid problems with stability and timing.

This technique guarantees the best reconstruction and design ways for researchers, the AL is efficient in reconstruction signals (synthesis), so we can get our information without loss information, from the other hand the denoising of AL is so fast (speed up) without forgetting how much it makes the calculations simplified, and also we can say in our experiment it was so practical and efficacy in the continuity of determining ECG pathologies in a large way, but if we want a good offers in the future we need to optimise this technique with advanced algorithms like the metaheuristics, and this optimisation would make the results of detecting pathologies more efficacy and exact as much possible, and we can say that this technique is not offered for the ECG only, it can work with the other biomedical signals and other signal branches and fields.

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