Flow depth computation at the toe of an over-flow dam in steeply-sloping case

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Abstract

Using a definition sketch, a theoretical development is proposed to define the functional relationship between the various parameters of the flow at the toe of an overspill dam. After conversion to dimensionless form, the USBR diagram for determining the velocity at the toe of a steeply-sloping spillway is replaced by a single curve. With the data collected *in situ* by USBR, the proposed theoretical equation is adjusted to allow for friction along the downstream face of the spillway. Finally, a convenient relationship for calculating the depth of flow at the downstream toe of an overspill dam with a steeply-sloping downstream face is proposed. It concerns steep slopes ranging from 1 on 0.6 to 1 on 0.8.

Keywords: Spillway, subcritical flow, supercritical flow, critical depth, head loss

1. Introduction

The need to calculate flow depth is a common problem for practising hydraulics engineers (Chow, 1981) and (Sinniger, 1989). Flow depth is a fundamental parameter in the design of free surface-flow pipes and canals, and several methods have been proposed in the past for such structures, for both uniform and gradually-varying non-uniform flow (Lencastre, 1999). The analysis is based on a rigorous theoretical development and generally produces very satisfactory results.

When the problem is to compute the flow depth at the toe of an overspill dam, it becomes more complex and there are practically no methods available. No theoretical development has been able to yield an acceptable estimate of the depth at this type of structure, the main reason being the geometry of the downstream face of the dam. Various shapes exist for different types of dam and they always result in a flow which is non-uniform spatially, making it a complex task to compute the head loss due to friction using the standard general hydraulics equations.

Among the spillway shapes encountered are the Creager (1929) based on experimental data collected by Bazin (1888-1898), Scimemi (1930), WES profiles (1952) and Creager and *al.* (1966).

As in the case of free-surface flow in conduits and canals, flow depth at the dam toe is an important feature in the design of any structure planned downstream of the dam itself (Smetana, 1948; 1949). Such is the case with the stilling basin (Achour, 2002) for the energy dissipation of the flow and every other type of discharge works. Designing a stilling basin requires the *Froude* number of the incident flow, which is strongly dependent on the initial flow depth. The velocity of the incident flow is another basic factor in estimating the tractive forces acting on the bottom surface of the stilling basin or discharge structure. In addition, computing the backwater curve of the non-uniform flow which may occur on the discharge structure downstream of a spillway is fundamentally dependent on knowing the initial depth of flow. This is the starting point of the backwater curve and coincides with the flow depth at the downstream toe of the dam. These examples illustrate the importance attaching to estimating the flow depth as accurately as possible. We decided to select the steeply-sloping overspill dam for our study because it represents a type in widespread use. This is not a chance decision nor an arbitrary one, but constitutes one of the key components in our contribution.

The only practical approach to computing the depth of flow at the downstream end of a steeply-sloping overspill dam is the USBR method (1955). Its special feature is its practicality, being based on field data collected from existing dams, including tests at Shasta and Grand Coulee dams in the USA. It estimates the incident flow velocity when the flow characteristics and the geometry of the overspill dam are known. The disadvantage of the method is certainly that it is a graphic approach which may give rise to errors in reading the required parameters, especially when interpolation is necessary. The method recommended by USBR (1955) is then a graphic method which must be used with great caution. But all the disadvantages that the use of the method can cause are particularly covered by our present study. The USBR method (1955) has always been considered a graphic method but our study aims to convert it to a semi-analytical method. The graph is replaced by a semi-empirical equation by combining a simplified theoretical approach with experimental data from the USBR field work (1955). By a suitable change of variable, the USBR graph (1955) is replaced by a single dimensionless curve, and then a generalised equation. A least squares fit confirmed the validity of our approach.

2. Theoretical Study

Taking an overspill dam and ignoring the steepness of the downstream face and the effect of friction on it, we can write a theoretical dimensionless equation containing all the hydraulic and geometrical parameters of the flow. It is dimensionless so that it can be generalised, under the same conditions in which it was originally determined. Figure 1 is a schematic representation of the flow over a parabolic dam crest. The flow is characterised by the head on the sill H, the upstream approach velocity head H_a, tailwater depth h_1 and mean velocity V_1. The head loss due to friction on the downstream face of the overspill dam is represented by h_f whereas Z is the vertical distance from reservoir level to the floor at the downstream apron. Friction head loss h_f can be ignored if the spillway always operates at design head. The total head line is also shown on Figure 1. Its general shape has been arbitrarily drawn because it is not possible to define it accurately. Friction head losses are also difficult to estimate in the current state of knowledge, because they are governed by the geometry of the downstream dam face which is not a straight line and the depth of flow on it. Assuming s as the geometric height of the overspill dam reckoned from the downstream apron to the crest sill, one may write from figure 1:

$$Z = s + H \tag{1}$$

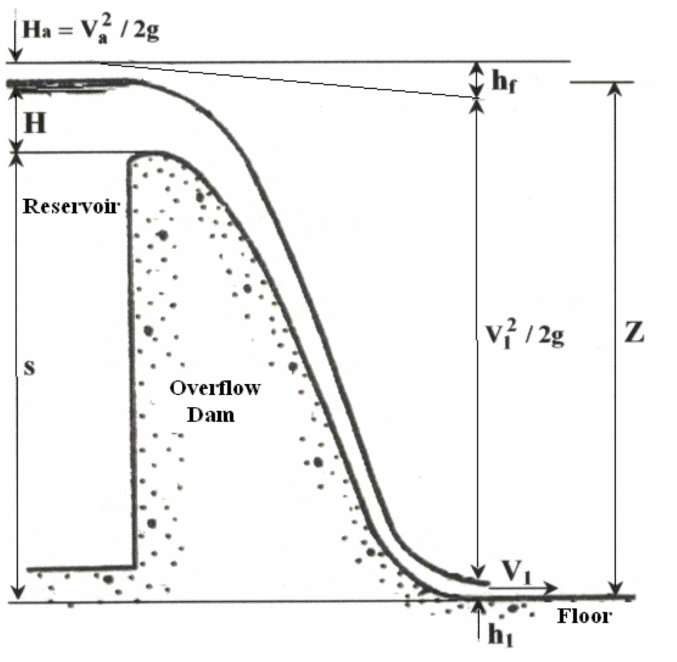

Fig.1. Components for writing the functional relationship f(Z, H, h_1) = 0

The flow is subcritical on the upstream side of the dam, in the reservoir, becomes critical at the dam crest and supercritical down the downstream face of the dam. The head H in Eq. (1) can therefore be considered as the critical head H_c provided the approach velocity of the flow is insignificant, meaning that the relationship between the geometric height s of the spillway and the design head H_d is $s/H_d > 1.33$ (USBR ,1948).

Applying Bernoulli's equation over the section from the reservoir to the point where depth is h_1 , yields the following relationship:

$$h_1 + V_1^2/2g + h_f = s + H + H_a \tag{2}$$

Where g is the acceleration due to gravity. Notice that Eq. (2) can also be established by equalising upstream and downstream heads. Neglecting the head loss h_f for the reasons previously mentioned and assuming $H_a \cong 0$, Eq. (2) can be simply rewritten as:

$$h_1 + V_1^2/2g = s + H \tag{3}$$

Inserting $V_1 = q/h_1$ into Eq. (3), where q is the discharge per unit width, yields:

$$h_1 + q^2/(2gh_1^2) = s + H \tag{4}$$

Since the head H is considered as the critical head, one may write:

$$H = 3h_c / 2$$

Where h_c is the critical depth.

Bearing in mind that:

$$h_c = (q^2/g)^{1/3}$$

The head H can be then written as:

$$H = (3/2)(q^2 / g)^{1/3} \tag{5}$$

Eliminating q between Eq. (4) and (5), results in:

$$h_1 + 4H^3/27h_1^2 = s + H \tag{6}$$

After simplification, Eq. (6) is reduced to the following cubic equation:

$$h_1^3 - (s + H)h_1^2 + (4/27)H^3 = 0 \tag{7}$$

Introducing the theoretical non-dimensional parameters $\psi_t = h_1/Z$ and $\phi_t = H / Z$,

Eq. (7) can be then written as:

$$\psi_t^3 - \psi_t^2 + (4/27)\phi_t^3 = 0 \tag{8}$$

It is obvious that both ψ_t and ϕ_t are less than unity since $h_1 < Z$ and $H < Z$. Furthermore, ψ_t and $\phi_t = H/Z$ vary within the following ranges $0 \le \psi_t < 1$ and $0 \le \phi_t < 1$ respectively. On the other hand, it is also evident that $h_1 < H$, implying $\psi_t < \phi_t$. Using trigonometric functions, Eq. (8) can be solved analytically and it found that its discriminant is given by:

$$\Delta = (2/27)^2 \phi_t^3 (\phi_t^2 - 1)$$

Bearing in mind that ϕ <1, one may write $\Delta < 0$ and Eq. (8) admits then three real roots. The latter were rigorously analysed and it is found that two of them have no physical meaning. The solution of Eq. (8) is finally obtained as:

$$\psi_t = (1/3)\left[1 + 2\cos(\alpha/3)\right] \tag{9}$$

Where:

$$\alpha = \pi + \cos^{-1}(2\phi_p^3 - 1) \tag{10}$$

3. Dimensionless practical relationship

The USBR diagram (1955) is very practical means of estimating the mean velocity V_1 from known values of H and Z. It was derived from field tests on existing operational overspill dams, which confers undoubted reliability and validity. The friction head losses down the downstream dam face are accounted for, since the data was collected *in situ*. The steeply-sloping downstream face of an overspill dam ranges from 1/0.6 to 1/0.8. The head on the sill is in the range $0.76\text{m} \le \text{H} \le 9\text{m}$. The geometric height s at overspill dams is $2.90\text{m} \le \text{s} \le 150\text{m}$. Mean flow velocity V_1 may be as high as 44 m/s approximately.

In this section, let us assume ψ_p and ϕ_p as the practical values of the non-dimensional parameters ψ and ϕ previously defined. By the use of USBR data (1955), ψ_p and ϕ_p can be computed as follows:

1. For a given head H, also considered as the critical head, the critical depth h_c is estimated using the well-known equation $h_c = 2H/3$. Once h_c is determined, the discharge $q = \sqrt{g h_c^3}$ per unit width follows then immediately. For the same value of H, USBR data give the main velocity V_1 for each value of the tested parameter Z.

2. Knowing q and V_1, the toe depth h_1 is easily evaluated, since $h_1 = q/V_1$. Thus, knowing the values of both Z and h_1, the non-dimensional parameter $\psi_p = h_1/Z$ is well defined.

3. Similarly, with the known values of both H and Z, the non-dimensional parameter ϕ_p is then computed.

4. Repeating the steps 1 to 4 for each value of the head H considered by the USBR, the corresponding values of ψ_p and ϕ_p are finally determined (Table 1).

Figure 2 shows the variation of ψ_p with respect to ϕ_p, for all considered values of the head H (solid signs) and one could easily drawn an average curve. This implies that the USBR diagram is reduced to a single dimensionless curve when considering the non-dimensional parameters ψ_p and ϕ_p. On the other hand, figure 2 indicates that ψ_p increases with the increase of ϕ_p. In table 1 are also indicated the values of $\psi_t(\phi_p)$ computed using Eqs. (9) and (10), after substituting ϕ_t by ϕ_p. The variation of ψ_t (ϕ_p) is also represented in Fig.1 (open signs).

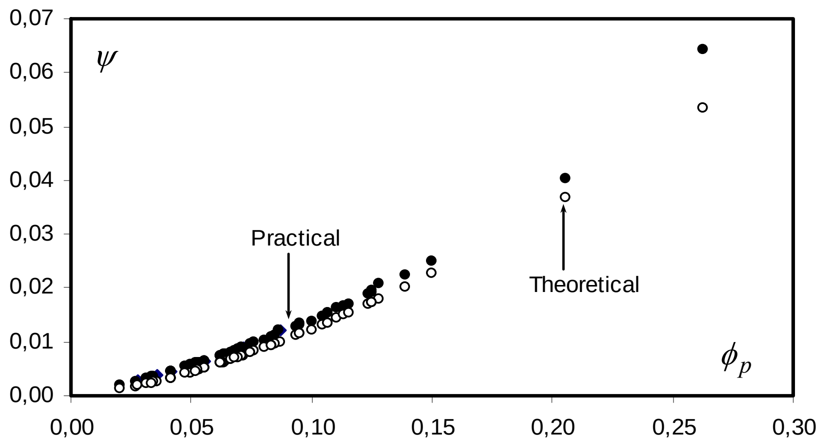

Fig.2. Variation of $\psi(\phi_p)$. (•) $\psi_p(\phi_p)$ resulting from USBR data.
(o) $\psi_t(\phi_p)$ resulting from the theoretical Eqs. (9) and (10).

Figure 3 compares then the theoretical values of ψ, obtained from Eq. (9) for $\phi_t = \phi_p$, with the practical values resulting from the USBR data. As it can be seen, deviations between the two may become significant for large ϕ_p. It can be observed that $\psi_p > \psi_t$ for all the considered range of ϕ_p. This confirms that, for a given value of ϕ_p, the practical depths h_1 are greater than the theoretical ones obtained from Eq. (9). This fact was expected, since the effect of the head losses was ignored when writing Eq. (9) which must be then corrected. This can be possible when regarding $\phi_p(\psi_t)$ plotted in figure 3, for all the considered values of ϕ_p.

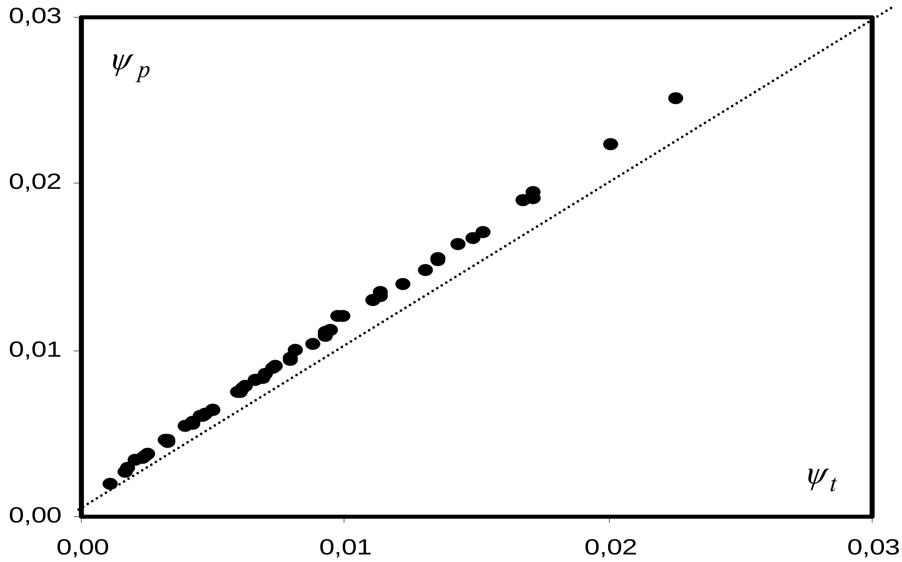

Fig.3. (•) Variation of ψ_p versus ψ_t; (.) For $\psi_p = \psi_t$.

As it can be seen, $\phi_p(\psi_t)$ is represented by a single curve which seems to be well correlated to a linear law. By a conventional statistical analysis, it is found that the non-dimensional parameter ψ_p may be approximated as:

$$\psi_p = 1.069\psi_t + 10^{-3} \tag{11}$$

Eq. (11), valid for the wide range $0.02 \le \phi_p \le 0.20$, was obtained with R^2 greater than 0.999. Furthermore, inserting Eq. (9) into Eq.(11), the following explicit $\psi_p[\alpha(\phi_p)]$ relationship is deduced :

$$\psi_p = 0.356[1 + 2\cos(\alpha/3)] + 10^{-3} \tag{12}$$

Where $\alpha(\phi_p)$ is given by Eq. (10). Provided H and Z are given, Eqs. (10) and (12) allow then a direct calculation of the non-dimensional parameter ψ_p, once $\phi_p = H/Z$ is determined. The toe depth $h_1 = Z\psi_p$ is then easily deduced.

4. Numerical illustration

To illustrate the simplicity of the present method, the use is shown using the following numerical example: The crest of an overspill dam, having a downstream slope of 1/0.7, is 60.96 m above the horizontal floor of the stilling basin. The head H on the crest is 9.144 m, then Z is 70.104 m and the maximum discharge q is 44.5639 m^2s^{-1}.

Using the given data, the values of ϕ_p , α according equation (10) and ψ_p according equation (12) are 0.13044 , 6.18894rd and 0.02056, respectively. The toe depth $h_1 = Z\psi_p$ is finally equal to 1.44 m. The Froude number is equal to 8.23.

5. Conclusions

Our contribution has focused primarily on calculating the depth h_1 of flow at the downstream toe of a steeply-sloping overspill dam face, within the range 1/0.6 to 1/0.8. With the proposed definition sketch, we defined the functional relationship $f(Z,H,h_1) = 0$ in which Z is the vertical distance from headwater elevation to downstream apron of the dam, and H is the head of water on the sill, considered as the critical head.

In a first approximation, we ignored the effect of head losses due to friction on the downstream overspill dam face. This led to a refinement of the above functional relationship and we demonstrated that Z, H and h_1 are related by a cubic equation. We showed that this relation is wholly composed of the non-dimensional parameters $\psi = h_1/Z$ and $\phi = H/Z$ and used trigonometric functions to find the real root of the equation.

We then proceeded to analyse the USBR diagram, which was constructed from field data collected at operational dams. The diagram shows the mean velocity V_1 of the flow at the downstream toe of an overspill dam versus the head on the sill H and the difference in elevation Z.

By means of an appropriate substitution of variables, we were able to transform this diagram into a single dimensionless curve defined by the non-dimensional parameters $\psi = h_1/Z$ and $\phi = H/Z$. We were able to show that $\psi_p > \psi_t$, where ψ_p is the practical value of ψ resulting from USBR data, whereas ψ_t is the theoretical value of ψ obtained from our formulation. We attributed this result to the friction head losses which were not accounted for in the theoretical development. Graphical representation of $\psi_p = f(\psi_t)$ shows clearly that ψ_p may be favourably related to ψ_t by a linear law. We were able to approximate $\psi_p = f(\psi_t)$, with a highly satisfactory coefficient of correlation. This allowed us to derive the explicit $\psi_p(\phi_p)$ relationship from which the toe depth h_1 can be easily evaluated provided H and Z are given.

Notation

H Total head (m)

H_a Upstream approach velocity head (m)

H_c Critical head (m)

H_d Design head (m)

H_e Total head including effect of approach velocity (m)

F_1 Froude number (-)

V_1 Mean velocity (ms^{-1})

V_a Approach velocity (ms^{-1})

Z Vertical distance from reservoir level to the floor at the downstream apron (m)

g Acceleration due to gravity (ms^{-2})

h_1 Toe depth (m)

h_c Critical depth (m)

h_f Friction head loss (m)

q Discharge per unit width (m^2s^{-1})

s Sill height (m)

ψ $\psi = h_1/Z$ (-)

ψ_p Practical value of ψ (-)

ψ_t Theoretical value of ψ (-)

ϕ $\phi = H/Z$ (-)

ϕ_p Practical value of ϕ (-)

References

Achour, B. (2002). "Ressaut hydraulique classique et contrôlé dans quelques profils de canaux", LARHYSS/Journal, (1), 37-85, Biskra.

Bazin, H. (1888-1898)."Expériences nouvelles sur l'écoulement en déversoir", Mémoires et documents, Annales des ponts et chaussées, Série 6,16(2ème Sem.) 393-448, Paris.

Chow,V.T.(1981). "Open-Channel Hydraulics", International Student Edition, McGraw-Hill, 17th Printing, Tokyo.

Creager, W.P. (1929). "Engineering for Masonry Dams", John Wiley and Soons, Inc., New York.

Creager, W.P., Justin, J.D., and Hinds, J.(1966). "Engineering for Dams". Volume I, General Design, ninth edition, Jhon Wiley & Sons, New York.

Lencastre, A. (1999). "Hydraulique générale", Editions Eyrolles,deuxième tirage, Paris.

Scimemi, E. (1930). "Sulla forma delle vene tracimanti" (The form of flow over weirs), L'Energia Elettrica, 7(4), 293-305, Milano.

Sinniger, R.O., and Hager, W.H. (1989). "Constructions hydrauliques – Ecoulements stationnaires", Traité de génie-Civil, EPFL, Editions Presses Polytechniques Romandes, 15, Lausanne.

Smetana, J. (1948)." Etude de la surface d'écoulement des grands barrages", Revue générale de l'Hydraulique, 14(46) ,185-194, Paris.

Smetana, J. (1949). "Etude de la surface d'écoulement des grands barrages", Revue générale de l'Hydraulique, 15(49) ,19-32, Paris.

U.S. Bureau of Reclamation. (1948). "Studies of crests for overfall dams.", Boulder Canyon Project Final Reports,4, Hydraulic Investigations, Bulletin 3.

U.S. Bureau of Reclamation. (1955). "Research Study on Stilling basins, Energy dissipators and associated appurtenances". Hydraulic Report, Hyd, 399.

U.S. Army, Corps of Engineers. (1952 – Revised in subsequent years), "Hydraulic Design Criteria" Army Waterways Experiment Station, Vicksburg, Mississipi.

Appendix 1

Table. 1. *Practical values of non-dimensional parameters ψ and ϕ , and the theoretical values of ψ computed using Equations. (9) and (10).*

H (m)	V_1 (m/s)	Z (m)	s (m)	$h_c = 2H/3$ (m)	q (m^2/s)	$h_1 = q/V_1$ (m)	ϕ_p =H/Z	ψ_p =h_1/Z	ψ_t	F_1 (-)
0,762	6,096	2,9	2,14	0,508	1,134	0,186	0,26276	0,06415	0,05328	4,51
	7,620	3,7	2,94	0,508	1,134	0,149	0,20595	0,04022	0,03665	6,30
	9,144	5,95	5,19	0,508	1,134	0,124	0,12807	0,02084	0,01780	8,29
	10,67	8,85	8,09	0,508	1,134	0,106	0,08610	0,01201	0,00977	10,46
	12,192	11,9	11,14	0,508	1,134	0,093	0,06403	0,00782	0,00626	12,76
	15,24	28,5	27,74	0,508	1,134	0,074	0,02674	0,00261	0,00168	17,89
	16,00	36,6	35,84	0,508	1,134	0,071	0,02082	0,00194	0,00116	19,17
1,524	15,24	17,5	15,98	1,016	3,208	0,210	0,08709	0,01203	0,00994	10,62
	16,764	21,3	19,78	1,016	3,208	0,191	0,07155	0,00898	0,00739	12,25
	18,288	27,4	25,88	1,016	3,208	0,175	0,05562	0,00640	0,00506	13,96
	18,90	30,5	28,98	1,016	3,208	0,170	0,04997	0,00556	0,00431	14,64
	19,66	36,6	35,08	1,016	3,208	0,163	0,04164	0,00446	0,00328	15,55
	20,117	42,7	41,18	1,016	3,208	0,159	0,03569	0,00373	0,00260	16,11
	20,421	54,8	53,28	1,016	3,208	0,157	0,02781	0,00287	0,00179	16,45
2,286	18,288	24	21,71	1,524	5,893	0,322	0,09525	0,01343	0,01138	10,29
	19,812	29,9	27,61	1,524	5,893	0,297	0,07645	0,00995	0,00817	11,61
	21,336	36	33,71	1,524	5,893	0,276	0,06350	0,00767	0,00618	12,97
	22,403	42,7	40,41	1,524	5,893	0,263	0,05354	0,00616	0,00478	13,95
	23,47	54,9	52,61	1,524	5,893	0,251	0,04164	0,00457	0,00328	14,96
	24,323	67	64,71	1,524	5,893	0,242	0,03412	0,00362	0,00243	15,79
	24,384	73	70,71	1,524	5,893	0,242	0,03132	0,00331	0,00214	15,83
3,048	22,555	36	32,95	2,032	9,072	0,402	0,08467	0,01117	0,00953	11,36
	24,384	45,7	42,65	2,032	9,072	0,372	0,06670	0,00814	0,00665	12,76
	25,908	54,9	51,85	2,032	9,072	0,350	0,05552	0,00638	0,00505	13,98
	27,737	73	69,95	2,032	9,072	0,327	0,04175	0,00448	0,00329	15,49
	28,53	91,5	88,45	2,032	9,072	0,318	0,03331	0,00348	0,00234	16,15
4,572	21,793	30,5	25,93	3,048	16,667	0,765	0,14990	0,02507	0,02260	7,96
	23,47	36,6	32,03	3,048	16,667	0,710	0,12492	0,01940	0,01714	8,89
	25,298	42,7	38,13	3,048	16,667	0,659	0,10707	0,01543	0,01358	9,95
	26,517	48,7	44,13	3,048	16,667	0,629	0,09388	0,01291	0,01113	10,67
	27,828	54,9	50,33	3,048	16,667	0,599	0,08328	0,01091	0,00929	11,48
	28,956	61	56,43	3,048	16,667	0,576	0,07495	0,00944	0,00793	12,18
	30,632	73	68,43	3,048	16,667	0,544	0,06263	0,00745	0,00605	13,26
	31,70	85,5	80,93	3,048	16,667	0,526	0,05347	0,00615	0,00477	13,96
	32,309	91,5	86,93	3,048	16,667	0,516	0,04997	0,00564	0,00431	14,36
	33,162	110	105,4	3,048	16,667	0,503	0,04156	0,00457	0,00327	14,93

H (m)	V_1 (m/s)	Z (m)	s (m)	$h_c = 2H/3$ (m)	q (m^2/s)	$h_1 = q/V_1$ (m)	$\phi_p = H/Z$	$\psi_p = h_1/Z$	ψ_t	F_1 (-)
6,096	27,432	49,4	43,3	4,064	25,660	0,935	0,12340	0,01894	0,01683	9,06
	28,651	55	48,9	4,064	25,660	0,896	0,11084	0,01628	0,01431	9,66
	30,48	64	57,9	4,064	25,660	0,842	0,09525	0,01315	0,01138	10,61
	32	73	66,9	4,064	25,660	0,802	0,08351	0,01098	0,00933	11,41
	33,833	85,5	79,4	4,064	25,660	0,758	0,07130	0,00887	0,00735	12,41
	34,595	91,4	85,3	4,064	25,660	0,742	0,06670	0,00812	0,00665	12,82
	35,357	97,5	91,4	4,064	25,660	0,726	0,06252	0,00744	0,00604	13,25
	36,576	109,7	103,6	4,064	25,660	0,702	0,05557	0,00640	0,00505	13,94
	37	115,8	109,7	4,064	25,660	0,694	0,05264	0,00599	0,00466	14,18
	37,49	128	121,9	4,064	25,660	0,684	0,04763	0,00535	0,00401	14,47
7,620	29,21	54,9	47,28	5,08	35,862	1,228	0,13880	0,02236	0,02011	8,42
	30,785	61	53,38	5,08	35,862	1,165	0,12492	0,01910	0,01714	9,11
	32	67	59,38	5,08	35,862	1,121	0,11373	0,01673	0,01487	9,65
	33,162	73	65,38	5,08	35,862	1,081	0,10438	0,01481	0,01307	10,18
	36,576	94,5	86,88	5,08	35,862	0,980	0,08063	0,01038	0,00885	11,80
	38,1	109,8	102,2	5,08	35,862	0,941	0,06940	0,00857	0,00706	12,54
	39,32	122	114,4	5,08	35,862	0,912	0,06246	0,00748	0,00603	13,15
	40,64	146,5	138,9	5,08	35,862	0,882	0,05201	0,00602	0,00458	13,82
9,144	33,833	73	63,86	6,096	47,141	1,393	0,12526	0,01909	0,01721	9,15
	34,9	79	69,86	6,096	47,141	1,351	0,11575	0,01710	0,01527	9,59
	35,966	85,5	76,36	6,096	47,141	1,311	0,10695	0,01533	0,01355	10,03
	37,185	91,4	82,26	6,096	47,141	1,268	0,10004	0,01387	0,01225	10,54
	39,624	110	100,9	6,096	47,141	1,190	0,08313	0,01082	0,00927	11,60
	41,148	122	112,9	6,096	47,141	1,146	0,07495	0,00939	0,00793	12,27
	42,367	134	124,9	6,096	47,141	1,113	0,06824	0,00830	0,00688	12,82
	43,434	146,5	137,4	6,096	47,141	1,085	0,06242	0,00741	0,00602	13,31