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Effect of sill in the hydraulic jump in a triangular channel

Effet du seuil sur le ressaut hydraulique dans un canal triangulaire

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ABSTRACT
Hydraulic jumps are experimentally examined in a horizontal symmetrical triangular channel of 90° central angle. This study investigates the main features of both the minimum-B jump and the sill-controlled jump under various inflow conditions. The development of jumps was ensured using either a thin-crested or a broad-crested sill. Based on a large experimental program, the data were fitted to empirical relations to detect the effect of the inflow Froude number on the relative sill height, the sequent depths ratio and the non-dimensional toe position of the sill. The relations obtained are recommended for designing irrigation ditches.

RÉSUMÉ
Les ressauts hydrauliques sont expérimentalement examinés dans un canal triangulaire symétrique horizontal à angle d'ouverture de 90°. Dans cette étude, des investigations ont été conduites sur les principales caractéristiques des ressauts contrôlés et B-minimum sous diverses conditions de l'écoulement incident. La formation des ressauts a été assurée par la mise en place aussi bien d'un seuil à parois mince que d'un seuil à paroi épaisse. En se basant sur un programme expérimental intense, des relations ont été déduites modélisant l'effet du nombre de Froude incident sur la hauteur relative du seuil, le rapport des hauteurs conjuguées et la positon relative du seuil. Les relations ainsi obtenues sont recommandées pour le dimensionnement des raies d'irrigation.

Keywords: Hydraulic jump, forced jump, minimum-B jump, controlled jump, triangular channel.

1 Introduction

A literature review reveals that hydraulic jumps were systematically investigated in rectangular channel. Among other studies, those of Forster and Skrinde (1950) and of Hager and Li (1992) relate to the controlled jump, and the study of Bretz (1988) for the forced jump. Only recently Achour and Debabeche (2003) elaborated an experimental study on the hydraulic jump controlled by a sill in a triangular channel of 90° central angle. The present study aims to experimentally investigate the effects of both the thin-crested and the broad-crested sills on the characteristics of the hydraulic jumps in a horizontal symmetric triangular channel. Particular attention is focused on both the minimum-B jump and the sill-controlled jump to estimate their main characteristics under various inflow conditions. To highlight the competitive configuration, a comparative study between each type of jumps will be proposed.

Notice that the triangular section does not satisfy the requirements of a stilling basin, but has some interesting practical applications when used as irrigation ditch section (Achour, 1989).

2 Position of problem

Hydraulic jump is the transition from supercritical to subcritical flow. If Q, h_1, g and θ are, respectively, the discharge, the inflow depth, the acceleration due to gravity and the angle inclination of the channel walls with regard to the horizontal, the inflow Froude number F_1 is expressed by $F_1 = [2Q^2/(gm^2h_1^5)]^{1/2}$, with $m = \cot g(\theta)$ where "cot g" denotes the cotangent.

Although the cross-sectional area is triangular, the hydraulic jump can be considered as a CHJ (classical hydraulic jump) since it occurs in a smooth, horizontal, prismatic channel. However, the flow pattern is strongly three-dimensional when compared to the rectangular channel which is almost two-dimensional.

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The hydraulic jump can be also controlled with a terminal thin-crested sill of height s as it is shown in Fig. 1(a). As usual, L_s denotes the distance between the toe and the upstream sill face, L_j is the length of the jump, h_1 and h_2 are the flow depths in the approaching and the tailwater regions. When the relative length L_j/L_s is greater or at least equal to unity, the hydraulic jump is completely located on the apron downstream from the sill beyond which the flow is supercritical. Otherwise, the flow over the sill is free and the energy dissipation is not accomplished. The increase of the inflow Froude number F_1 due to the increase of the discharge Q or the decrease of the inflow depth h_1 moves the jump upstream and the A-jump formation happens when the end of the roller is just above the front face of the sill. At the same time, an increase of the distance x_o can be observed (Fig. 1a).

Increasing tailwater by adding a second sill at the channel extremity and increasing simultaneously the inflow Froude number, the so-called minimum B-jump is thus formed (Fig. 1b). This is a forced jump, characterized by: a main roller of length L_R, a bottom roller of length L_{RB} beyond the sill and the formation of a second surface roller. Thus, one may write the length of the basin as $L_B = (L_s + L_{RB})$.

All these forms of hydraulic jumps were experimentally analysed by Bretz (1988) in a rectangular channel and his results were well summarized by Hager and Bretz (1988), and Hager and Li (1992).

What is needed in our study is to examine the aforementioned forms of hydraulic jumps when they happen in a symmetrical triangular channel. The effect of both continuous thin-crested and broad-crested sills on the ratio Y of the sequent depths will be observed and quantified. The relationships $Y(F_1)$ and $S(F_1)$ will be also determined. Our attention will be focussed on the minimum B-jump which may be considered as the flow with minimum acceptable tailwater. This configuration is of practical interest in the irrigation field as it can be observed in SIG region located in north-west of Algeria. The capacity of the hydraulic jump to raise tailwater depth is used to prime a hose drawing siphon designed for the required discharge, whereas the small depths upstream the sill in the A-jump are unable to put into motion the priming of the siphon. That is the main practical interest of the hydraulic jump in triangular channel which is worth evoking.

3 Sequent depths ratio

When a hydraulic jump is not forced, a sill has theoretically not a reducing effect on the downstream flow depth h_2. Figure 2 shows the experimental variation of $Y(F_1)$, for both the sill-controlled and the forced hydraulic jump. Y is the sequent depth ratio h_2/h_1. The theoretical curve $Y^*(F_1)$ for the classical jump in a triangular channel is also plotted as (Hager and Wanoschek, 1987):

$$F_1^2 = \frac{2Y^{*2}(Y^{*2}+Y^*+1)}{3(Y^*+1)}; \qquad Y^* = \frac{h_2^*}{h_1} \tag{1}$$

Figure 2 shows that the equality $Y = Y^*$ is satisfied only for the range $2 \leq F_1 \leq 7.5$. For $F_1 > 7.5$, the influence of a sill on the sequent depths ratio decreases Y as compared to Y^*. Even if ΔY increases with increasing F_1, the relative deviation $\Delta Y/Y^*$ is good. For $F_1 = 11$, $\Delta Y/Y^*$ is approximately equal to 5% for the thin-crested sill and equal to 7% for the broad-crested sill. It can also be observed that the sequent depth ratio of the forced hydraulic jump is lower than for controlled jumps for $F_1 > 3$. From an analysis of data (Appendixes 1 and 2) and for $1.60 \leq F_1 \leq 11$, the explicit relationship is:

$$Y = 1 + \alpha \ln\left[\frac{(F_1+4)}{5}\right] \tag{2}$$

in which, the constant α takes the following values: $\alpha = 3.96$ for the hydraulic jump controlled by a thin-crested sill (Achour and Debabeche, 2003); $\alpha = 3.78$ for the hydraulic jump controlled by a broad-crested sill; $\alpha = 3.33$ for the minimum-B forced jump by a thin-crested sill.

Figure 3 shows the experimental data of the sequent depths ratio (Appendixes 1 and 2) Y with respect to the relative sill height $S = s/h_1$ for the two types of the tested hydraulic jumps. As it can be seen, Y may be related linearly to S as:

$$Y = 1 + \beta S \tag{3}$$

where: $\beta = 0.96$ for the hydraulic jump controlled by a thin-crested sill (Achour and Debabeche, 2003); $\beta = 1.02$ for the

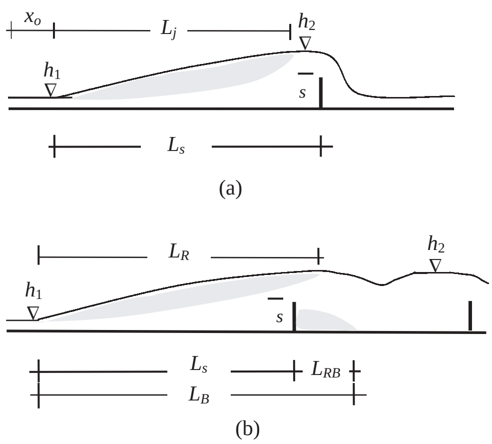

Figure 1 Formation of hydraulic jumps: (a) controlled hydraulic jump by a singular thin-crested sill, (b) forced minimum-B jump.

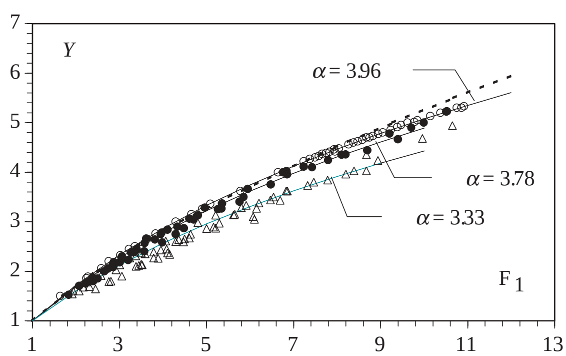

Figure 2 Experimental variation of $Y(F_1)$ for the hydraulic jump controlled by (○) thin-crested sill (Achour and Debabeche, 2003), (•) broad-crested sill, (Δ) minimum-B forced jump by a thin-crested sill, (- - -) Eq. (1).

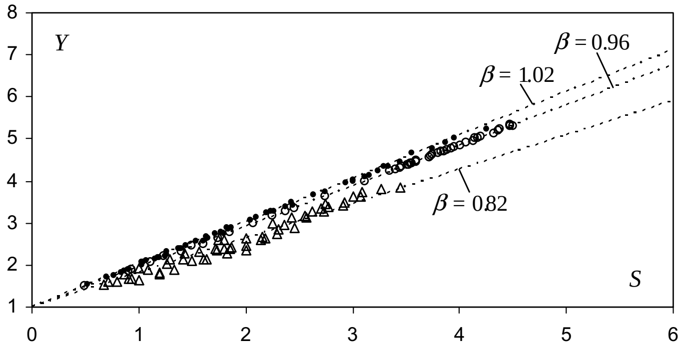

Figure 3 Experimental variation of $Y(S)$ for the hydraulic jump controlled by (○) thin-crested sill (Achour and Debabeche, 2003), (•) broad-crested sill, (Δ) minimum-B forced jump by a thin-crested sill, (- - -) Eq. (3).

hydraulic jump controlled by a broad-crested sill; $\beta = 0.82$ for the minimum-B forced jump by a thin-crested sill.

4 Relative sill height

The required relative sill height $S(F_1)$ follows from Eqs (2) and (3) as:

$$S = \gamma \ln \left[\frac{(F_1 + 4)}{5} \right] \tag{4}$$

where: $\gamma = 4.13$ for the hydraulic jump controlled by a thin-crested sill (Achour and Debabeche, 2003); $\gamma = 3.71$ for the hydraulic jump controlled by a broad-crested sill; $\gamma = 4.06$ for the minimum-B forced jump by a thin-crested sill.

Figure 4 compares Eq. (4) with the experimental data (Appendixes 1 and 2).

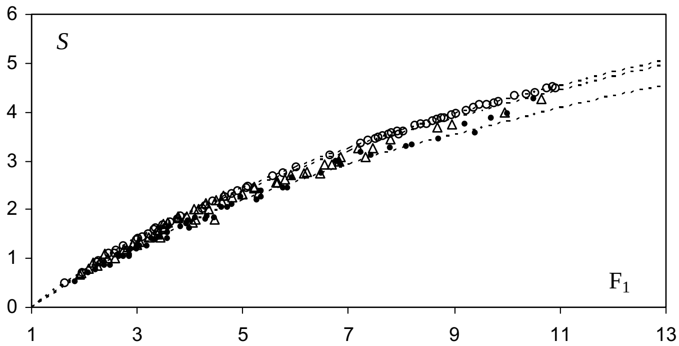

Figure 4 Experimental variation of $S(F_1)$ for the hydraulic jump controlled by (○) thin-crested sill (Achour and Debabeche, 2003), (•) broad-crested sill, (Δ) minimum-B forced jump by a thin-crested sill, (- - -) Eq. (4).

5 Relative basin length

As illustrated in Fig. 1, the basin length is different for the two types of jumps. For the controlled hydraulic jump by either a thin-crested or a broad-crested sill, it corresponds to the position L_s of the sill relative to the toe of the jump. Moreover, L_s can be defined as the length L_j of the controlled jumps. For the minimum-B forced jump, the basin length L_B is the sum of the sill position L_s and the length of bottom roller L_{RB} beyond the sill, that is $L_B = (L_s + L_{RB})$. Figure 5 shows the experimental variation of both relative length $\lambda = L_s/h_1$ of the controlled hydraulic jumps and $\lambda = L_B/h_1$ of the minimum-B forced jump by a thin-crested sill with respect to the inflow Froude number F_1. Experimental data (Appendixes 1 and 2) follow the relationship:

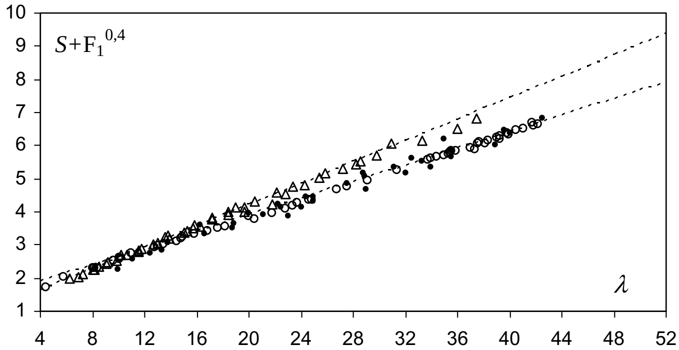

Figure 5 Experimental variation of the basin relative length $\lambda(F_1)$ for the hydraulic jump controlled by (○) thin-crested sill (Achour and Debabeche), (•) broad-crested sill, (Δ) minimum-B forced jump by a thin-crested sill, (- - -) Eq. (5).

$$S + F_1^{0.4} = a\lambda + b, \quad \lambda \geq 4 \tag{5}$$

Table 1 gives the corresponding values of a and b.

Table 1 Values of constants a and b in Eq. (5)

	Controlled jump		Forced minimum-B jump
	Thin-crested sill	Broad-crested sill	
a	0.127	0.125	0.161
b	1.278	1.410	1.000

The data for the controlled hydraulic jumps by either the thin-crested or the broad-crested sill are almost identical. For the same inflow Froude number F_1 and for the same relative sill height s/h_1, the relative length λ is practically invariable, therefore. Concerning the minimum-B jump, its relative length λ is smaller than the controlled hydraulic jumps, provided $F_1 \geq 4$. Furthermore, one may observe that for $(S + F_1^{0.4}) = 6.68$, deviation between L_s/h_1 and L_B/h_1 is about 17%.

6 Conclusion

The controlled and the minimum-B forced hydraulic jumps in a symmetrical triangular channel of 90° central angle are described. Based on experimentation, the influence of both the inflow Froude number F_1 and the relative sill height S on the sequent depths ratio was quantified. The reduction of tailwater level ΔY due to a presence of a sill was investigated. Comparing Y to Y^* of the triangular classical jump, practically no deviation may be observed. Moreover, the relative position λ of the sill remains invariable, provided the identical inflow Froude number F_1 and relative sill height S are considered. Concerning the minimum-B forced hydraulic jump by a thin-crested sill, the obtained results seem to be more interesting towards the sill-controlled basin efficiency. It found that a less tailwater level is required and a reduced length is needed when compared to a triangular classical jump basin.

From an analysis of data, the non-dimensional governing equations of both the sequent depths ratio Y and the relative sill height S are presented for the three types of the tested hydraulic jumps. It found that Y could be related linearly to S, whereas both Y and S are linked to the inflow Froude number F_1 by a logarithmic law. The generalized relationship $f(S, F_1, \lambda) = 0$ is well defined and permits particularly a direct determination of the relative position λ of the sill, provided one of S or F_1 is given.

Notation

F_1 = Inflow Froude number [–]
h_1 = Inflow depth [m]
h_2 = Tailwater depth [m]
h_2^* = Tailwater depth of classical hydraulic jump [m]
L_j = Length of jump [m]
m = Tangent of the half central angle of a triangular channel [–]
s = Sill height [m]
S = Relative sill height ($S = s/h_1$) [–]
L_RB = Length of the bottom roller [m]
L_B = Basin length [m]
λ = Relative basin length ($\lambda = L/h_1$) [–]
Y = Sequent depth ratio ($Y = h_2/h_1$) [–]
Y^* = Sequent depth ratio of triangular classical hydraulic jump ($Y^* = h_2^*/h_1$) [–]

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Appendix 1 Experimental data of minimum-B forced hydraulic jump by a thin-crested sill in triangular channel

F_1	Y	S	λ
1,91	1.54	0.67	6.20
1.93	1.60	0.71	6.94
2.07	1.60	0.80	7.30
2.17	1.68	0.91	8.00
2.24	1.78	0.86	8.21
2.31	1.69	0.93	8.47
2.38	1.90	1.09	9.91
2.45	1.64	1.00	9.10
2.57	1.91	1.00	9.20
2.75	1.79	1.20	10.2
2.80	1.80	1.20	10.67
2.92	2.03	1.27	11.55
3.00	2.13	1.29	11.53
3.05	1.90	1.33	11.8
3.23	2.27	1.43	12.71
3.37	2.31	1.57	13.93
3.38	2.10	1.50	
3.44	2.12	1.42	13.07
3.50	2.37	1.71	15.01
3.50	2.14	1.60	13.60
3.52	2.13	1.64	13.84
3.59	2.35	1.73	15.24
3.78	2.39	1.86	16.31
3.78	2.27	1.82	15.73
3.89	2.26		
3.95	2.43	1.87	15.87
4.06	2.61	1.75	
4.08	2.45	2.00	17.23
4.10	2.37	1.80	12.90
4.15	2.34	2.00	17.22
4.29	2.60	2.14	18.43
4.36	2.64	2.00	17.20
4.47	2.59	1.80	15.30
4.49	2.65	2.18	18.45
4.60	2.67	2.15	19.69
4.63	2.75	2.29	19.64
4.80	2.98	2.25	19
5.00	2.86	2.31	21.85
5.16	2.89		18.2
5.21	3.13	2.43	20.86
5.21	2.87	2.46	23.38
5.29	2.97	2.36	20.45
5.56		2.00	17.2
5.62	3.14	2.57	22.14
5.64			19.20
5.65	3.15	2.55	22.82
5.80	3.28	2.62	25.85
5.91	3.33	2.71	23.43

Appendix 1 (*Continued*)

F_1	Y	S	λ
6.07	3.10	2.40	21.13
6.10	3.05		
6.10		2.20	18.80
6.15	3.27	2.73	24.25
6.20	3.38	2.77	28.00
6.47	3.44	2.75	
6.54	3.50	2.92	29.38
6.65		2.53	21.80
6.69	3.43	2.91	25.45
6.83	3.62	3.00	25.88
6.86	3.62	3.08	30.92
7.17		3.25	28.25
7.32	3.73	3.09	27.18
7.46	3.80	3.27	28.55
7.78	3.84	3.45	29.82
8.20	3.96	2.40	20.80
8.39	4.03	2.60	22.56
8.67	4.35		30.88
8.67	4.03	2.80	24.90
8.94	4.24		33.25
9.96	4.68		36.00
10.65	4.94		37.50

Appendix 2 Experimental data of hydraulic jump controlled by a broad-crested sill in triangular channel

F_1	Y	S	λ
3.01	2.18	1.18	12.50
3.57	2.40	1.40	13.75
2.85	2.11	1.07	11.11
2.39	1.80	0.84	10.00
4.29	2.75	1.80	16.25
4.80	3.13	2.10	20.00
2.86	2.08	1.04	13.00
5.26	3.25	2.20	22.50
3.98	2.58	1.60	16.67
3.06	2.30	1.26	15.60
2.07	1.70	0.70	9.17
5.86	3.50	2.43	25.00
3.40	2.44	1.44	18.00
4.48	2.87	1.82	18.89
4.71	3.04	2.04	21.11
3.62	2.66	1.64	20.00
6.76	4.00	3.00	28.75
2.50	1.85	0.87	11.67
5.34	3.27	2.27	22.22
4.10	2.84	1.84	22.00
2.74	2.05	1.03	13.33
5.76	3.40	2.44	24.44
2.87	2.18	1.20	14.17
4.60	3.06	2.04	23.00
8.10	4.35	3.30	32.50

Appendix 2 (*Continued*)

F_1	Y	S	λ
4.96	3.28	2.24	24.00
9.20	4.78	3.75	35.00
6.85	3.96	2.93	28.89
3.34	2.38	1.37	16.67
9.71	4.90	3.88	40.00
7.23	4.11	3.16	31.11
3.58	2.57	1.53	18.33
1.84	1.53	0.53	9.38
7.80	4.24	3.24	33.33
10.51	5.23	4.25	42.50
3.95	2.75	1.72	20.00
8.20	4.36	3.33	35.56
8.69	4.44	3.44	35.56
6.83	4.02	3.00	32.00
4.33	2.90	1.87	21.67
2.21	1.75	0.78	10.00
9.40	4.67	3.56	38.89
7.43	4.10	3.12	34.00
2.38	1.88	0.90	13.13
5.36	3.37	2.38	25.00
2.65	2.00	1.03	13.75
2.86	2.15	1.15	14.38
3.06	2.25	1.26	15.00
3.27	2.38	1.39	16.25
3.82	2.64	1.65	16.88
3.97	2.78	1.78	18.75