

## Modeling of Induction Machines with Skewed Rotor Slots Dedicated to Rotor Faults

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**Abstract** - A model permitting the simulation of induction machine with skewed rotor slot is presented. The model is based on the Extension of the Modified Winding Function Approach, which allows for all harmonics of MMF to be taken into account. The effects of rotor skew which have not been able to be analyzed accurately by 2-D finite element technique were investigated. The model is used to predict the characteristics frequency components which are indicative of rotor bar faults. The results obtained showed the importance of the MWFA for the analysis of the induction motors.

### I. INTRODUCTION

The induction motor is a key component of the majority of the industrial plants, because of its great robustness and its low cost. It is indeed, omnipresent in the industrial sectors like aeronautics, the nuclear power, chemistry. In spite of these qualities, stresses of various natures (thermal, electrical, mechanical or environment) can affect the life span of this one by involving the occurrence of stator and/or rotor faults [1]. These faults cause considerable economic losses. Thus, it is imperative to implement monitoring systems in order to avoid the unforeseen stops. The usual method for stator and rotor faults detection is based on the motor current signature analysis (MCSA) [2]. For that, a model closer to reality must be established. Several studies was published in this axis, and made possible the detection of the various defects, while analyzing the frequencies higher or lower than the fundamental stator frequency. A model taking into account the real distribution of FMM is presented, it is based on an extension in 2D of the modified winding function approach, with the effects of the rotor skew and the eccentricity [3], [4], [5].

The expressions of the magnetizing inductances and mutual inductances are derived from an extension of the modified winding function approach. In the case of multiple coupled circuit model, an accurate calculation of the electromagnetic torque depends on the accuracy of inductances derivatives. The inductances matrixes as well as

its derivative must be calculated at each integration step, it well known that the other method consume a high time, the proposed method is based in a simple and clever analytical calculation of a double integration witch will be the base of inductances calculation. This method can reduce the simulation time. The simulation tools are implemented using MTLAB language. Simulation and experimental results confirm the validity of this model.

### II. INDUCTION MACHINE MODEL

Knowing that the cage can be viewed as identical and equally spaced rotor loops, it is possible to establish voltage equations of stator and rotor loops as [7]:

$$[U_s] = [R_s][I_s] + \frac{d[\psi_s]}{dt}, \quad (1)$$

$$[0] = [R_r][I_r] + \frac{d[\psi_r]}{dt}, \quad (2)$$

$$[\psi_s] = [L_{ss}][I_s] + [L_{sr}][I_r], \quad (3)$$

$$[\psi_r] = [L_{rs}][I_s] + [L_{rr}][I_r]. \quad (4)$$

The vector  $[U_s]$  corresponds to the stator voltages,  $[I_s]$  and  $[I_r]$  to the stator and rotor currents.  $m$  is the number of stator phases and  $N_b$  the number of rotor bars.  $[R_s]$  is an  $m$  dimensional diagonal matrix,  $[L_{ss}]$  is an  $m \times m$  symmetric matrix,  $[L_{sr}]$  is an  $m \times (N_b + 1)$  matrix, and  $[R_r]$  and  $[L_{rr}]$  are  $(N_b + 1) \times (N_b + 1)$  matrix. Adding to these equations the mechanical expression and the equations electromagnetic torque yields to:

$$C_e - C_r = J_r \frac{d\omega_r}{dt}, \quad C_e = \left( \frac{dW_{co}}{d\theta_r} \right) \Big|_{(I_s, I_r = \text{constant})}, \quad (5)$$

$$W_{co} = \frac{1}{2} \left( [I_s]^T [L_{ss}] [I_s] + [I_s]^T [L_{sr}] [I_r] + [I_r]^T [L_{rr}] [I_r] + [I_r]^T [L_{rs}] [I_s] \right), \quad (6)$$

where  $W_{co}$  is the coenergy,  $C_e$  the electromagnetic torque,  $C_r$  the load torque,  $J_r$  the rotor load inertia, and  $\omega_r$  the mechanical speed of the rotor.

### III. MODELING OF INDUCTION MOTOR WITH SKEWED ROTOR SLOTS USING AN EXTENSION OF MODIFIED WINDING FUNCTION APPROACH (MWFA)

#### A. Formulation

To formulate the problem, we refer to Fig. 1 which gathers two cylindrical masses separated by an air-gap, one of it hollow and represent the stator, and the other represents the rotor. Given  $abcd$  an arbitrary contour definite in comparison to a reference of phase fixed on the stator, to an axial reference and to the mechanical position of the rotor measured by respecting a fixed stator reference. For a position  $\theta_r$  of the rotor, and at  $\varphi_0 = 0$  and  $z_0 = 0$  are the points  $a$  and  $b$ , and at  $\varphi$  and  $z$  are  $c$  and  $d$ . On another side,  $a$  and  $d$  are located on the stator inner surface, and  $b$  and  $c$  on the external rotor surface. According to the Gauss's law and Amper's law applied to the circuits of the machine, it will be possible to lead to the expression of winding function [3]:

$$N(\varphi, z, \theta_r) = n(\varphi, z, \theta_r) - \frac{1}{2\pi l \langle g^{-1}(\varphi, z, \theta_r) \rangle} \int_0^{2\pi l} n(\varphi, z, \theta_r) g^{-1}(\varphi, z, \theta_r) dz d\varphi. \quad (7)$$

where,  $g$  is the air gap and  $l$  is the effective length. It is to be noticed that this new expression does not hold any restriction as for the axial uniformity, in particular in term of skewing slots and axial air-gap non uniformity. In this study, we take into account only the rotor skew and the opening of the slots, in the case of a constant air-gap.

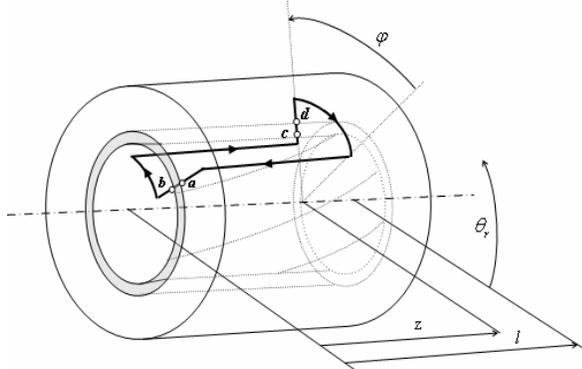


Fig. 1. Elementary induction machine

By supposing that the machine is symmetrical, the air-gap length  $g$  is reduced to  $g_0$  which is the radial air-gap length in the case of no eccentricity. If  $F$  is the MMF distribution in the air-gap due to the current  $i_{A_i}$  flowing in an arbitrary coil  $A_i$ , and knowing that the elementary flux corresponding in the air-gap is measured in comparison to an elementary volume of section  $ds$  and length  $g_0$  such as

$$d\phi = \mu_0 F g_0^{-1} ds. \quad (8)$$

The calculation of total flux is made through a calculation of a double integral, by carrying out the change of variable  $x = r\varphi$  and  $x_r = r\theta_r$ , the study is transformed to a reference with axes  $X$  and  $Z$  where we can imagine a plane representation of the machine. It is clear that, in this case,  $x$  translate correctly the linear displacement along the arc corresponding to the angular opening  $\varphi$ . It is the same thing concerning  $x_r$  and  $\theta_r$ .

Knowing that  $N$  is the MMF per unit of current, the expression giving the flux seen by all the turns of coil  $B_j$  of winding  $B$  due to  $i_{A_i}$  flowing in coil  $A_i$  will be reduced as

$$\phi_{B_j A_i} = \frac{\mu_0}{g_0} \int_{x_{1j}(x)}^{x_{2j}(x)} \int_{z_{1j}(x)}^{z_{2j}(x)} N_{A_i}(x, z, x_r) n_{B_j}(x, z, x_r) i_{A_i} dz dx. \quad (9)$$

This is due to the fact that by taking into account the axial asymmetry  $n_{B_j}(x, z, x_r)$  will be defined so as to be able to translate the skew of the slots. In 2-D, it will be written in the following way:

$$n_{B_j}(x, z, x_r) = \begin{cases} w_{B_j} & x_{1j}(x) < x < x_{2j}(x) \text{ and } z_{1j}(x) < z < z_{2j}(x) \\ 0 & \text{in the remaining interval} \end{cases} \quad (10)$$

where  $w_{B_j}$  is the number of turns of coil  $B_j$ . It is equal to 1 in the case of a rotor loop. Generally, the total flux  $\psi_{BA}$  related to all coils composing winding  $A$  and  $B$ , holds its general expression by integrating over the whole surface. Then knowing that the mutual inductance  $L_{BA}$  is the flux  $\psi_{BA}$  per unit of current, yields

$$L_{BA}(x_r) = \frac{\mu_0}{g_0} \int_0^{2\pi r} \int_0^l N_A(x, z, x_r) n_B(x, z, x_r) dz dx. \quad (11)$$

Note that a rearrangement of (12) makes possible to define an inductance per unit of length as described in [6]:

$$L_{BA}(x_r) = \int_0^l L'_{BA}(z, x_r) dz. \quad (12)$$

According to the manner of connecting the coils translated by the sign in (13), this inductance can be obtained by summing all mutual inductances between the  $q$  and  $p$  coils of windings  $A$  and  $B$  respectively, such as:

$$L_{BA}(x_r) = \sum_{i=1}^q \sum_{j=1}^p \pm L_{B_j A_i}(x_r) \quad (13)$$

### B. Bars skewing

Fig. 2 depicts the crossing of a rotor loop  $r_j$  under the field of a stator coil  $A_i$ . The skew is written thanks to the definition of  $z(x)$  in (10) which will be a function describing the uniform skew, or particularly, the case of spiral skew.

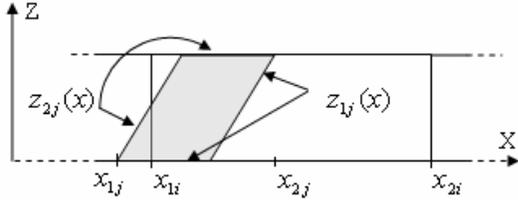


Fig. 2. Representation of the skew

We can notice that the pitch  $\alpha_{A_i}$  of the coil  $A_i$  is defined in comparison to its sides placed at  $x_{1i} = r \cdot \varphi_{1i}$  and  $x_{2i} = r \cdot \varphi_{2i}$ , and that the effect of linear rise of MMF across the slot is not represented in the figure.

### C. Slot opening.

Let us examine the case of coil  $A_i$  with  $w_{A_i}$  turns placed in slots which, according to cases, can present an opening of width  $\beta$  according to the considered configuration. Fig. 3 shows the turns function of coil  $A_i$  if the slot opening is taken into account in the calculation of the resultant linear rise of MMF across the slot.

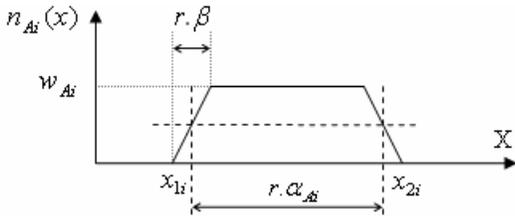


Fig. 3. Turns function of coil  $A_i$

## IV. SIMULATION RESULTS

The described method was applied to a three phases induction machine 3 kW, tow poles, 230/400v, 2800tr/mn, 50Hz, Nb=28, skewed rotor. Machines parameters are given in the Appendix. Fig. 4 illustrates the functions which describe the mutual inductances  $L_{r1A}$  between the first stator phase A, and the first rotor loop, in the different cases

considered, without and with taking into account the slots opening and the skewing of rotor bars. A rotor loop is regarded as being a coil with one turn. Note that the mutual inductance between phase and the second rotor loop is the same as it was given in Fig. 5, but shifted to the left by  $2\pi/Nb$ . As for the other inductances,  $L_{r1B}$  and  $L_{r1C}$  are identically reproduced, but shifted to the right by  $2\pi/3$ . The mechanical angle of skewing of the rotor bars is  $\gamma = \pi/14$  rad, which is selected equal to one stator slot pitch, and the width of slot opening  $\beta = \pi/36$ .

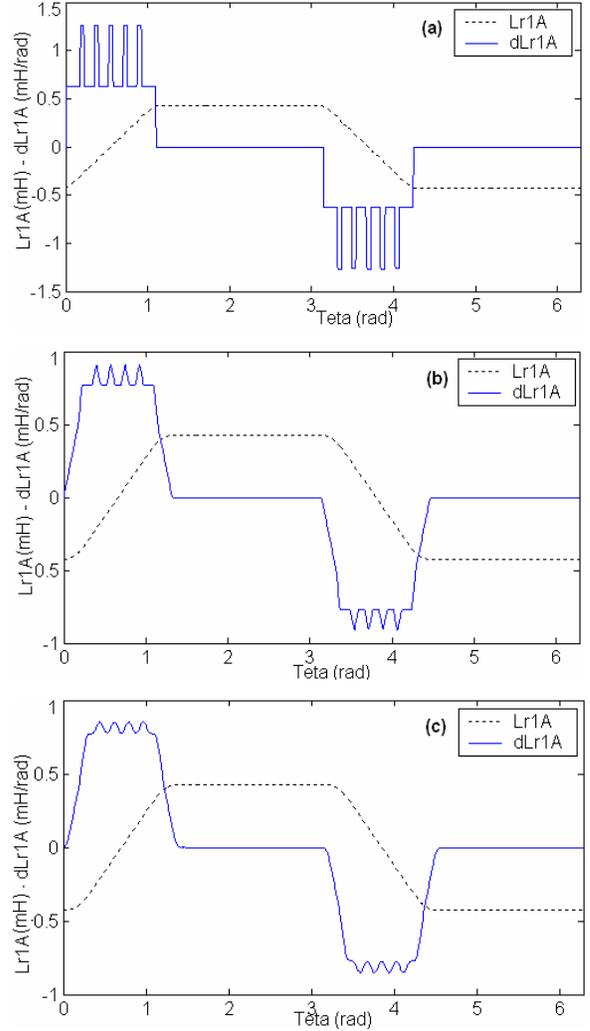


Fig. 4. Mutual inductance between stator phase A and rotor loop  $r_1$ .

- a - without skew and slot opening.
- b - with skew and without slot opening.
- c- with skew and slot opening.

By solving the system of differential equations representing the induction machine, it is possible to simulate the dynamic operation of the machine. Fig. 5 shows the transient dynamic start up behaviors of the induction motor under no load condition. At  $t=0.6s$  load is applied. The use of

skewing reduces the average value and the oscillations of the speed at steady state.

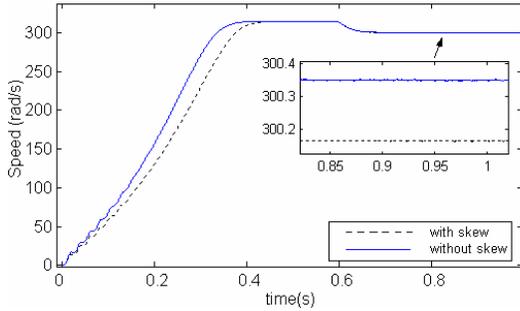


Fig.5. Speed without rotor skew *and with* rotor skew.

Fig. 6 shows the transient dynamic start up behaviors of the induction motor under no load condition. At  $t=0.5s$  load is applied. It is clear that the use of skewing reduce the parasitic torque but it does not eliminate completely [8], [9], [10]. It is of course, to eliminate such effects that skew is employed in the real machines. The average torque has conserved the same value; it may be due to additional forces that the model can't take into account.

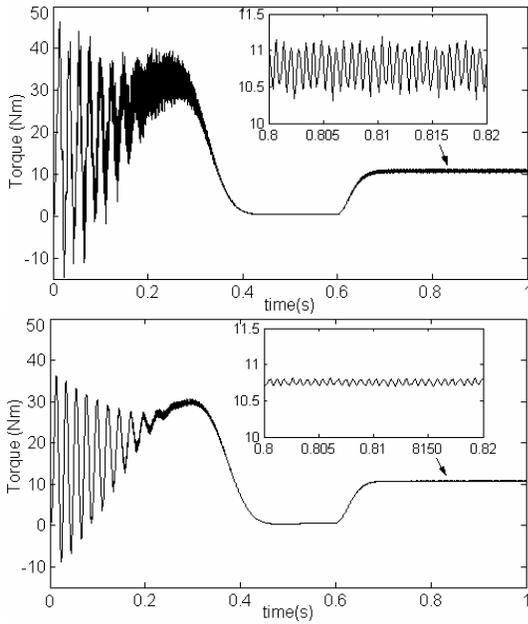


Fig.6. Torque without rotor skew (top) and with rotor skew (bottom).

Fig. 7 shows the stator current at steady state for a loaded machine as a result of simulation analysis. Once again, if skew is neglected a slot ripples rise in the computed results. The spectra of the stator current of a loaded machine are shown in Fig. 8. It is obvious that besides the supply frequency component, only two higher frequency components exist around the PSH1,2 as was predicted by:

$$f_{slot} = \left[ \frac{N_b}{p} (1 - s) \pm 1 \right] f_s \quad (14)$$

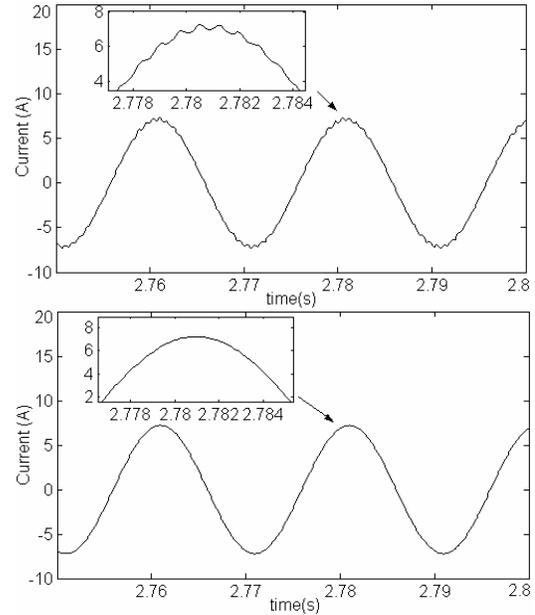


Fig.7. Stator current without rotor skew (top) and with rotor skew (bottom) at steady state.

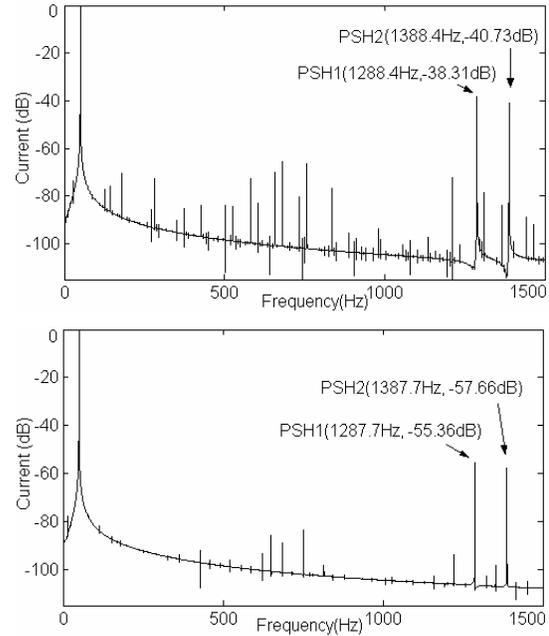


Fig.8. Spectra content of stator current without rotor skew (top) and with rotor skew (bottom) for a healthy motor.

where  $f_s$  represents the supply frequency,  $s$  the slip,  $N_b$  the number of rotor slots, and  $p$  is the pole-pair number. The magnitude of frequency associated to 1288.4Hz (1287.7Hz) and 1388.4Hz (1387.7Hz) are modified with the skewing.

The use of skew reduces clearly the amplitude of slot harmonics [11].

In fig. 9, the spectra content of stator current with one broken bars are presented, for the skewed and unskewed rotor. In case of broken rotor bars, the rotor is electrically asymmetric and the backward rotating field is created. The current spectrum reveals sidebands expected around the supply frequency given by:

$$f_{bb1} = [(1 \pm 2s)]f_s \quad (15)$$

In order to have a better understanding of rotor broken bar, it may be necessary to examine the higher frequency components of the frequency spectra. When we took into account the space harmonics, additional sidebands appear as was predicted by [12]:

$$f_{bb2} = \left[ \frac{k}{p} (1 - s) \pm s \right] f_s \quad (16)$$

where,  $k/p = 1,3,5,7, \dots$

It is noticed that the amplitude of the harmonics was reduced by the rotor skew, especially at high frequencies. Moreover, the slot harmonics were not affected clearly by the broken bars.

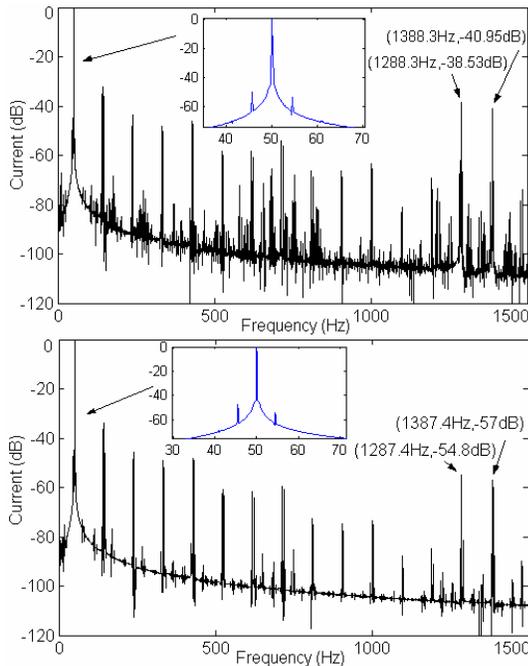


Fig.9. Spectra content of stator current without rotor skew (top) and with rotor skew (bottom) for one broken rotor bars.

For including the harmonics of the power supply, we have injected the real signal resulting from the acquisition. In fig. 10, the spectrum of stator current with one broken bar is presented, for the skewed rotor.

In fig. 10, we can notice the presence of the principal slots harmonics (PSH) and harmonics due to the power supply.

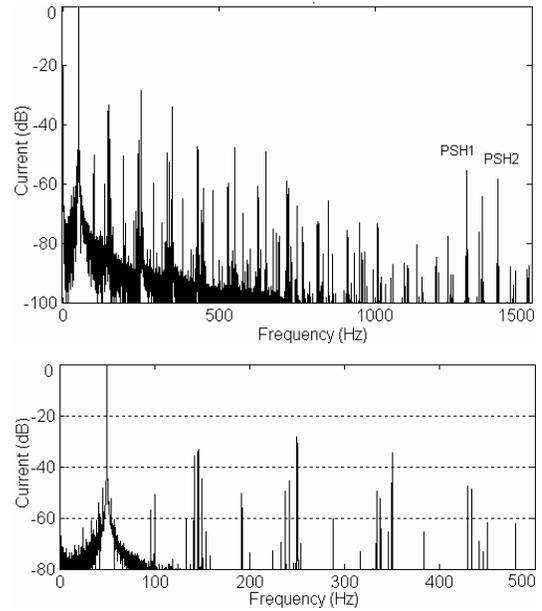


Fig.10. Spectra content of stator current with one broken bars (top), frequencies 0 to 500Hz (bottom).

## V- EXPERIMENTAL RESULTS

The experimental tests were carried out on an experimental bench. Fig. 11 shows the simulation results of the operation of the same induction motor used in simulation.

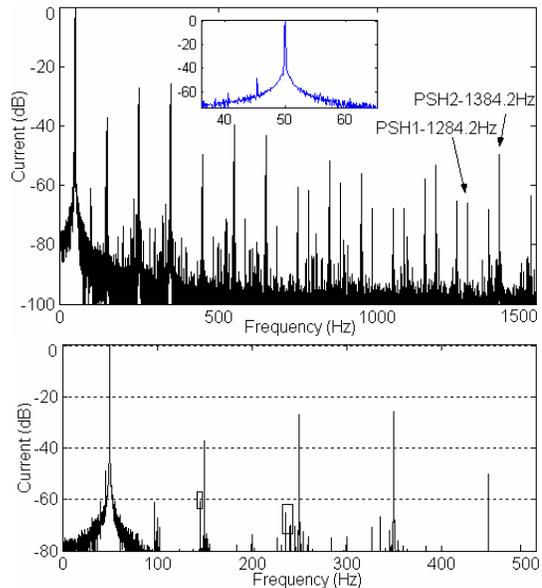


Fig.11. a-Spectra content of stator current for a healthy motor (top), frequencies 0 to 500Hz (bottom).

The spectrum of fig. 11 shows that even for a motor in a healthy state, there are always frequency components but of low amplitudes, this is due to the natural asymmetry of the motor, and on the other hand from the power supply. We notice the presence of the slot harmonics in addition to the harmonics due to saturation. As envisaged during simulation, we show the presence of the harmonics components of high frequencies. The spectra of figure. Fig. 12 shows that for a motor with one broken bar, The stator current frequency described by (15) and (16) can be detected over the observation bandwidth between 0 Hz and 500 Hz.

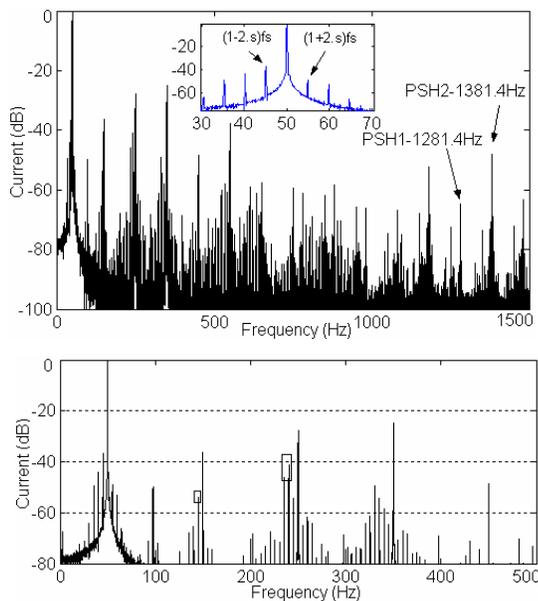


Fig.12. Spectra content of stator current with one broken bar (top), frequencies 0 to500Hz (bottom).

By comparing the spectra resulting from the motor with one broken bar of the experimental test and simulation, we can notice that the additional frequencies which appear in the spectrum resulting from the simulation of the motor supplied with the real signal resulting from acquisition are due primarily to the harmonics of times contained in the supply voltage and in particular frequencies 150Hz and 250Hz what enables us to say that the differences with simulation and the practice results are mainly in the approximations adopted at the level of the power supply.

## VI. CONCLUSION

This paper has presented an analytical approach applied to the induction motor with an extension of modified winding function approach taking into account the influence of rotor skew, and the broken rotor bars. Comprehensive experimental results verifying the developed theory are also presented. The effects of rotor skew were investigated. The

torque ripples are reduced by the skewing, but not eliminate completely. The presence of skew in an induction motor produce an axial variation in the magnetic field witch is not taken into account by the model. Permeance variation due to saturation, rotor and stator slotting, loading have strong influence on the magnitude of the detected harmonics components.

## APPENDIX

- Machine parameters:  $g = 0.000172m$ ,  $Nb=28$ ,  $Ne=36$ ,  
 $r = 0.0516m$ ,  $w = 80$ ,  $l = 0.125m$ ,  $L_b = 0.000172H$ ,  
 $L_e = 0.009594H$ ,  $R_s = 2.86\Omega$ ,  $R_b = 2.856e-5\Omega$ ,  
 $R_e = 1.8560e-005\Omega$ ,  $J = 0.023976 \text{ kgm}^2$ ,  $\gamma = \pi/14rad$ .  
 $\beta = \pi/36$ .

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