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Réf. :

Robust Control for uncertain Non-Linear Quadrotor

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Dedications

We dedicate this modest work to :

Our beloved parents for their encouragement, affection, advices, support and sacrifice.

All our families.

All our friends and colleagues.

All our teachers.

Acknowledgements

We offer our heartfelt gratitude to **ALLAH** for giving us both the perseverance and resolve which enabled us to finish this work.

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The evaluation jury members receive appreciation for accepting their role and reviewing this work :

Pr. Terki Nadjiba professor at the University of Biskra and the jury president.

Dr. Mihi Assia professor at the University of Biskra and the jury examiner.

Finally, We thank our families , our teachers and our friends for their patience, encouragement, and support which were very useful to us during our work.

Abstract

This work is concerned with modeling and control of Quadrotor Unmanned Aerial Vehicles (UAVs) that are rapidly used in civil and military domains because of their mobility and flexibility. However, their nonlinear , underactuated nature and sensitivity to disturbances make control strategy an open issue. In this work, a dynamic model of a quadrotor is derived using Newton-Euler approach with external disturbances to reflect real word conditions.Two control strategies are presented and compared: the widely used Proportional-Integral-Derivative (PID) controller and a more advanced controller which is Sliding Mode Control (SMC) one, including a Second-Order Sliding Mode to face the chattering issue. Simulations using MATLAB/SIMULINK are executed within both normal and disturbed scenarios. Results indicate that although PID operate satisfactorily in normal situations, SMC offers high robustness and better tracking accuracy when disturbances and uncertainties act. These results demonstrate the benefits of SMC for reliable and fault-resistant quadrotor control, especially in dynamic and uncertain environments.

Keywords : Quadrotor, UAV, nonlinear, disturbances, uncertainties, Newton-Euler, PID control, SMC control, Robustness.

Résumé

Ce sujet s'intéresse à la modélisation et au contrôle des drones quadrotors, rapidement utilisés dans les domaines civil et militaire en raison de leur mobilité et flexibilité. Cependant, leur nature non linéaire, sous-actionnement et sensibilité aux perturbations rendent la stratégie de contrôle un sujet ouvert. Dans ce travail, un modèle dynamique de quadrotor est dérivé par l'approche Newton-Euler avec perturbations externes pour refléter les conditions réelles. Deux stratégies de contrôle sont présentées et comparées : le contrôleur proportionnel-intégral-dérivé (PID), largement utilisé, et un contrôleur plus avancé, le contrôle par mode glissant (SMC), incluant un mode glissant du second ordre pour gérer le problème de l'oscillation. Les simulations par MATLAB/SIMULINK sont réalisées dans des scénarios normales et perturbés. Les résultats indiquent que, bien que le PID fonctionne de manière satisfaisante en situation normale, le SMC présente une grande robustesse et une meilleure précision de suivi en cas de perturbations et d'incertitudes. Ces résultats démontrent les avantages du SMC pour un contrôle fiable et résistant aux pannes des quadrotors, notamment dans les environnements dynamiques et incertains.

Mots-clés : quadrotors , UAV, non linéaire, perturbations, incertitudes. Newton-Euler, PID contrôle, SMC contrôle, robustesse.

ملخص

يتناول هذا الموضوع نمذجة والتحكم في الطائرات بدون طيار رباعية الدوارات (UAVs) التي تُستخدم بكثرة في المجالات المدنية والعسكرية نظرًا لقدرتها على الحركة والمرونة. ومع ذلك، فإن طبيعتها غير الخطية وغير النشطة وحساسيتها للاضطرابات تجعل استراتيجية التحكم قضية مفتوحة. في هذا العمل، تم اشتقاق نموذج ديناميكي لطائرة رباعية الدوارات باستخدام نهج نيوتن-أويلر مع اضطرابات خارجية تعكس ظروف العالم الحقيقي. يتم تقديم استراتيجيتين للتحكم ومقارنتهما: وحدة التحكم التناسبية التكاملية المشتقة (PID) المستخدمة على نطاق واسع ووحدة تحكم أكثر تقدمًا وهي وحدة التحكم في وضع الانزلاق(SMC) ، بما في ذلك وضع الانزلاق من الدرجة الثانية لمواجهة مشكلة التذبذب. يتم تنفيذ عمليات المحاكاة باستعمال MATLAB/SIMULINK في كل من السيناريوهات العادية والمضطربة. تشير النتائج إلى أنه على الرغم من أن PID يعمل بشكل مرضٍ في المواقف العادية، إلا أن SMC تُظهر متانة عالية ودقة تتبع أفضل عند حدوث الاضطرابات والشكوك. توضح هذه النتائج فوائد SMC للتحكم الموثوق به والمقاوم للأخطاء في الطائرات الرباعية الدوارة، وخاصة في البيئات الديناميكية وغير المؤكدة.

الكلمات المفتاحية : طائرة رباعية الدوارات، طائرة بدون طيار، غير خطية، اضطرابات، تحكم، نيوتن-أويلر، PID، SMC، المتانة، الشكوك.

Table of Contents

List of Tables

List of figures

List of Acronyms

UAV	Unmanned Aerial Vehicule.
PID	Proportional-Integral-Derivative.
DOF	Degrees Of Freedom.
SMC	Slidng Mode Control.
SOSMC	Second-Order Sliding Mode Control.
USA	United States of America.
VTOL	Vertical Take Off and Landing.
CW	Clockwise.
CCW	CounterClockWise.
LQR	Linear quadratic regulators
QUAV	Quadrotor Unmanned Aerial Vehicle.
HOSM	High-Order sliding modes
MATLAB	Matrix Laboratory.

General Introduction

General introduction :

Quadrotors, a subset of UAVs, have developed into a remarkable technology in military, civilian, and industrial applications. Within the context of technological innovation rapidly evolving along numerous fronts, UAVs come to the forefront. They are sophisticated aerial platforms that enable unparalleled surveillance, infrastructure inspection, renewable energy monitoring, cinematography, among many others. However, quadrotor's complex dynamics and agile operational environments require advanced development in control methods for guaranteeing stable, accurate, and robust performance. The design that quadrotors have gives great flexibility due to the four-rotor configurations. However, despite their potential in maneuverability prowess, these systems are challenged with :

* Varying system dynamics that are complex and nonlinear in nature.
* Weakness to external disturbances.
* Complicated control requirements.
* Precise Attitude and position tracking is needed.

A Traditional control method like Proportional-Integral-Derivative (PID) controller, while widely used, it shows some limitations when confronted with the dynamics of quadrotor systems under uncertain and disturbed conditions.

Objectives :

Focus areas this research will concentrate on include:

✓ **Complete Quadrotor Modeling**

- Create a quadrotor dynamics mathemetical model using Newton-Euler formalism.
- Capture the complex behavior of the system in six degrees of freedom (DOF).
- Include possible external disturbances and uncertain parameters of the system.

✓ **Evaluating Control Strategy Effectiveness**

- Assess and evaluate the effectiveness of classical PID control.
- Examine the Slidng Mode Control (SMC) for alternative control approach feasibility.
- Second-Order Sliding Mode Control (SOSMC) study.

✓ **Performance Based Evaluation**

- Test the conditions on the controllers under optimal and test conditions.
- Examine the parameters of tracking precision, robustness and disturbance rejection.
- Determine the benefits of using advanced control approaches.

Work plan :

Chapter 1 : State of the Art :

- Comprehensive literature review on UAV technologies.
- History of UAVs
- Control Methodologies.

Chapter 2 : Modeling of Quadrotor :

- Study of Quadrotor dynamics.
- Mathematical Modeling of Quadrotor systems.

Chapter 3 : Sliding Mode Control (SMC):

- Sliding Mode Control (SMC) design.
- Develop Second Order Sliding Mode Control (SOSMC)
- Chattering and its causes and solutions.

Chapter 4 : Simulation Results : Comparison Between PID and SMC :

- MATLAB/Simulink simulation
- Comparative analysis of controllers performance

CHAPTER 1 : State of the Art.

I-1. Introduction :

In today's rapidly evolving era, advancements in science and technology are leading the charge in driving innovation and reshaping industries. From healthcare and manufacturing to everyday devices, these technologies are shaping the future, making life more convenient, faster, and more efficient.

Over the past decade, technological advances have enabled the design and construction of miniaturized airplanes and helicopters, with increasingly sophisticated capabilities for autonomous flight. These devices, commonly referred to as drones, are gaining growing interest from both industrial sectors and universities due to their applications in both civilian and military fields.In this section, we provide a brief overview of UAVs (Unmanned Aerial Vehicles) in general, with a particular focus on quadrotors.

I-2. Unmanned Aerial Vehicles (UAVs) :

Unmanned Aerial Vehicles UAVs as shown in **Figure I.1** or as commonly known as **drones** are remotely controlled or autonomously programmed aerial vehicules that carry out multiple tasks (surveillance, data collection, delivery, mapping, military use ...) without the need of human boarding direct control, UAVs range in size from small simple use ones to large military-level ones. Although these evolving aircrafts are seeing enormous growth, However, there still exists many technical points and trials during the development of UAVs, which still remain completely unstudied and unsolved. Also, many requirements related to these aircraft are neglected, such as their high controlled robustness , their restricted-energy capacity, and their optimal tracking. Consequently, a strong knowledge of the current UAV assisted solutions is needed to address the actual issues and to improve the functionality of applications [1].

Figure I.1 : UAV example

I-3. History and background of UAVs :

I-3-1. Beginnings of Drones :

The history of UAVs begins in 1883 when Douglas Archibald attached an anemometer to a kite. He managed to measure wind speed at altitudes of 400m . Five years later, Arthur Batut equipped a kite with a camera and took the first aerial photo on June 20, 1889 in Paris (**Figures I.2, I.3**) . These were the first flying machines equipped for surveillance or detection [2].

Figure I.2 : The kite used by Arthur Batut. **Figure I.3** : Aerial view of Labruguière in 1889.

I-3-2. Military History :

The development of pilotless vehicles began in Britain and the USA during World War I. Britain's "**Aerial Target"** (**Figure I.4**) a small radio-controlled aircraft, underwent its first test in March 1917, while the American "Kettering Bug," an aerial torpedo, had its initial flight in October 1918. Although promising in tests, neither was operationally deployed during the war.

Between the wars, unmanned aircraft technology continued to advance. In 1935, Britain produced several radio-controlled planes as training targets. It's believed the term "drone" originated then,inspired by one of these models named the **DH 82B Queen Bee** (**Figure I.5**). The United States also manufactured radio-controlled drones during this period, using them for target practice and training exercises.

Large-scale deployment of reconnaissance UAVs began during the Vietnam War, where drones took on a variety of new functions, including acting as decoys in combat, launching missiles at stationary targets, and distributing leaflets for psychological warfare.

After the Vietnam War, other nations beyond Britain and the United States began exploring unmanned aerial technology. Newer models became increasingly advanced, with longer endurance capabilities and the ability to fly at greater altitudes. In recent years, innovations like solar power have been introduced to address the challenge of sustaining longer flights.

Today, drones serve diverse purposes, from monitoring climate change and conducting search operations after natural disasters to photography, filming, and delivering goods. However, their most recognized and contentious use is in the military, particularly for reconnaissance, surveillance, and targeted strikes. Since the 9/11 attacks, the U.S. has notably expanded its drone operations, often using them for surveillance in regions too hazardous for ground troops. Drones are also deployed as weapons and have been involved in strikes on suspected militants. Their use in ongoing conflicts, especially in populated areas, has sparked ethical debates due to civilian casualties, whether from inaccurate data or from proximity to intended targets [3].

Figure I.4 : Aerial Target

Figure I.5 : The Queen Bee

I-3-3. Civil History and Applications :

UAV drones are multipurpose devices that offer the potential to reimagine some of the most critical ways humanity operates. Below, we glance at a wide range of applications that are harnessing UAV technology for several purposes [4] :

- **Infrastructure Development and Maintenance with UAV :**

The infrastructure industry faces significant inefficiencies, with large projects often taking 20% longer and costing up to 80% more than planned. Drones offer a solution by improving efficiency, cutting planning and survey costs, providing precise records, and reducing disputes over project status. Investing in drone technology has become highly beneficial for infrastructure companies, as it allows for safer, faster, and more cost-effective inspections of existing structures [4].

- **Continuing Renewable Energy :**

Renewable energy reduces reliance on polluting fossil fuels, but maintaining large wind turbines and solar panels is challenging. Drones help by inspecting turbines, capturing real-time video of power cables, and creating

3D images of turbine blades. They also record hydroelectric dam walls and support the installation and inspection of solar panels using thermal imaging. Drones complete these tasks quickly, efficiently, and with minimal environmental and financial costs [4].

- **Security, Monitoring, and Surveillance :**

UAVs are now essential to military surveillance missions, with numerous countries involving them into defense strategies. These aerial systems support operations like enemy detection, anti-poaching efforts, border security, and maritime surveillance of key sea routes. UAVs offer a low-cost, dependable, and adaptable solution for aerial surveillance and monitoring, helping to prevent illegal activities in designated areas. For example, drones can detect potential threats and monitor movements in restricted zones, automatically alerting authorities with minimal human intervention [5].

- **Cinematography :**

Drones are commonly used in Cinema as they are the director's eyes in the sky for shooting high angle scenes [6].

I-4. Drones classifications :

I-4-1. Classification by size :

Size	Length	Propeller diameter	Weight	Use
Very small drones (Micro)	15 cm or less	5cm or less	200 g or less	Military surveillance.
Small drones (Mini)	Up to 30 cm	7.5 - 15 cm	200 g – 1 kg	• Indoor equipment inspections. • Recreation and photography.
Medium drones	30 - 120 cm	15 - 64 cm	1 kg – 20 kg	• Professional applications. • Amateur photography.
Large drones	120 cm and up	64 cm and up	20 kg and up	• Enemy detection and combat capabilities. • Civil applications such as drone deliveries or filmmaking.

Table I.1: Drones Classifications (By size).

Figure I.6 : **a)** Very small drone, **b)** Small drone, **c)** Medium drone, **d)** Large drone.

I-4-2. Classification by propulsion mode :

- **Fixed wing :**

As we can see in the figure below (**Figure I.7**) fixed-wing drone features a single rigid wing designed to resemble and function like an airplane wing, generating lift without relying on vertical lift rotors. As a result, this type of drone only requires energy to propel forward rather than to stay airborne, making it highly energy-efficient [7].

Figure I.7 : Fixed wing drone.

- **Fixed-Wing Hybrid VTOL (Vertical Take Off and Landing) :**

Hybrid VTOL drones (**Figure I.8**) combine the advantages of fixed-wing and rotor-based designs. With rotors attached to fixed wings, these drones can hover, take off, and land vertically. Although there are currently only a few hybrid models on the market, advancements in technology are likely to make this option increasingly popular in the years ahead. A notable example of a fixed-wing hybrid VTOL drone is Amazon's Prime Air delivery drone [7].

Figure I.8 : Fixed wing hybrid VTOL drone.

- **Single-Rotor :**

Single-rotor drones (**Figure I.9**) are robust and durable, resembling helicopters in both structure and design. They feature a single large rotor, functioning as a spinning wing, along with a tail rotor that provides directional control and stability [7].

Figure I.9 : Single rotor drone.

- **Multi-Rotor :**

Multi-rotor drones are the most accessible and affordable way to achieve an aerial perspective. They provide excellent control over positioning and framing, making them ideal for aerial photography and surveillance. Named "multi-rotor" due to their multiple motors, these drones include types like tricopters (3 rotors), quadrotors (4 rotors as shown in **Figure I.11**), hexacopters (6 rotors as shown in **Figure I.10**), and octocopters (8 rotors), among others. Quadrotors, with four rotors, are by far the most popular type of multi-rotor drone which is the one we are focusing on.far the most popular type of multi-rotor drone which is the one we are focusing on.

Figure I.10 : Multi rotor drone (Hexarotor)

Figure I.11 : Multi rotor drone (Quadrotor)

I-5. Quadrotor :

Quadrotors operate with two pairs of identical fixed-pitch propellers, two rotating clockwise (CW) and two counterclockwise (CCW) as shown in **Figure I.12**. For effective steering, rotors spinning in the same direction are positioned opposite each other [8].

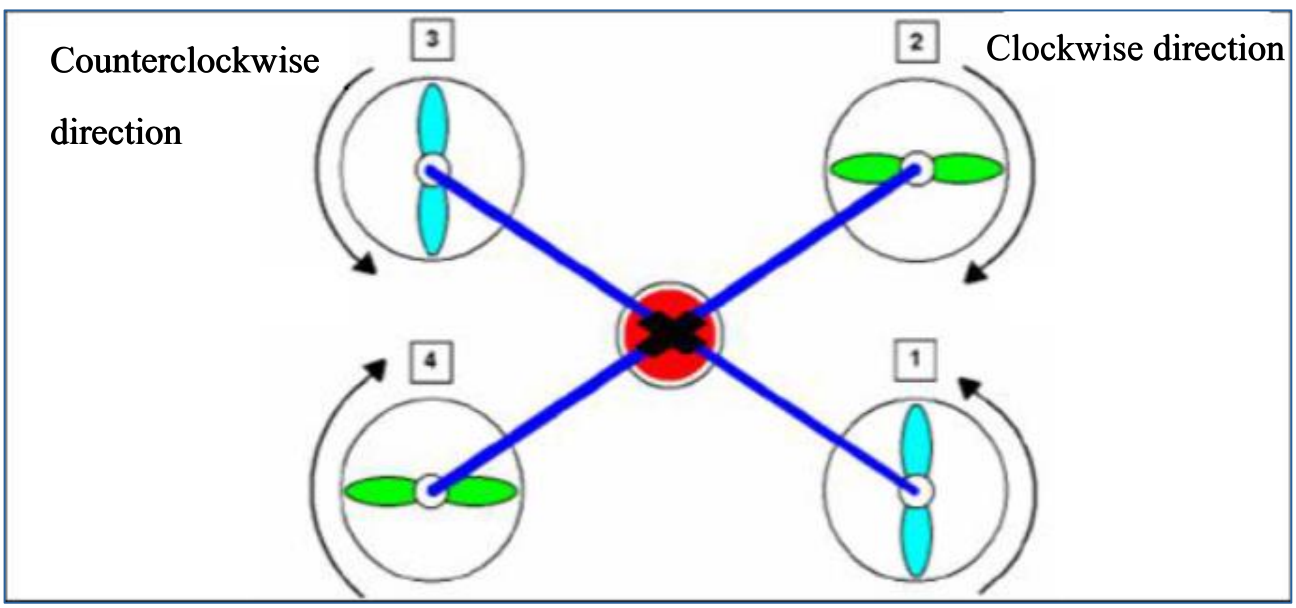

Figure I.12 : Rotors rotations directions.

- I-5-1. Quadrotor advantages and drawbacks :

- **Advantages :**

The quadrotor offers several advantages compared to other configurations:

- Its small size and high maneuverability allow it to move in both closed and open environments while avoiding obstacles.
- Its simple mechanical design makes maintenance easier.
- No clutch is required between the motor and rotors, and there is no need to adjust the rotor blade angle.
- It enables vertical takeoff and landing.
- Control is achieved solely by adjusting the rotational speed of the four motors.
- Its lift capacity is increased due to the presence of four rotors instead of one.
- Its dynamics are less complex than those of a helicopter, reducing the need for rapid reaction times.
- The four small rotors replace the helicopter's large rotor, significantly reducing stored kinetic energy and minimizing damage in the event of accidents [8].

- **Drawbacks :**

- Limited lift capacity: Quadrotors have a limited lift capacity, meaning they can only carry small payloads such as cameras or sensors. This limits their utility for industrial and commercial applications.
- Complex Control : Require complex flight control algorithms,due to the non-linear actuators aerodynamic [9].
- Short flight times.

- I-5-2. Quadrotor control techniques :

Quadrotor UAVs encounter several challenges while performing tasks, including environmental disturbances, obstacles, and both parametric and non-parametric perturbations. These factors make it necessary to apply robust and effective control strategies to stabilize the UAVs and improve their performance. Although various control methods can yield satisfactory results under ideal conditions, their effectiveness and performance levels vary. Among the control methods that have attracted research interest for quadrotor UAVs are both linear and nonlinear control methods, we mention : Linear quadratic regulators (LQR), proportional-integral-derivative (PID) controller, H-infinity ($H\infty$), feedback linearization, sliding mode controllers (SMC), backstepping, and adaptive control. Since these approaches are widely used for attitude stabilization and position control in quadrotors, this thesis provides an overview of these controllers.

- I-5-2-1. Linear control :

- **Proportional-Integral-Derivative Controller (PID) :**

PID controller is a classical control scheme used for several electrical and mechanical systems. It is the most widely used control technique in the industry due to its simplicity, ease of implementation, and acceptable performance with relatively small control efforts. Nowadays, many researchers are employing the PID controller for commercial quadrotor systems [10].

- **Linear Quadratic Regulator (LQR) :**

LQR is an optimal control strategy designed to minimize a cost function, typically a combination of control effort and state error. It is effective for stabilizing quadrotors and improving response time by finding the optimal control inputs to maintain a desired trajectory, and it can handle multiple input and output variables and is suitable for applications requiring precise control.

- **H-infinity (H∞):**

H∞ (i.e. "H-infinity") methods are used in control theory to synthesize controllers to achieve stabilization with guaranteed performance. To use H∞ methods, a control designer expresses the control problem as a mathematical optimization problem and then finds the controller that solves this optimization. [11][21].

- I-5-2-2. Nonlinear control :

- **Feedback Linearization:**

Feedback linearization is one of the common nonlinear control techniques. In this approach, thenonlinear dynamic system is transformed into a linear dynamic system by model inversion. Then, a stabilizing controller can be designed for the linearized system to keep it stable using the linear systems. Many successful applications of this method are reported in the literature [12] [10].

- **Backstepping Control :**

Backstepping is a recursive strategy for controlling nonlinear dynamics systems. This strategy partitioned the control design into several steps that ensure the asymptotic stability of the system at each step. Notably, backstepping requires the exact system dynamics and uncertainties to provide excellent performance [10].

- **Sliding Mode Control (SMC) :**

Sliding mode control is a robust nonlinear control strategy that compensates for system uncertainties and perturbations [13]. The control signal is discontinuous and switches from one state to another to ensure convergence of the states to the reference state. During the flight operation, the quadrotors encountered environmental disturbances and parametric perturbations. They require a very agile and robust control system [14]. Which is the control technique that we will be focusing in this research.

I-6. Conclusion :

The use of quadrotor UAVs is rapidly growing across both military and civilian sectors due to their flexibility and versatility. However, these applications bring several challenges that arise during flight or while performing specific tasks and require careful resolution. Key challenges include dynamic uncertainties, environmental disturbances, underactuation, and the complex nonlinear nature of the quadrotor's dynamics. As such, developing reliable and effective control mechanisms is essential. This work surveys various control approaches applied to quadrotor UAVs, each with unique advantages, limitations, and algorithms. The choice of a suitable controller depends on the desired performance and intended application. As quadrotor applications continue to expand, so does the need for hybrid control techniques that build on established control methods. Notably, some studies have combined multiple control algorithms to enhance controller performance. This thesis reviews recent research employing hybrid control techniques to ensure optimal quadrotor performance, with future studies set to explore cloud-based control, guidance, and navigation for quadrotors.

CHAPTER 2 : Modeling of Quadrotor

II-1. Introduction :

The Quadrotor responds to some very specific flight dynamics, it is the combined effect of the four motors that allows it to orientate and move. It is therefore essential to understand the operation of this type of drone in order to hope to control it. In order to design such a system to ensure that the simulations of the behavior of the Quadrotor are as close as possible to reality, we must first understand the different movements of it, its dynamics and consequently its dynamic equations. In this chapter, we study the flight and movement possibilities of a Quadrotor system with six degrees of freedom, by mathematically modeling its dynamics, as well as the necessary means to express its orientation , the definition of all the physical quantities for the modeling and the identification of all the forces and moments having an impact on the motion model.

II-2. Functioning of the quadrotor :

The Quadrotor has four (04) rotors defined in space by six (06) Degrees of Freedom (DOF), three rotational movements and three translational movements like we see in the next figure (**Figure II.1**). The four rotors are usually placed at the ends of a cross, and the control electronics are usually placed in the center of the cross. The opposite propellers turn in one direction, and the other two turn in the opposite direction in order to prevent the device from rotating on itself [15]. The Quadrotor is an under-actuated system (the number of actuators is less than the number of DOFs to be able to achieve) and its operation is quite particular. By varying cleverly the rotational speeds of the motors, it is possible to make it rise/fall (heave), tilt it left/right (roll) or forward/backward (pitch) or even pivot it on itself (yaw) [16].

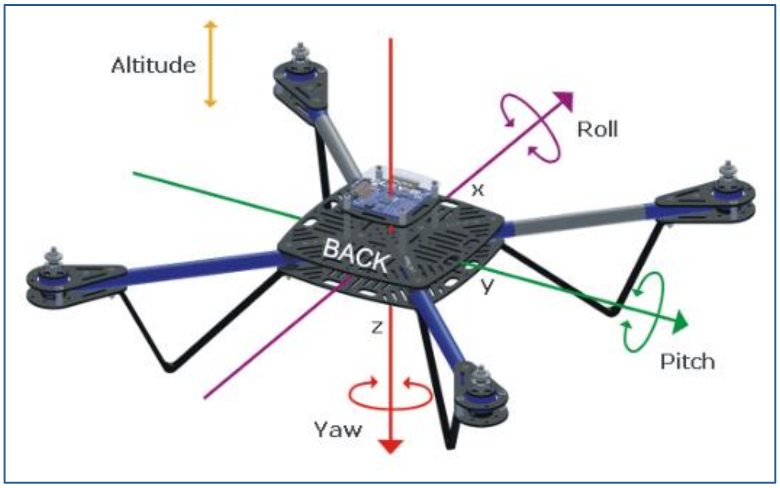

Figure II.1 : Drone's DOFs (Degrees of Freedom)

II-2-1 : Heave movement :

The quadrotor's Heave movement is controlled by uniformly increasing or decreasing the speeds of all propellers. This action creates a vertical force along the z-axis in body frame, causing the quadrotor to either ascend or descend [18] (**Figure II.2**).

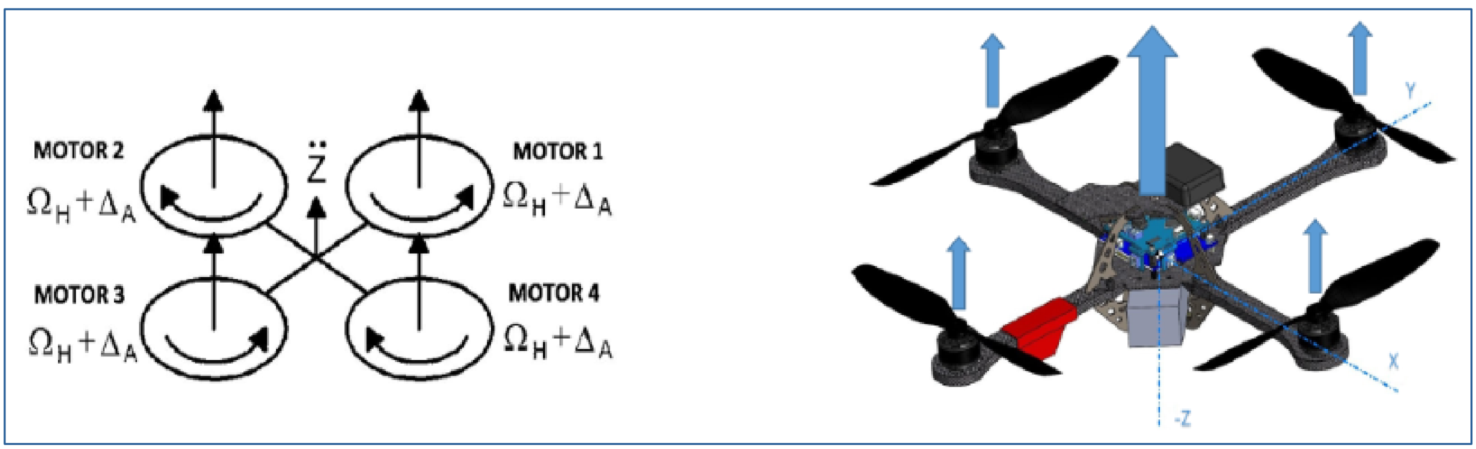

Figure II.2 : Heave movement of a quadrotor.

II-2-2 : Roll movement :

The quadrotor achieves roll movement by adjusting the speeds of its propellers: increasing the speed of the left propeller while decreasing the speed of the right propeller, or vice versa. This variation in speed creates torque along the x-axis in body frame, causing the quadrotor to turn and resulting in roll angle acceleration [20] (**Figure II.3**).

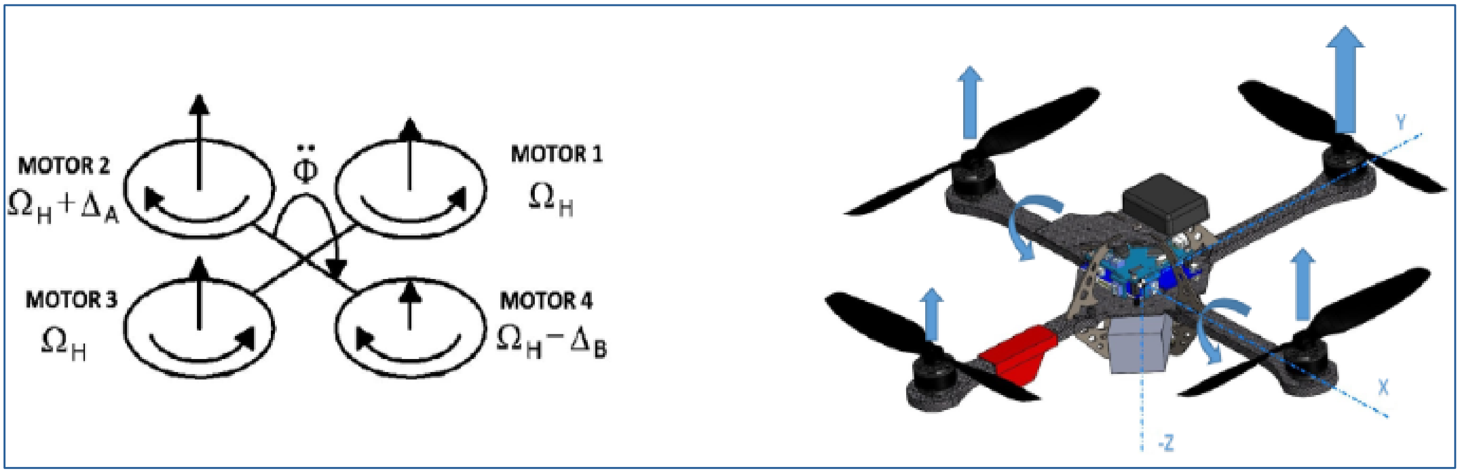

Figure II.3 : Roll movement of a quadrotor.

II-2-3 : Pitch movement :

This movement is almost the same as the roll one and it works by increasing or decreasing the rear propeller speed and by decreasing or increasing the front one. This lead to a torque with respect to the Y-axis in body frame that makes the quadrotor turn [18] (**Figure II.4**).

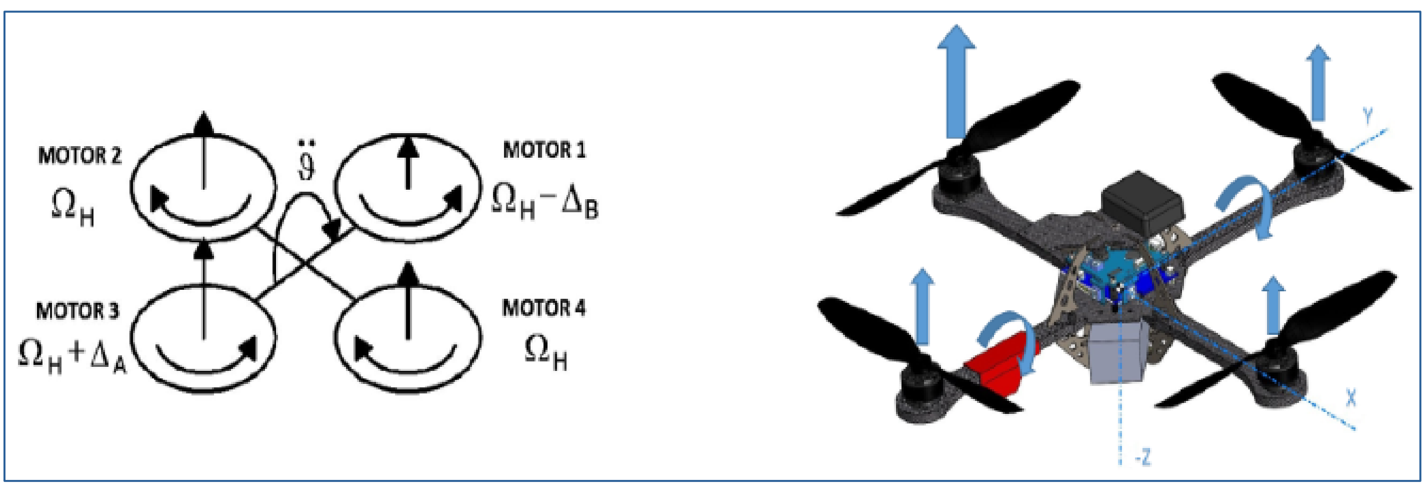

Figure II.4 : Pitch movement of a quadrotor.

II-2-4 : Yaw movement :

Yaw movement in a quadrotor is controlled by adjusting the speeds of the front-rear and left-right propellers. Increasing the speed of the front-rear propellers while decreasing the speed of the left-right propellers, or its opposite, creates torque along the Z-axis in the body frame (**Figure II.5**). This torque causes the quadrotor to turn. The yaw movement is facilitated by the opposite rotation directions of the propellers: the left-right propellers rotate clockwise, while the front-rear propellers rotate counterclockwise.

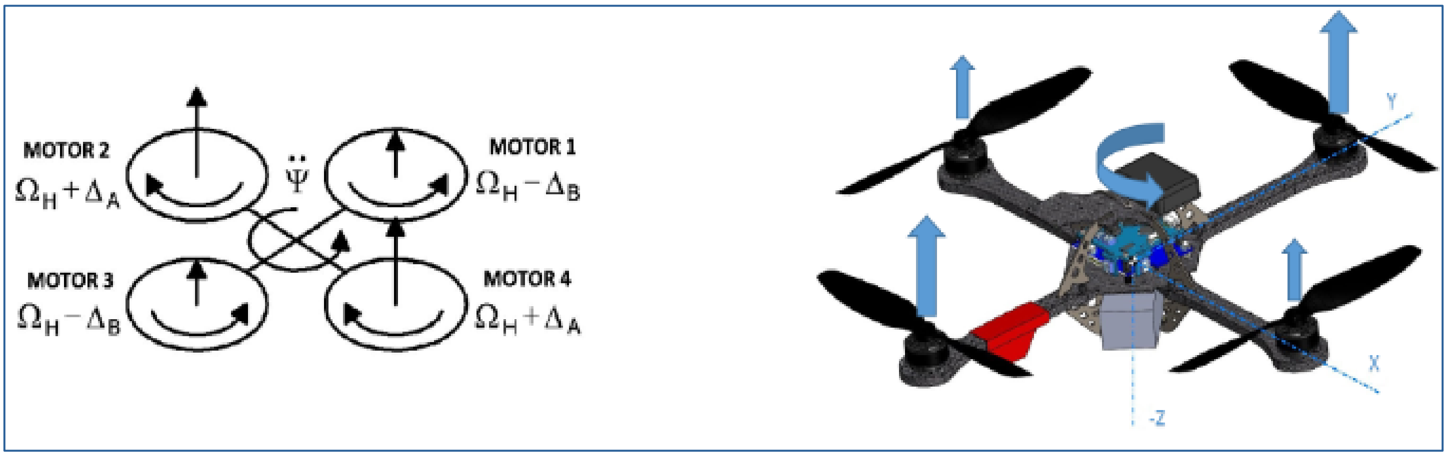

Figure II.5 : Yaw movement of a quadrotor.

II-3. System Modeling :

In this work, the Newton-Euler formalism is employed to derive the dynamics of the Quadrotor. The following assumptions are made for the design :

- The structure is assumed to be rigid and symmetrical.
- The propellers are considered rigid.
- Thrust and drag are proportional to the square of the propellers [17].

So , Let us introduce the Newton-Euler equation, describing the translational and rotational dynamics of a rigid body:

$$\begin{bmatrix} f^{W} \\ \boldsymbol{\tau}^{B} \end{bmatrix} = \begin{bmatrix} m\boldsymbol{I}_3 & 0 \\ 0 & \mathcal{J} \end{bmatrix} \begin{bmatrix} \boldsymbol{a}^{W} \\ \boldsymbol{\alpha}^{B} \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{\omega}^{B} \times \mathcal{J}\boldsymbol{\omega}^{B} \end{bmatrix} \quad \text{(II.1)}$$

f^w : applied total forces expressed in the world frame.

τ^B : applied total torques expressed in the body frame.

m : body mass.

J = moment of inertia about the center of mass.

I_n : n × n identity matrix.

a^w : translational acceleration of the center of mass expressed in the world frame.

α^B : angular acceleration of the body expressed in the body frame.

w^B : angular velocity of the body expressed in the body frame.

II-3-1. Dynamics modeling :

The Quadrotor UAV movement has two subsystems :

- Rotational subsystem : Roll (ϕ), Pitch (θ) and Yaw (ψ).
- Translational subsystem : Altitude (z) and (x,y) positions.

The rotational subsystem is entirely operated as the translational subsystem is under operated [17].

And for our interest in developing a quadrotor controller that encounters problems that we may find in attitude tracking such as parametric uncertainties, external disturbances… , we take in consideration the existance of external disturbances in the modelling of quadrotor dynamic model [18].

II-3-2. Rotational equations of movements :

The link between the torques given to the system and the quadrotor's angular accelerations is described by the rotational dynamics of a quadrotor, which is developed using the Newton-Euler formalism. These formulas take into consideration the propellers' aerodynamic effects as well as the dynamics of rigid bodies. Here are the Rotational equations of movement :

$$J\dot{\omega} + \omega \times J\omega + M_G = M_B \qquad \text{(II.2)}$$

J : diagonal inertia Matrix of QUAV.
ω : angular body rates.
M_G : Gyroscopic moments produced by inertia of rotors.
M_B: moments affecting the QUAV of the body frame.

The first two terms, J$\dot{\omega}$ and ω × Jω, in (II.2) catch the rate of change of the system's angular movement. Therefore the Gyroscopic moments are found to have a value of ω × [0 0 J_r Ω_r] . Thus the Rotational equation for the Quadrotor movement can be written as [17] :

$$J\dot{\omega} + \omega \times J\omega + \omega \times [0\ 0\ J_r\ \Omega_r] = M_B \qquad \text{(II.3)}$$

Where:
J_r : inertia of rotors.
Ω_r: relative speed of rotors: $\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4$
The reason why the equations of movement-rotation under body coordinates, instead of the inertial coordinates, are derived is to obtain an inertial matrix which is time-independent [17].

- **Matrix of inertia :**

For a quadrotor, symmetry instructs that the products of inertia are zero. Consequently, its inertia matrix is diagonal.

$$J = \begin{bmatrix} Ix & 0 & 0 \\ 0 & Iy & 0 \\ 0 & 0 & Iz \end{bmatrix} \qquad \text{(II.4)}$$

Ix, Iy and Iz : inertia around the principle axes of the frame of the body.

II-3-3. Translational equations of movements :

The Quadrotor translational equations of movements use Newton's 2nd law and we take the earth’s inertial frame as a reference :

$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + R\, F_B \qquad \text{(II.5)}$$

$r = [x, y, z]^T$: distance between the quadrotor and the inertial frame.

m : the mass of QUAV.
g : acceleration of gravity $g = 9.81 m/s^2$.
F_B : non-gravitational forces that affect the Quadrotor in the bodywork.

II-4. State space representation :

Formulating the mathematical model obtained for Quadrotor by a state-space model formulates the control problem very simply [17].

II-4-1. State vector X :

The state vector of the QUAV is described by:

$$\chi = [x^1 x^2 x^3 x^4 x^5 x^6 x^7 x^8 x^9 x^{10} x^{11} x^{12}]^T \quad \text{(II.6)}$$

That is mapped to the DOF of the QUAV by:

$$\chi = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T \quad \text{(II.7)}$$

The state vector determines the QUAV position in space with its angular and linear speed [17].

II-4-2. Control input vector U_i :

Control input vector U_i , contains four inputs :

$$U_i = [U_z, U_\phi, U_\theta, U_\psi] \quad \text{(II.8)}$$

$$\begin{aligned} U_z &= k_f\ (\Omega_2^2 + \Omega_2^2 + \Omega_2^2 + \Omega_4^2) \\ U_\phi &= k_f\ (-\Omega_2^2 + \Omega_4^2) \\ U_\theta &= (\Omega_1^2 - \Omega_3^2) \\ U_\psi &= k_M(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{aligned} \quad \text{(II.9)}$$

U_z : the upward force of the four rotors which is the main cause of the QUAV altitude and its rate of change $(z, \dot{z})$.

U_ϕ : the difference in force between rotors two and four that is accountable for the roll rotation with its rate of change (ϕ, $\dot{\phi}$).

U_θ : the variation in thrust between rotors one and three therefore producing the pitch rotation with its rate of change (θ, $\dot{\theta}$).

U_ψ : the difference in torque among the two clockwise rotating rotors with the two counterclockwise rotating rotors producing the yaw rotation with ultimately its rate of change (ψ, $\dot{\psi}$).

Ω_i^i : the specified rotors angular speed.

- We can express the general state space representation of the system model by :

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = F_i(\chi) + G_i(\chi)U_i \\ y_i = x_{i,1}, i \in \{x, y, z, \phi, \theta, \psi\} \end{cases} \quad \text{(II.10)}$$

$x_i \in [x_{i,1}, x_{i,2}]$ is the vector of local state of each subsystem, then $y_i = x_{i,1}$ and $x_{i,2}$, its derivative. the entire six different outputs that this system gives can be split into two sections : the coordinates of location (x,y,z) and orientation (ϕ, θ, ψ) But being that there's only four independent inputs for this complicated system to work on, it isn't at all simple control and individualiaries in each of those six subsystems. To resolve this, two virtual control inputs (u_x, u_y) will be generated to activate the positional subsystems x and y.

$$\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} u_z(c\phi s\theta c\psi + s\psi s\phi) \\ u_z(c\phi s\theta c\psi - s\psi s\phi) \end{bmatrix} \quad \text{(II.11)}$$

The quadrotor model can be considered as a large-scale system, which is composed of six Single-Input Single-Output (SISO) interconnected subsystems. It can be rewritten in interconnected SISO state space equations as follow :

$$\ddot{x} = m^{-1}(U_z(c\psi s\theta c\phi + s\psi s\phi) + p_x)$$
$$\ddot{y} = m^{-1}\big(U_z(c\psi s\theta s\phi - c\psi s\phi) + p_y\big)$$
$$\ddot{z} = m^{-1}U_z(c\theta s\phi) - g + m^{-1}p_z$$
$$\ddot{\phi} = I_x^{-1}\big(lU_\phi + I_y - I_z\dot{\theta}\dot{\psi} - I_r\Omega_r\dot{\theta} + p_\phi\big)$$
$$\ddot{\theta} = I_y^{-1}\big(lU_\theta + I_z - I_x\dot{\phi}\dot{\psi} - I_r\Omega_r\dot{\phi} + p_\theta\big)$$
$$\ddot{\psi} = I_z^{-1}\big(lU_\psi + I_x - I_y\dot{\psi}\dot{\phi} + p_\psi\big)$$

II-5. Conclusion :

This chapter seeks to offer a comprehensive understanding of what a QUAV is, how it works and how best they might be controlled. It starts with quadrotor motion theory, and readers learn why this is very important. Once six degrees of freedom (DOF) and the four rotors thrust interactions are taken into account, understanding whole quadrotor movement patterns becomes clear. The modeling process divides the translational and rotational subsystems: It shows how variations in rotor speed dictate movements such as heave, roll, pitch and yaw.
Using the Newton-Euler method, the equations of motion whose assumptions are structural rigidity, symmetry, and quadratic relationship between propeller thrust or drag force and one's angular velocity are derived as well as their analytical emotive equivalences. These equations are then used in formation of a state-space linearized model for the quadrotor, specifying both state vector representation and input control vectors. Special emphasis is given to the analysis of external disturbances that are necessary in all real control system designs, but more especially when you need control excellent attitude performance as described. However, This is a general study about all the DOF because we are taking only 4 DOF in this thesis : (altitude, roll, pitch and yaw).

CHAPTER 3 : Sliding Mode Control (SMC)

III-1. Introduction :

Exerting control over uncertain and nonlinear systems is one of the most enduring challenges in today's control engineering. A system with considerable nonlinearity, time variant characteristics, or even external perturbations usually does not perform well under traditional linear control approaches. Sliding Mode Control is one of the most preferred because to its simplicity of implementation, strong theoretical structure, and unmatched robustness features.

According to SMC, the system states must cross and remain on a certain region of the surface and perform sliding movements along it. Motion on the sliding surface is scheduled in a way that once the system is on the surface, it undergoes the dynamics of the system designed for it. SMC is well known for providing robustness since during sliding mode, certain predefined uncertainties and external system perturbations can be neglected. An SMC has been used and researched since it was introduced by Utkin back in the 1950s for different applications in robotics, aerospace, automotive systems, and more modern applications in power electronics.

In this thesis, Sliding Mode Control forms an important technique to develop robust controllers for complex nonlinear systems. But this doesn't mean that SMC is unexposed to difficulties. This chapter presents a comprehensive overview of SMC theory, design methods, its major challenge (Chattering), to highlight the practical importance of Sliding Mode Control.

III-2. Concept of SMC :

Sliding mode control uses switching control law to drive the nonlinear system trajectory on to a chosen surface and to maintain the state trajectory on the surface. When system state trajectory is above or below the surface, the switching control law should drive the trajectory back toward the surface as shown in (**Figure III.1**) [19].

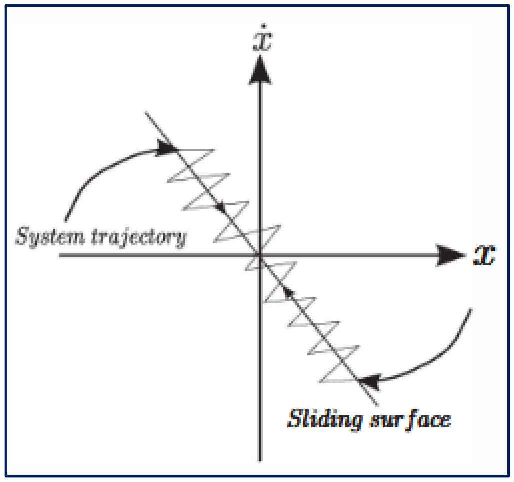

Figure III.1 : Possible trajectories around sliding surface.

This interface is referred to as a sliding surface or sliding manifold. Lyapunov theorem techniques are used for defining the oscillatory motion of the state trajectory mapping onto the sliding surface. With a lyapunov function limit a selected gains of the switching control law such that the derivative of a lyapunov function is guaranteed to be negative definite for increasement ensure movement of state trajectory, and system stability is maintained.

III-3. Controller Design :

SMC objective involves two parts: in the first one, a control law to enforce the error vector toward a decision rule, called sliding surface, during the reaching phase is designed, this part the control is switching on the different sides of this sliding surface and secondly, once the error vector is restricted in the sliding surface, it tracks the dynamics imposed by the equations describing the sliding surface, this second part of the controller is called equivalent control [20].

III-3-1. Sliding Mode Control Design steps :

- **Determine the system model :**

We can apply SMC on systems defined by :

$$\dot{x} = f(x) + B(x)u \tag{III.1}$$

x : state vector

u : control input

$f(x)$: known nonlinear dynamics

$B(x)$: control effectiveness matrix

- **Determine a sliding surface :**

The sliding surface $s(x)$ defines the desired system behavior. For a tracking problem:

$$s = \frac{d}{dt}^{(n-1)} e + \lambda_{n-2} \frac{d}{dt}^{(n-2)} e + \cdots + \lambda_0 e \tag{III.2}$$

While :

$$e = x - x_{\text{desired}} \tag{III.3}$$

λ_i are positive constants chosen to make $s = 0$ stable

- **Design the control law**

The control is split into two parts:

$$u = u_{\text{eq}} + u_{\text{sw}} \tag{III.4}$$

- Equivalent control u_{eq}: keeps the system on the sliding surface (ideal behavior).
- Switching control u_{sw}: brings the system to the sliding surface and counters disturbances.

A typical form of the switching control:

$$u_{sw} = -K \cdot \text{sign}(s) \qquad \text{(III.5)}$$

To reduce chattering (undesirable high-frequency switching), the sign function can be replaced by a saturation or sigmoid function:

$$\text{sat}(s/\phi) \quad \text{or} \quad \tanh(s/\phi)$$

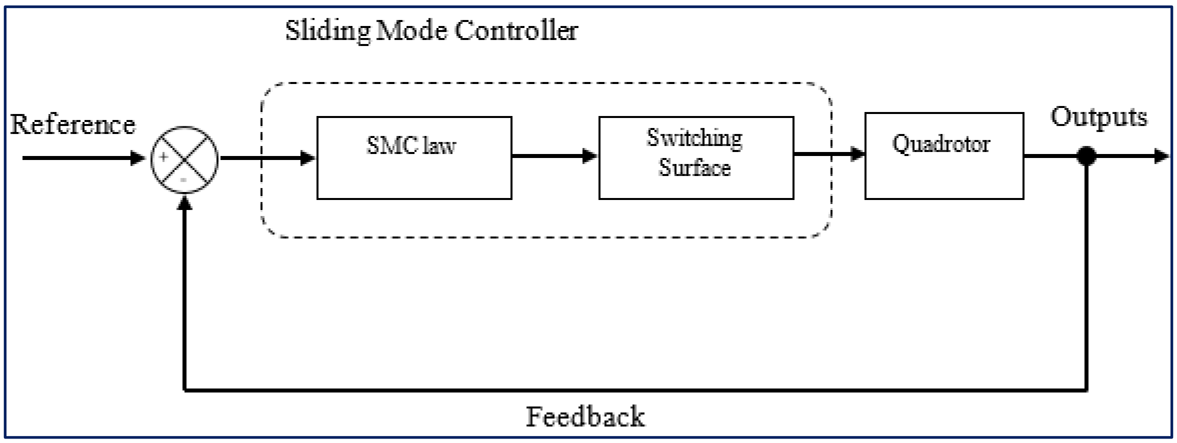

Figure III.2 : Sliding Mode Control block diagram.

III-3-2. Chattering :

Chattering is a phenomenon characterized by rapid oscillations between states in a system or device (**Figure III.3**), often leading to unwanted noise, vibrations, or reduced efficiency. It frequently arises in control systems, mechanical components, and electronic circuits due to excessive switching or instability, that’s why it is counted as the major drawback of SMC.

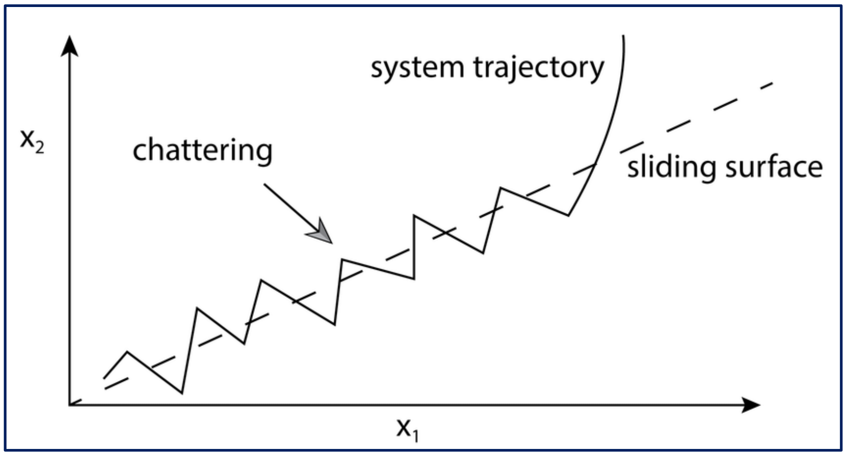

Figure III.3 : Chattering effect on a certain system.

III-3-2-1. Causes of Chattering :

- High-Frequency Switching :

Occurs when the controller tries to correct errors too aggressively or rapidly, leading to rapid on and off transitions.

- Mechanical issues :

loose components or play between parts can cause frequent bouncing or vibrations.

III-3-2-2. Effect of Chattering on a Quadrotor :

In quadrotor control with a Sliding Mode Control (SMC) system, the chattering issue appears as oscillations having a certain frequency within the thrust command for the motors. This results in erratic and jerky movements. Such abnormal control decreases the flight stability of the quadrotor, and increases the energy expenses, and also increases the mechanical wear on the motors and propellers. Furthermore, the unwanted oscillation associated with chattering can harm on-board sensors causing decreased sensor navigation accuracy, and in a worst-case scenario may lead to control failure altogether.

III-3-2-3. Chattering Solution in Quadrotor SMC :

To remove chattering effect, one effective approach to reduce chattering is to use a smooth approximation to the discontinuous signum function, such as a saturation (sat) function or a sigmoid function. This places a thin boundary layer around the sliding surface. Also, through adaptive gain scheduling, the aggressiveness of the controller is altered yielding lower tracking errors, thus reducing the effectiveness of unneeded control switches. The combination of SMC with some low pass filters or even high-order sliding modes (HOSM) helps to enhance the smoothing process of the control signal to still retain robustness against both internal and external disturbances. All these changes help maintain the benefits offered by the SMC in terms of disturbance rejection, while removing the high-frequency oscillations in signals.

III-4. Second Order Sliding Mode Control:

Second-Order Sliding Mode Control (SOSMC) is applied to quadrotor attitude stabilization, building upon the principles of Sliding Mode Control (SMC) [20]. This method was developed to face the chattering issue from the classical SMC. Among the various SOSMC strategies proposed for nonlinear systems, one prominent approach employs a Proportional-Integral-Derivative (PID) sliding surface to enhance tracking performance and robustness :

$$\begin{aligned}\sigma_i &= \dot{s}_i + \beta s_i \\ &= k_d \dot{e}_i(t) + k_p e_i(t) + k_I \int_0^t e_i(v)dv\end{aligned} \qquad \text{(III.6)}$$

e_i : Tracking error.

$k_p, k_I\ and\ k_d$: the proportional, integral and derivative positive gains.

β : a strict positive constant that defines the slope of the sliding line σ_i .

i : $\in \{z, \psi, \theta, \phi\}$.

This also makes the SOSMC subject to the previous control law equation (III.4).

The derivative of the sliding line is :

$$\dot{\sigma}_i = \ddot{s}_i + \beta_i \dot{s}_i \\ = k_{Di}\ddot{e}_i + k_{Pi}\dot{e}_i + k_{Ii}e_i \quad \text{(III.7)}$$

Which takes us to this next equation :

$$\ddot{s}_i = -\beta_i \dot{s}_i + k_{Pi}\dot{e}_i + k_{II}e_i + k_{Di}\left[\ddot{y}_i^d - \mathcal{F}_i - \mathcal{G}_i u_i - \mathcal{D}_i\right] \quad \text{(III.8)}$$

If the system dynamics are well known and the quadrotor system is exposed to uncertainties and external disturbances ($\mathcal{D}_i = \Delta\mathcal{F}_i + \Delta\mathcal{G}_i + h_i$).

$\mathcal{F}_i$: the non linear dynamic function.

$\mathcal{G}_i$: the input control gain of the subsystem.

In this case, u_{eq} for each subsystem is :

$$u_{eq} = \frac{1}{k_D \mathcal{G}_i}\left[-\beta_i \dot{s}_i - k_{Di}\mathcal{F}_i + k_{Di}\ddot{y}_i^d + k_{Pi}\dot{e}_i + k_{Ii}e_i\right] \quad \text{(III.9)}$$

The following Lyapunov function :

$$V_i = \frac{1}{2} \dot{s}_i^2 + \frac{k_{1i}}{2} s_i^2 \quad \text{(III.10)}$$

k_{1i} : a positive constant.

The Lyapunov function derivative :

$$\dot{V}_i = \dot{s}_i \ddot{s}_i + k_{1i} s_i \dot{s}_i \quad \text{(III.11)}$$

To satisfy the reaching condition, the switching control term becomes :

$$u_{swi} = \frac{1}{k_D \mathcal{G}_i}\left[k_{1i}s_i + k_{2i}\text{sgn}(\dot{s}_i)\right] \quad \text{(III.12)}$$

k_{2i}: the control gain.

$\dot{V}$ can be upper bounded as :

$$\dot{V}_i = k_{2i}|\dot{s}_i| + k_{Di}D_i\dot{s}_i \leq -|\dot{s}_i|\{k_{2i} - k_{Di}\overline{D}\} \quad \text{(III.13)}$$

So the quadrotor control system is globally stable according to Lyapunov's method with k_{2i} must be selected as $k_{2i} > k_{Di}\overline{D}$ [22].

III-5.Conclusion :

This chapter gave a general technical background for the development and application of sliding mode control (SMC) for the control of nonlinear and uncertain dynamic systems. The basic principle of SMC is that the sliding mode solution has to reach and remain on a predefined sliding manifold, which provides the ability to successfully control the system under parametric uncertainties as well as external disturbances. The controller design process in this chapter is illustrated with reference to the step-by-step controller design including Lyapunov-based stability analysis to guarantee asymptotic convergence to the sliding surface. The decomposition of the control problem into equivalent and switching components was explained, the switching law to reject disturbances and finite-time convergence.

The key problem of chattering is discussed in both theoretical and practical contexts in the form of high-frequency oscillations that adversely affect actuator longevity and system stability. A variety of mitigation strategies are presented including boundary layer methods (e. g., saturation and sigmoid functions), adaptive gain strategies and filtering. Second-Order Sliding Mode Control (SOSMC) is introduced as a higher-order approach that not only preserves robustness but substantially reduces chattering by continuous control input formulations and tracking accuracy via PID-type sliding surfaces.

In summary, this chapter demonstrates rigorously that while classical SMC has strong robustness characteristics, in practical applications it requires special design approaches (such as SOSMC and smooth switching approximations) to effectively avoid chattering and maintain high performance in real-world nonlinear systems such as quadrotors.

CHAPTER 4 : Simulation Results : Comparison between PID and SMC.

IV-1. Introduction :

In control engineering, the choice of a control method is critical to maintaining system stability, accuracy, and robustness. Between numerous modes of control, the most widely used and fundamentally different are Proportional-Integral-Derivative (PID) control and Sliding Mode Control (SMC). Optimal PID controllers are widely used in linear systems because they are easy to implement and effective. They are widely used in industrial applications too.

A side effect is that Sliding Mode Control is an effective modern nonlinear control method capable of handling system uncertainties and external disturbances. It is a robust and well-conducted form of control that is especially suitable for nonlinear systems with mismatches between model parameters or rapid state changes (such as robotic manipulators, unmanned aerial vehicles and automotive systems).

This chapter provides a systematic and control comparison between the PID controllers as well as SMC controllers, covering the theoretical aspects of each, and the practical aspects of each approach. The analysis is constrained by simulation studies of a quadrotor non linear system, thereby providing a direct comparison with respect to stability, transient response, disturbance rejection and control effort.

With this comparison, readers will gain an understanding of the strengths and weaknesses of each approach to control, and the ability to know which controller is best suited for certain classes of systems and operating conditions.

IV-2. Proportional-Integral-Derivative (PID) controller :

Proportional-Integral-Derivative (PID) controller is one of the most widely used control algorithms for industrial applications due to its simplicity, well-defined design, and operational performance across a wide variety of systems (like a Quadrotor as (**Figure IV.1**)). A PID controller continuously calculates an error value (which is the difference between a desired setpoint and an associated measured process variable) and applies correction using proportional, integral, and derivative terms. Mathematically, the PID control law is expressed as equation (III.7) :

$$u(t) = k_d \dot{e}_i(t) + k_p e_i(t) + k_I \int_0^t e_i(v) dv$$

e_i : Tracking error.

k_p, k_I and k_d : the proportional, integral and derivative gains.

$u(t)$: control input.

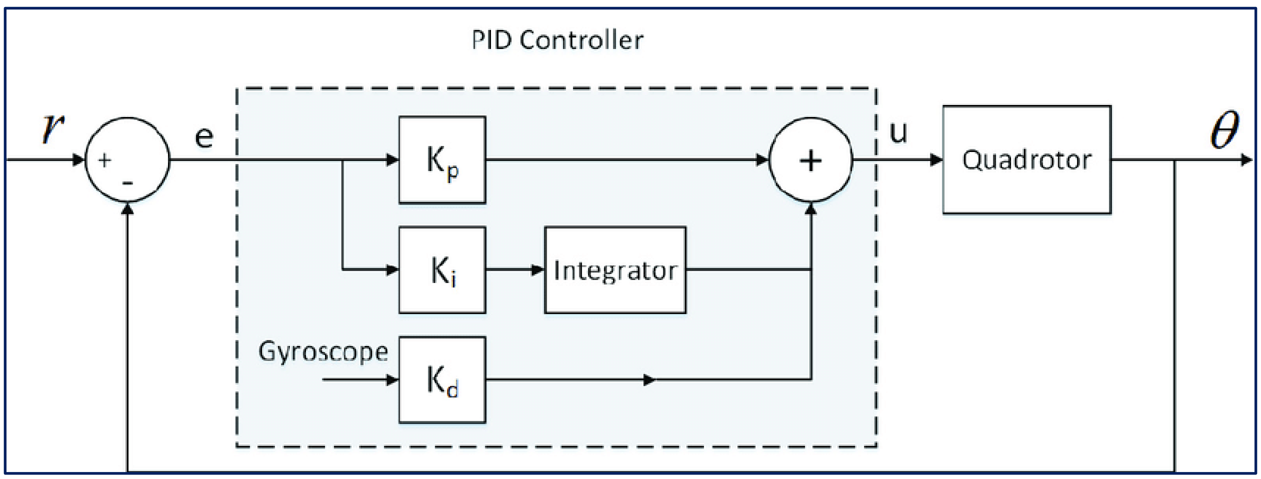

Figure IV.1 : PID control structure for quadrotor.

- Each term in the PID controller plays a distinct role:

Proportional (P): Gives an output proportional to the current error. It will reduce the rise time and steady state error. However it will introduce overshoot.

Integral (I): accumulates error over time, reduces steady state error. But it also slows the response a bit and results in oscillations if it 's not tuned right.

Derivative (D): predicts the definite error of the future function of its change rate. Improves stability of system and reduces overshoot. Not sensitive to noise.

- Though often used in linear and well modeled systems, PID controllers are generally not very effective at:

- Nonlinear dynamics,
- Time-varying parameters,
- External disturbances,
- Systems with significant delays or uncertainties.

While PID control is generally preferred because it 's easy to use, other methods such as Sliding Mode Control are considered for more challenging applications where robustness and adaptability are important.

IV-3. Simulation Results:

In this section , we shall report some simulations (MATLAB / SIMULINK) to show how efficient is the system within the proposed approach (SOSMC that is inspired from SMC) for the attitude dynamic model of a 4 DOF (roll, pitch, yaw, Altitude) quadrotor. Proposed control scheme is tested in two scenarios : First scenario in which there are no external disturbances and uncertainties. Second scenario is where we add external disturbance forces of about 2(N) (**Figure IV.8**) and parameter uncertainties ΔIxx, ΔIyy and ΔIzz are applied at (t > 30s). The desired signal employed is a sine wave with a magnitude of 1(rad) and 1(Hz) Frequency for roll (ϕ), pitch (θ), and yaw (ψ). As for the Altitude (Z), we employed a step signal as a desired signal with a step time of 1(s) and a final value of 1(m) . All this while comparing with the PID controller under the same circumstances in both First and Second scenarios.

Parameters	Value
Ix	7.5×10^{-3} (kg. m^2)
Iy	7.5×10^{-3} (kg. m^2)
Iz	1.3×10^{-2} (kg. m^2)
Jr	6.5×10^{-5} (kg. m^2)
b	3.13×10^{-5} (N. s^2)
d	7.5×10^{-7} (N. $m.s^2$)
l	0.23 (m)
m	0.65 (kg)
g	9.81 ($m.s^2$)

Table IV.1: quadrotor model parameters.

Ix : Quadrotor moment of inertia around X axis.

Iy : Quadrotor moment of inertia around Y axis.

Iz : Quadrotor moment of inertia around Z axis.

Jr: Total rotational moment of inertia around the propeller axis.

b: Thrust factor.

d : Drag factor.

l : Distance to the center of the Quadrotor.

m : Mass of the Quadrotor in Kg.

g : Gravitational acceleration.

- **<u>First scenario :</u>**

A simulation which contained a control comparison between SMC and PID controllers WITHOUT any interference of disturbances and uncertainties using MATLAB / SIMULINK :

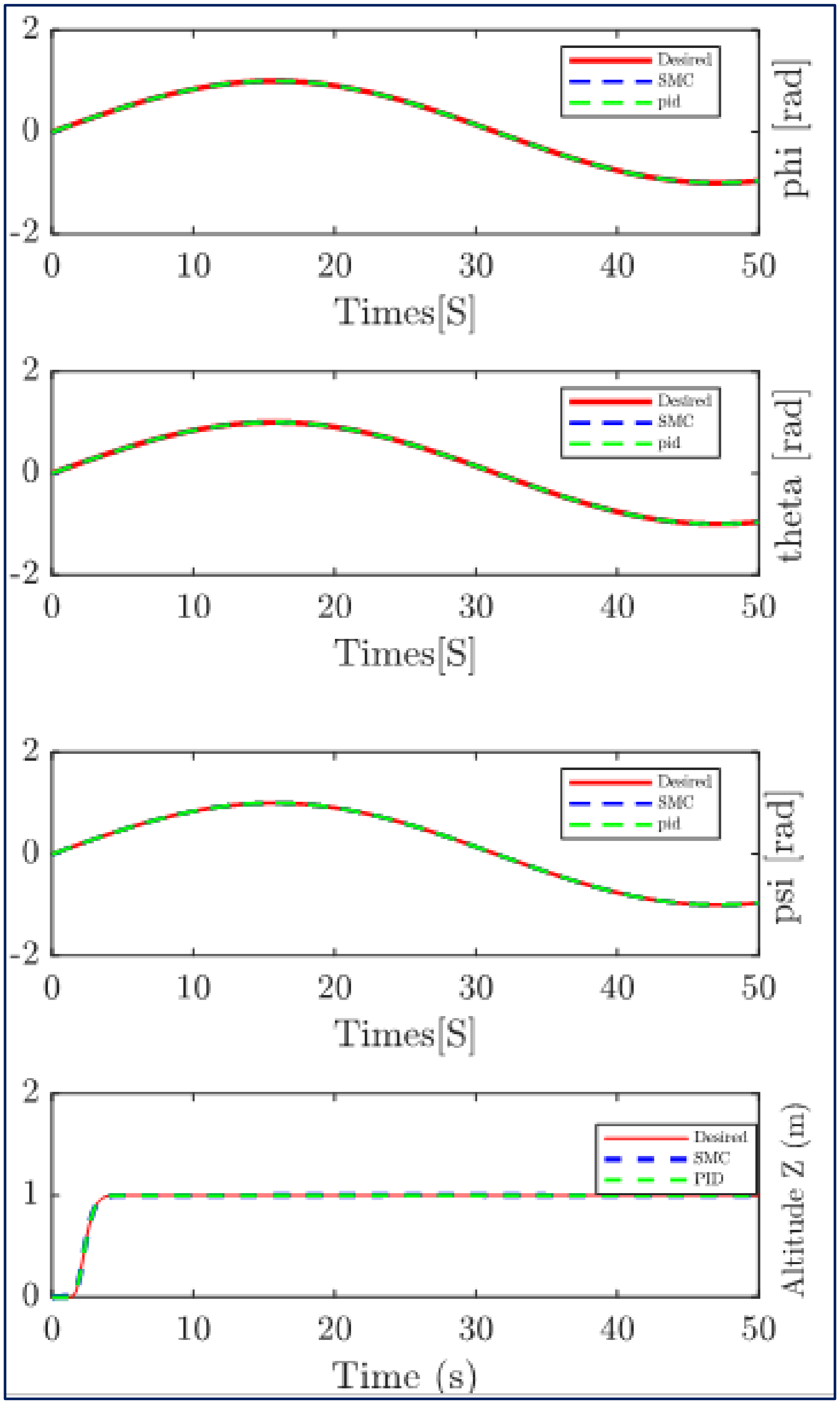

Figure IV.2 : Attitude tracking (First scenario).

- **Interpretation (Attitude tracking : First scenario):**

Both the SMC and PID controllers as shown in **Figure IV.2** make the quadrotor system follow the desired trajectory very closely. The blue and green dashed lines are perfectly superimposed on the solid red line, indicating excellent tracking performance for both controllers due to perfect conditions with no disturbances or uncertainties in all 4 DOF.

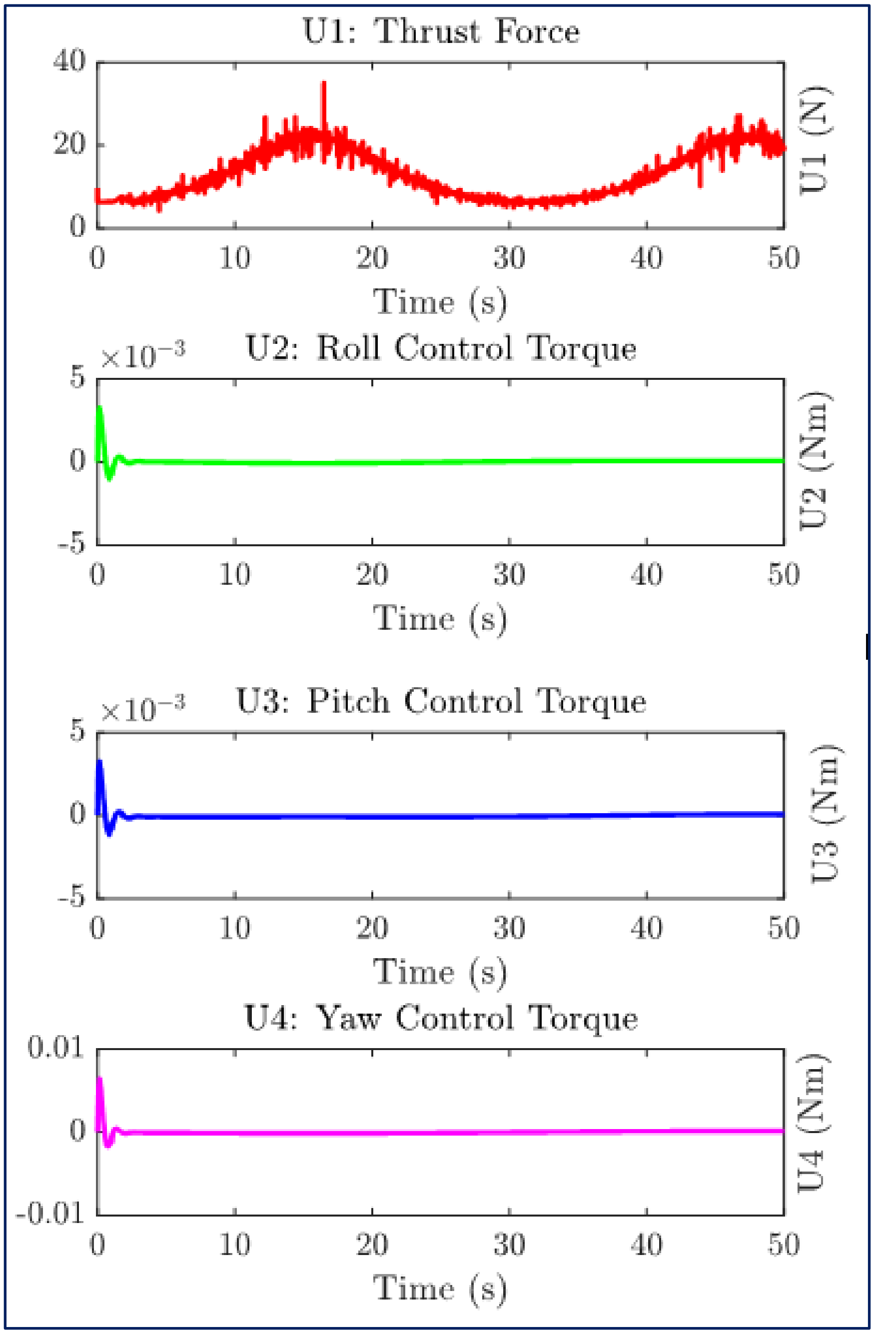

Figure IV.3 : Control inputs (First scenario)

- **Interpretation (Control inputs : First scenario):**

The control input plots in **Figure IV.3** show that the focus is on managing altitude (U1), and the inputs for controlling the position (U2, U3, U4) play a lesser role and adjust quickly. This pattern is common in steady quadrotor flight, where some initial adjustments are needed, but not much control is necessary once things settle down.

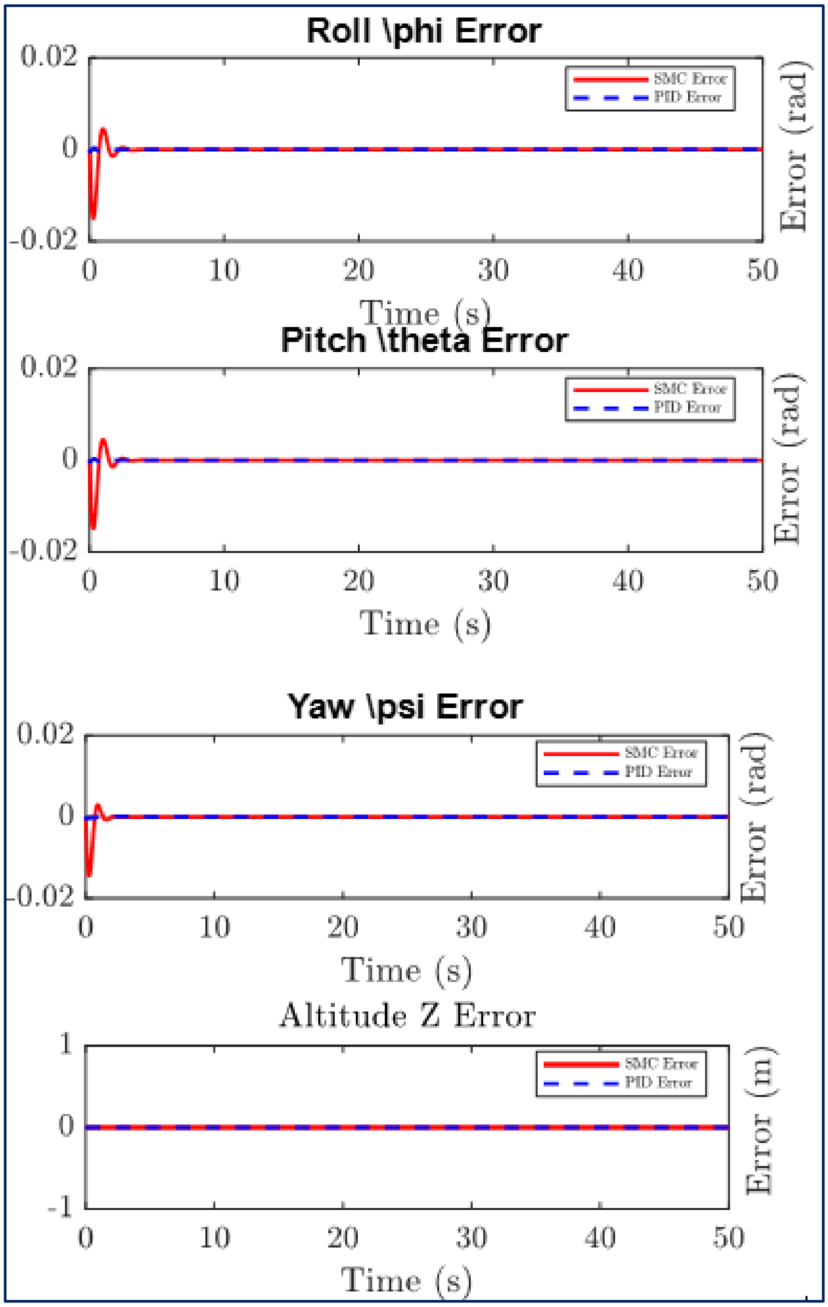

Figure IV.4 : Tracking Errors (First scenario)

- **Interpretation (Tracking errors : First scenario):**

These plots in **Figure IV.4** show how much error there is (compare the attitude tracking described before). They also prove that SMC and PID controllers can get very high accuracy. Errors tend to converge to near-zero values very quickly (within 1-3 seconds) for all 4 DOF, that is to say they have no obvious tracking error at all for both controllers in this simulation. With these error plots you can still not make an argument that one controller is better than the other, both of them are able to reduce and maintain errors at very low levels because there are not any kind of Disturbances or uncertainties applied on the Quadrotor system.

- **Second scenario :**

A simulation which contained a control comparison between SMC and PID controllers WITH the interference of disturbances and uncertainties using MATLAB / SIMULINK :

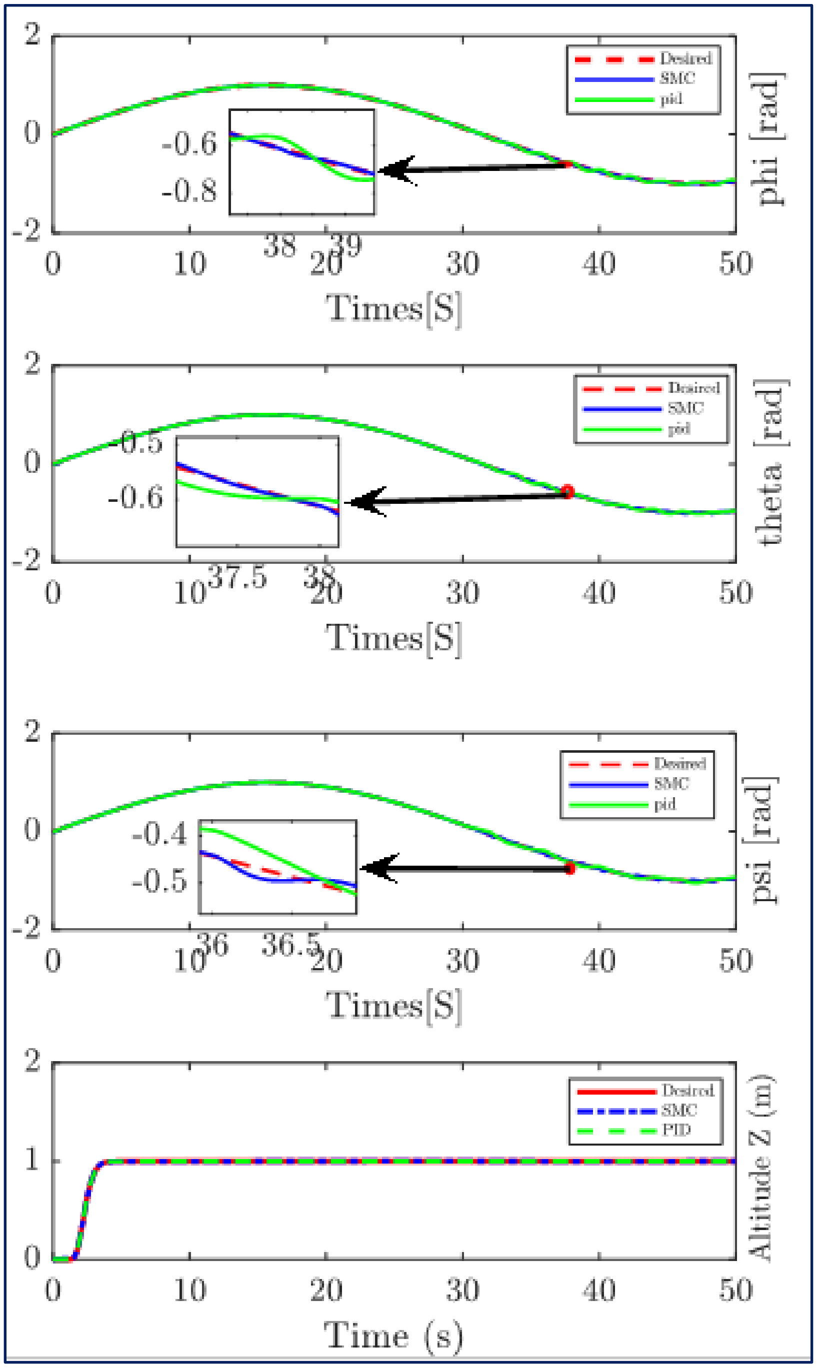

Figure IV.5 : Attitude tracking (Second scenario).

- **Interpretation (Attitude tracking : Second scenario):**

Both SMC (Sliding Mode Control) controllers and PID controllers succeed in reaching the desired altitude about 1m and demonstrate similar performance in the step response with smooth initial rise. On the other Degrees Of Freedom, SMC controls better attitude tracking accuracy than PID, as shown by the magnified insets (blue), which tracks the desired values (red), better than PID (green) (**Figure IV.5**). Precisely, after 30s because of the interference of external disturbances and uncertainties , The relative accuracy of attitude tracking is explained as a result of the resilience of SMC to nonlinear dynamics and uncertainties compared to PID.

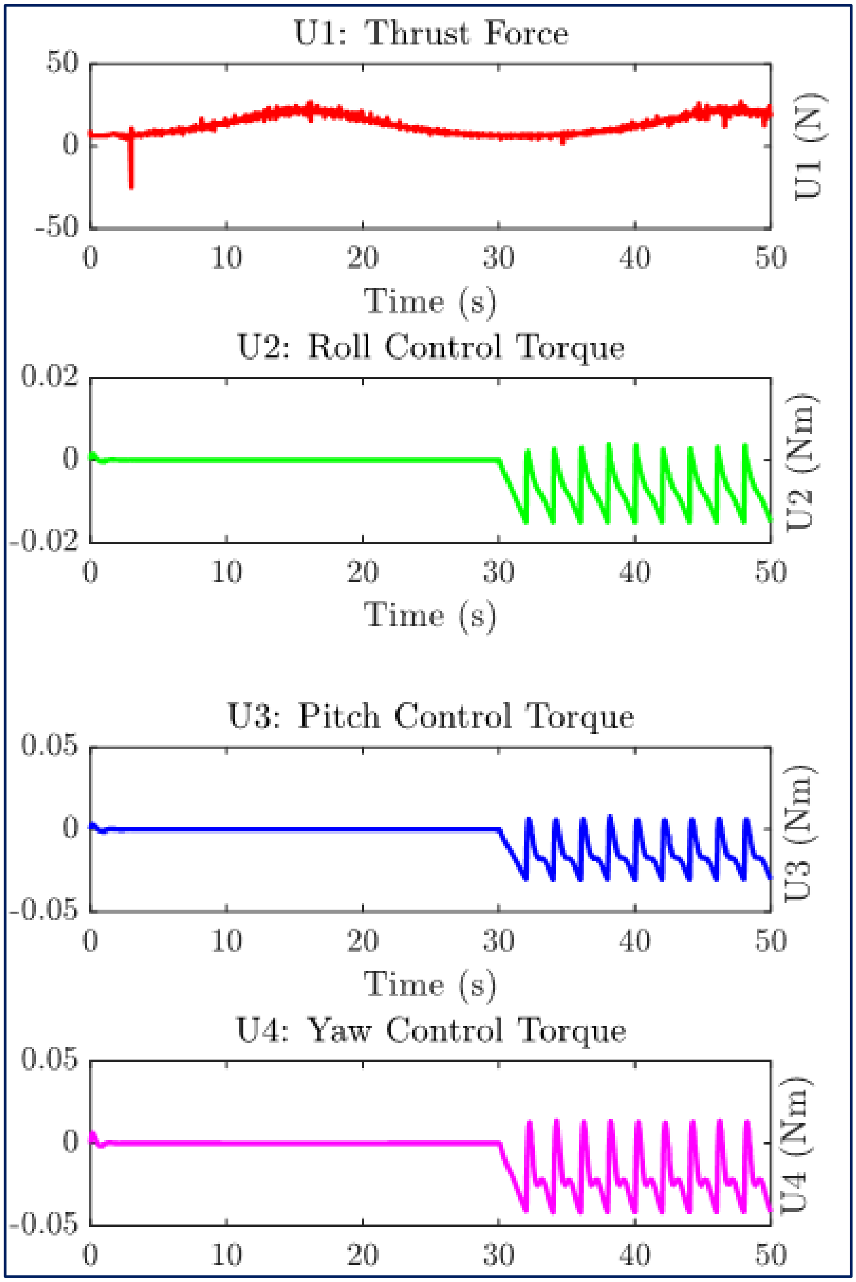

Figure IV.6 : Control inputs (Second scenario).

- **Interpretation (Control inputs : Second scenario):**

All attitude control torques (roll / pitch / yaw) show little activity prior to the 30s and large oscillations after. This clearly indicates that at 30s, A more aggressive maneuver was commanded due to external disturbances that were intentionally introduced which made the control strategy changed. As for thrust force U1, it Maintained a positive value around 10-20N throughout the test, which is expected due to counteract gravity and a slight variation pattern that corresponds to altitude maintenance. The oscillations in the control torques are the controller 's attempt to hold the desired attitude under hard conditions. This plot supports the conclusion that the control conditions experienced after 30 seconds were more difficult in the system and the SMC controller was much more robust in sustaining attitude control. (**Figure IV.6**)

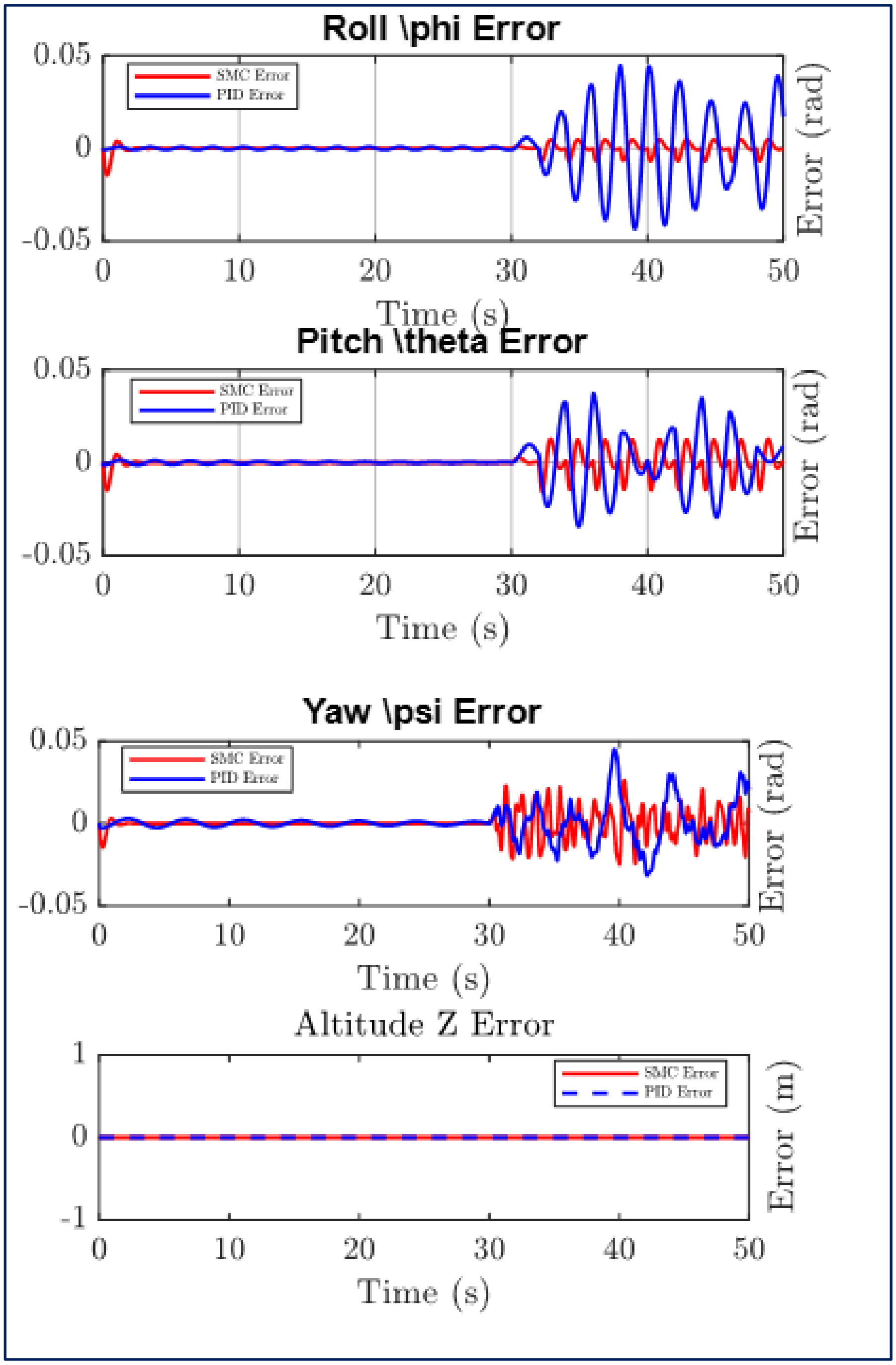

Figure IV.7 : Tracking Errors (Second scenario).

- **Interpretation (Tracking errors : Second scenario):**

The error plots shown in **Figure IV.7**, show a very sharp performance difference between sliding mode control (SMC) and PID control for quadrotor, particularly after 30 seconds of use under difficult conditions. While both controllers exhibit excellent altitude tracking and no significant Z-axis error during the whole test period, attitude control (roll, pitch and yaw) performance changes, SMC exhibits significantly better containing/vibrational error behavior with no fluctuations, while the PID controller develops significant oscillational behavior (reaches about 0. 05 radians) in each of the three rotational axes after 30 seconds. This divergence in performance suggests that SMC is considerably more resilient if disturbances or during more demanding flight conditions be introduced.

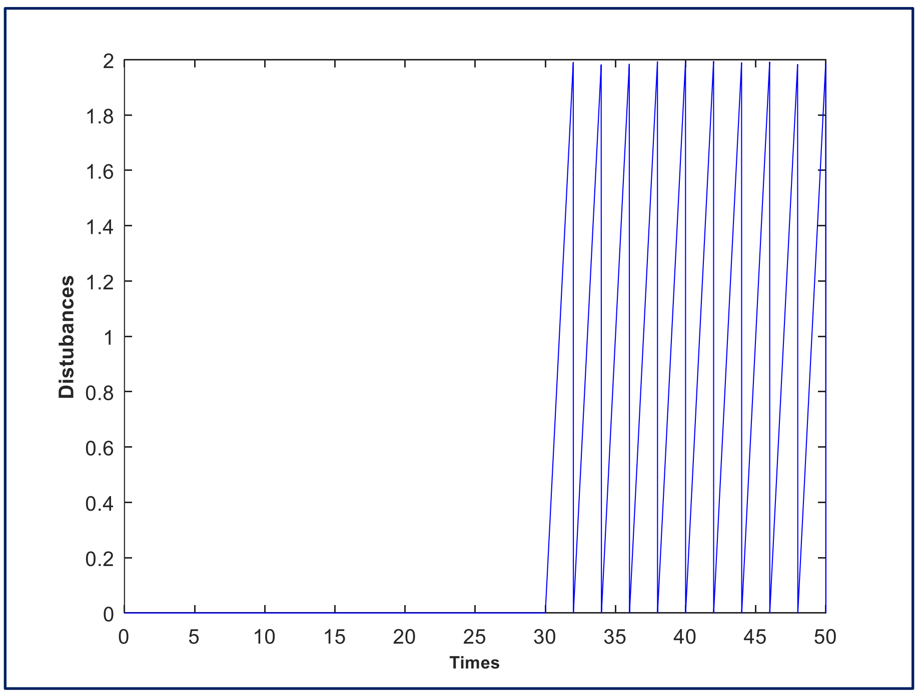

Figure IV.8 : External Disrurbances applied on the quadrotor.

IV-4. Conclusion :

The comparison revealed major findings about the performance of SMC vs PID control for quadrotor systems. Given that both controllers showed similar tracking performance under the best tracking conditions in first scenario , yet when external disturbances and parameter uncertainties were introduced in second scenario , their performances diverged significantly. The SMC controller proved to have better robustness, since it was able to maintain an almost negligible tracking error in roll, pitch, yaw, and altitude. The PID controller developed prominent oscillations with tracking error in the rotational axes (pitch and roll), with error magnitude approaching ±0.05 radians. With this performance differential we have seen SMC's absolute advantage to be able to handle nonlinear dynamics, time-varying parameters, and external disturbances, which supports the theoretical limitations of PID controllers when it comes to non-linear systems under disturbances and uncertainties. The results of the control input measure also supported these results, indicating how that SMC generated a more acceptable disturbance rejection, while managing control efforts more adequately. The ability demonstrated by the SMC favorably provide additional confidence in regards to both previous theoretical implications of SMC to be a strong and efficient approach to tackle challenging control applications, and importantly, those control applications whose real strength is its robust performance . This is critically important and appears to be an inherent advantage with nonlinear systems like quadrotors when functioning in a time-varying environment or unpredictable space.

Final Conclusion.

Final Conclusion :

A detailed analysis of quadrotor UAVs using advanced control methods for modeling and control and performance evaluation has been presented. The research foundation started with an extensive mathematical model which depended on Newton-Euler formalism to build operational control systems addressing quadrotor dynamics complexities and disturbances. Traditional PID controller performance reveals fundamental constrains through simulation results during disturbances and parameter uncertainty events when system performance becomes substantially worse.

Sliding Mode Control (SMC) functions as an extremely robust control method which delivers outstanding tracking results despite challenging operational conditions. The system obtains enhanced stability characteristics and more continuous control functions through the implementation of Second-Order Sliding Mode Control (SOSMC) and smoothing functions to eliminate chattering effects. The simulation analysis confirms that SMC-based methods exceed traditional approaches through their superior performance in disturbance compensation along with efficient error reduction capabilities and reduced control costs. The experimental findings demonstrate that SMC represents an excellent choice when designing systems which must function precisely and remain resilient.

The future advancement of quadrotor systems' reliability depends on implementing observer-based techniques like Sliding Mode State Observer and Disturbance Observer Sliding Mode for better system autonomy. The research validates SMC's usefulness for high-demand control while creating a foundation for future development of hybrid adaptive systems that may introduce artificial intelligence and cloud control capabilities for dynamic environments.

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