Université Mohamed Khider de Biskra
Faculté des Sciences et de la Technologie
Département de génie électrique

MÉMOIRE DE MASTER

Sciences et Technologies
Automatique
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Présenté et soutenu par :
Bougherara mohmed alaa eddinne

Le : lundi 2 juin 2025

Robust design of PID controller by a multi-objectif meta-heuristic method

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Avis favorable de l'encadreur :
-Prof. Megherbi Hassina

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ABSTRACT:

The current work demonstrates how the use of modern optimization methods, namely, differential evolution algorithm, greatly improves PID controller design, thus effectively overcoming traditional limitations found in mathematically modeled and hand-designed approaches.

The use of a multi-objective optimization approach enabled the balance between temporal efficiency and accuracy while ensuring resilience and robustness against disruptions.

The major findings from this research are outlined below Evolutionary algorithms such as DE enable efficient exploration of the solution space without the need for derived information or precise models.

Design engineering can potentially generate an optimal response even in complex or dynamic systems.

Implementing resilient control principles in this intelligent tuning ensures the continued effectiveness of the controller, even in adverse conditions.

The deployment of these specifications, including ITAE specifications and requirements for robustness, created a soundly solid basis for multi-parametric assessment, ensuring a very high degree of reliability for resulting completed tuning.

The approach utilized here shows a level of flexibility, allowing for adaptation to complex systems, nonlinear dynamics, or industrial contexts requiring adaptive predictive control, thus providing a sound foundation for future educational or career development. [24], [27], [48]

Acknowledgments

I am providing my sincere thanks and deepest gratitude to everyone who contributed towards the completion of this study.

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I also extend my gratitude to my friends and colleagues who helped and supported me at every stage of this study.

I pray God Almighty to make this work in good faith for His sake and make it a contributive science.

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ABBREVLATIONS:

PID – *Proportional–Integral–Derivative*

Meaning: A controller based on three parameters: proportional, integral, and derivative.

DE – *Differential Evolution*

Meaning: An evolutionary algorithm used to discover the optimal solution via a difference between solutions.

GA – *Genetic Algorithm*

Meaning: An algorithm inspired by natural evolution and selection.

PSO – *Particle Swarm Optimization*

Meaning: Improvement using the behavior of flocks of birds or fish.

MOO – *Multi-Objective Optimization*

Meaning: Optimize for more than one conflicting goal at once.

MO-DE – *Multi-Objective Differential Evolution*

Meaning: A version of the DE algorithm that supports multiple objectives.

NSGA-II – *Non-dominated Sorting Genetic Algorithm II*

Meaning: An advanced genetic algorithm for multi-objective optimization.

ABC – *Artificial Bee Colony*

Meaning: An optimization algorithm inspired by bees' behavior in search of food.

Performance metrics:

ISE – *Integral of Squared Error*

Meaning: Integration of the error box over time.

IAE – *Integral of Absolute Error*

Meaning: Integral absolute value of error.

ITAE – *Integral of Time-weighted Absolute Error*

Meaning: Error integration multiplied by time; gives weight to late errors.

ITSE – *Integral of Time-weighted Squared Error*

Meaning: Error square integration multiplied by time.

Modeling and analysis:

CR – *Crossover Rate*

Meaning: The rate of mating between individuals in evolutionary algorithms.

F – *Mutation Factor*

Meaning: The factor of variation used in the DE algorithm.

NP – *Number of Population*

Meaning: The number of individuals in a generation within an evolutionary algorithm.

D – *Dimension of the Problem*

Meaning: The number of variables or dimensions of the problem to be improved.

MIMO – *Multiple Input, Multiple Output*

Meaning: A system that has more than one input and output.

INTRODUCTION

As industrial and computer systems continually evolve, the need for designing efficient and high-performance control systems has become a necessity. Among these control systems, the proportional-integral-differential (PID) controller is the most common and widely used in various distinct applications due to its ease of implementation and ability to yield acceptable performance across a wide class of systems. But the main problem for the engineers is how to calibrate the parameters of this controller reliably and with high efficiency, especially in complex or nonlinear systems.

Here, artificial intelligence and evolutionary application algorithms have been considered to have a high potential more and more, mainly the differential evolution (DE) algorithm, which is one of the most powerful metaheuristic algorithms for searching the solution space and system performance optimization without the need for precise mathematical models.

This paper will explore and apply the DE algorithm to reduce PID controller parameter tuning in a simulation environment through software such as MATLAB/Simulink to obtain a balance of speed of response, error reduction, and increased robustness to disturbances or change. The work involves the use of single- and multi-objective optimization techniques to unveil the ability of the algorithm to tackle modern systems and provide correct and effective solutions.

Chapter One

Introduction to Optimization and PID Control

1.1 Introduction

1.1.1 General introduction of the control system

Control systems are one of the most important components of modern engineering systems. They are used to regulate the behavior of dynamic systems to achieve some desired performance or maintain some desiredstability. The systems are used to control the outputs of a system so that they follow a reference value or respond to particular inputs in a specific and efficient manner. [1]

Control systems find application in a very wide range of industrial and everyday uses, from the extremely simple, such as household thermostats, to the extremely complex, such as space navigation and aircraft control systems. The operation principle of control systems is based on the measurement of certain variables (such as speed or temperature), comparison with set-point values, and adjustment of the system inputs based on this difference (also referred to as error) through a control unit.

Control systems are of two types:

Open-loop control systems: These do not use feedback, the output does not affect the input. This type is simple but sensitive to errors due to changes in the environment or system characteristics.

Closed-loop control systems: These are feedback principle-based, where the output is continuously measured and compared with a reference signal to regulate the inputs. It is more accurate and stable and has widespread application in those requiring precise control. [14]

One of the most popular control tools is the proportional-integral-differential (PID) controller, which plays a significant part in industrial systems due to its efficiency and simplicity. Computing and algorithm development also brought about optimization techniques and intelligent

strategies such as metaheuristics to further improve the performance of control systems, especially in nonlinear or complicated settings.

1.1.2 Types of control systems

Control systems are divided into several types based on their structure, function, and response method. They can be divided as follows:

First: Based on the presence or absence of feedback:

Open-Loop Control Systems:

The control command is sent irrespective of the current state of the system. An example is an automatic washing machine that operates its cycle regardless of whether clothes have been washed or not. [14]

Closed-Loop Control Systems

They are feed back based wherein the output is sensed, with reference to a reference value and input is regulated. They are used in the pursuit of acquiring high accuracy as well as high stability.

Second: Depending upon the type of signal:

Analog control systems: are signal continuous and are used in conventional systems.

Digital control systems: are digital based and are utilized through computers as well as microcontrollers.

Third: Depending on response time:

Continuous-Time: Works with signals that continuously flow through time.

Discrete-time: Processed for specified time intervals.

Fourth: Based on the complexity level:

Linear systems: Modeled using linear equations.

Nonlinear systems: More in agreement with the way the world works, but more difficult to analyze.

1.1.3 History of the development of control systems

Control systems have gone through different stages over the centuries, growing enormously due to industry and technological needs.

Ancient Times:

- Heron of Alexandria used simple methods to control steam.
- Al-Jazari made automatic devices driven by water during the eighth century.

Industrial Revolution:

- James Watt developed the centrifugal regulator for the regulation of steam engines (Dorf R. C., 2021). [3]

Twentieth Century:

- Classical control theory was formed with the help of analytical methods such as the Laplace transform and Bode diagrams.
- In the latter half of the century, modern control theory evolved on the basis of state space models [2], [3] and control of multivariable, multiple-input, multiple-output (MIMO) systems.

Modern Era:

- Evolution of digital control, adaptive control, and optimal and robust control.
- Intelligent control evolved on the basis of artificial intelligence, such as neural networks, fuzzy logic, and evolutionary algorithms.

1.1.4 The importance of control systems in engineering and industrial applications

Control systems are essential in ensuring safe and stable functioning in industrial and engineering systems. Their greatest benefits include:

- **Dynamic stability:** preventing system oscillation.
- **Enhanced performance and accuracy:** reducing error between the reference value and the output.
- **Economy and reduced waste:** maximum resource utilization.
- **Flexibility in adapting:** performing under diverse environmental conditions.
- **Reducing human intervention:** ensuring automation.
- **Compliance with quality standards:** adhering to technical specifications for production. [12], [14]

Practical Application Examples:

Field	Application	Role of the Control System
Energy	Turbine control	Voltage and frequency regulation
Automotive	ABS system	Enhancing safety and reducing skidding
Robotics	Industrial robotic arms	Precise motion and positioning
Aerospace	Autopilot systems	Maintaining aircraft stability
Water Treatment	Desalination and processing plants	Controlling flow and pressure
Water Treatment	Desalination and processing plants	Controlling flow and pressure

Table 1:Representative Industrial Applications of Control Systems

1.1.5 Basic Control System Components

A control system consists of interconnected components that perform specified functions:

1. Plant: Controlled system (e.g., motor or furnace).
2. Sensor: Supplies measurements of output variables.
3. Setpoint: Desired reference value for the output.
4. Comparator: Compares output and reference value to determine the difference (error).
5. Controller: Processes the error and supplies the control command (e.g., PID).
6. Actuator: Implements the control command on the system.
7. Feedback: Connects the output to the comparator to adjust performance. [2]

Basic example of a closed loop control system:

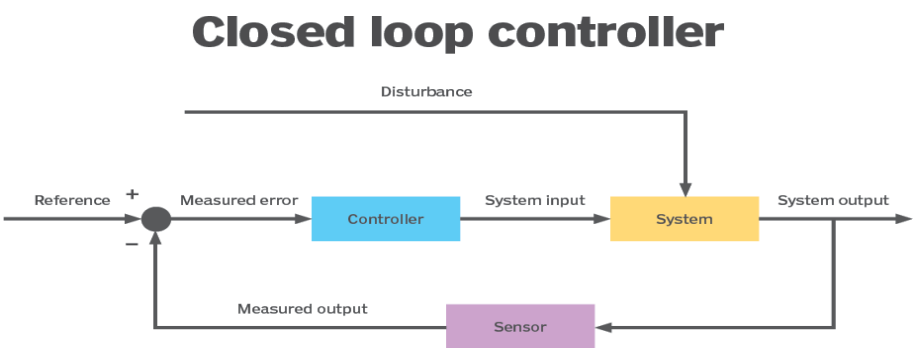


Figure 1: closed loop control system

1.1.6 Types of controllers

Controllers play a pivotal role in control systems, determining how the system reacts to the resulting error between the reference value and the actual output. The types of controllers vary in complexity, accuracy, and responsiveness, and the following are the most important ones:

1. Proportional Controller – P

It depends on the current error only. The output is calculated as the product of the proportionality coefficient (Kp) multiplied by the error.

Formula:

$$u(t) = Kp \cdot e(t)$$

Benefits:

- Simple and fast.
- Reduce error quickly.

Disadvantages:

You can't always eliminate the whole error (a constant error remains).

---

2. Proportional-Integrative Control Unit (PI)

It adds to proportional control an integral element based on the sum of the error over time, allowing the elimination of constant error.

Formula:

$$u(t) = Kp \cdot e(t) + Ki\int e(t)\,dt$$

Benefits:

Reduces stable error to zero.

Widely used in systems that do not bear permanent fault.

Disadvantages:

They may cause delays or fluctuations if not adjusted well.

---

3. Proportional-differential (PD) controller

It integrates proportional control with error differential, i.e. depends on the speed of error change, improving the system's response without affecting the stable error.

Formula:

$$u(t) = Kp \cdot e(t) + Kd \cdot \frac{de(t)}{dt}$$

- **Benefits:**
- Reduces oscillation and improves stability.
- Quickly responds to changes.

Disadvantages:

Do not eliminate the stable error.

---

4. PID controller (Proportional–Integral–Derivative)

It is the most common and commonly used, combining all three elements: proportional, integrative, and differential, to achieve an accurate and stable response.

Full equation:

$$u(t) = Kp \cdot e(t) + Ki\int e(t)\,dt + Kd \cdot \frac{de(t)}{dt}$$

where:

- **e(t):** error (difference between reference value and output).
- $\boldsymbol{Kp}$**:** Proportional Gain :
- **ki:**Integral Gain.
- **kd:** Derivative Gain.

Properties:

- Elimination of the final error (thanks to integration).
- Improved dynamic response (thanks to differential).
- Strike a balance between speed, accuracy, and stability.

Benefits:

Comprehensiveness and high flexibility.

They can be adjusted to fit most physical systems.

Disadvantages:

- Tuning requires expertise, or using optimization algorithms such as **differential development** or **artificial intelligence**.

Comparison of different controllers:

Control Type	To eliminate the final error	Improved responsiveness	Easy adjustment
P	✗	✓	✓
PI	✓	✗	✓
PD	✗	✓	✓
PID	✓	✓	✗

Table 2:Effectiveness of P, PI, PD, and PID Controllers on System Objectives. [4]

1.2 the concept of PID controller:

The PID controller (Proportional–Integral–Derivative Controller) is one of the most widely used control tools in engineering and industrial applications due to its ideal balance between simplicity and effectiveness. It is applied in various systems such as temperature, speed, pressure, position, and flow control.

1. **What is a PID Controller?**

A PID controller is an algorithm that calculates the control signal based on the error between a desired setpoint and the actual output of the system. It consists of three main components:

2. **Proportional Control (P):**

3. Generates a control signal proportional to the current error. The larger the error, the stronger the corrective action. Controlled by the gain $\boldsymbol{Kp}$.

4. **Integral Control (I):**

5. Accumulates the error over time. If an error persists, the integral term increases the control signal gradually until the error is eliminated. Controlled by the gain $\boldsymbol{Ki}$.

6. **Derivative Control (D):**

7. Depends on the rate of change of the error. It predicts system behavior and reduces sudden changes in error, improving system stability [4]. Controlled by the gain $\boldsymbol{Kd}$.

8. **General Time-Domain Equation of the PID Controller:**

$$u(t) = Kp \cdot e(t) + Ki \int_0^t e(\tau)\, d\tau + Kd \cdot \frac{de(t)}{d(t)}$$

- $u(t)$: Control output signal.
- $e(t)$: Error = Setpoint – Actual value.
- Kp**:** Proportional gain.
- Ki **:** Integral gain.
- Kd**:** Derivative gain.

Function of Every Term:

Term	Primary Function	Effect on System Performance
P	Improves current error	Improves speed of response, might leave steady-state error
I	Eliminates built-up error	Eliminates steady-state error
D	Estimates upcoming error	Reduces oscillations and improves system stability

Table 3:Roles and Effects of PID Controller Components

- **Advantages of Using a PID Controller**
- **Flexibility:** Highly configurable for a wide variety of systems.
- **Precision:** Can reduce error to zero or as close as possible.

- **Dynamic Response Improvement:** Reduces rise time and settling time.
- **Simplicity in Implementation:** Easily implemented in most modern digital and analog control systems.

- **Disadvantages of the PID Controller**
- **Requires Tuning:** Optimal values of **Kp**, **Ki**, and **Kd** depend on system characteristics.
- **Sensitive to Noise:** Especially in the derivative term, which can amplify high-frequency noise.
- **Limited Performance in Nonlinear or Time-Varying Systems:** May require adaptive control or intelligent tuning methods.

- **Techniques for PID Parameter Tuning**

Several tuning techniques are used to find optimal PID gains:

1. **Manual Tuning:**
    - Based on trial and error.
    - Suitable for simple or linear systems.
2. **Classical Methods (e.g., Ziegler–Nichols):**
    - Use the system's step response to estimate tuning parameters.
    - Provides a systematic starting point.
3. **Auto-Tuning Using Intelligent Algorithms:**
    - Includes **Differential Evolution (DE)**, **Genetic Algorithms (GA)**, and **Particle Swarm Optimization (PSO)**.
    - Useful for complex, nonlinear, or poorly modeled systems. [4], [14], [24]

- **Examples of PID Controller Applications**
- **Heating and Cooling Systems:** Precise temperature regulation.

- **Automotive Systems:** Engine speed control, cruise control.
- **Robotics:** Accurate position and path control.
- **Chemical Processing Plants:** Control of pressure, concentration, and flow rate.

1.2.1 - Mathematical equation of the PID controller

he PID (Proportional–Integral–Derivative) controller is mathematically formulated to generate a control signal based on the error between a system's desired reference value and its actual output. This formulation enables the controller to dynamically adjust its output to minimize the error over time using three distinct actions: proportional, integral, and derivative.

- **Time-Domain Representation**

The continuous-time PID control law is expressed as:

$$u(t) = \frac{de(t)}{dt} Kd + Ki \int_0^t e\,(\tau)d\tau + e(\tau).kp$$

Where:

- **u(t):** Control signal applied to the system.
- $y(t) - r(t) = e(t)$: Tracking error (difference between reference input r(t) and system output y(t).
- $\boldsymbol{Kp}$**:** Proportional gain, amplifies the current error.
- $\boldsymbol{Ki}$**:** Integral gain, eliminates steady-state error by accumulating past errors.
- $\boldsymbol{Kd}$: Derivative gain, predicts future error based on the rate of change.

Each term contributes to improving different aspects of the system's response, such as speed, accuracy, and damping.

Laplace-Domain Representation

Applying the Laplace transform to the time-domain equation (assuming zero initial conditions), the PID controller becomes:

$$u(s) = (Kp + \frac{ki}{s} + kd^s s)E(s)$$

Thus, the PID transfer function is:

$$G_{PID}(s) = \frac{U(s)}{E(s)} = Kp + \frac{Ki}{s} + Kd.s$$

This design is commonly utilized in control system analysis and synthesis, especially for simulated systems like MATLAB/Simulink. [2], [4]

Role of Each Term per Individual:

- **Proportional Term (P)**:

  Reacts to the immediate error. Elevated proportional gain gains sensitivity but could lead to overshoot and instability.

- **Integral Term (I)**:

  Reacts to the history of past errors. Helps eradicate steady-state error but may delay the system response or cause oscillations if over-tuned

- **Derivative Term (D):**
  Reacts to the predicted future error based on its rate of change. It enhances system damping and reduces overshoot but is sensitive to high-frequency noise.

Practical Considerations:

- The effectiveness of the PID controller relies heavily on proper tuning of the gains $\boldsymbol{Kp}$, $\boldsymbol{Ki}$, and $\boldsymbol{Kd}$.
- In real-world implementations, derivative action is often filtered to mitigate the effect of noise.
- Digital implementations typically use discrete-time approximations of the PID equation.
-

1.2.2 - Effect of each coefficient (Kp, Ki, Kd)

The operation and behavior of a PID controller are characterized by properly tuning its three gain parameters: proportional gain (Kp), integral gain (Ki), and derivative gain (Kd). Each of the parameters has an individual role to characterize the system response:

1. **Proportional Gain (Kp):**

Purpose: Multiples the current error.

Influence on System:

- Makes the system more reactive.
- Reduces the rise time (faster response).
- Higher value may cause overshoot as well as instability.
- Cannot correct the steady-state error by itself.

2. **Integral Gain (Ki):**

Role: Accumulates past errors to eliminate long-term bias.

System Effect:

- Eliminates steady-state error through integration over time of error.
- Doubles the system type and accuracy in steady state.
- Too much integral action can generate overshoot and oscillations.
- Can increase settling time and probability of instability.

3. **Derivative Gain (Kd):**

Role: Forwards future error by predicting rate of change.

System Effect:

- Lessens overshoot and damping time.
- Increases system stability and reduces oscillations.
- Has little effect on steady-state error.
- Very sensitive to measurement noise, which may have to be filtered. **[4]**
- Practical Insight

Gain	Primary Role	Effect on System Performance
Kp	Reacts to current error	Faster response, reduced rise time, possible overshoot
Ki	Reacts to accumulated error	Eliminates steady-state error, may cause instability
Kd	Reacts to rate of error change	Reduces overshoot and oscillations, improves damping

Table 4: Functional Roles and Effects of PID Controller Gains

Careful tuning of these three gains is required to achieve a desired balance between speed, accuracy, and stability. The interaction of the terms can be complex:

- Increasing Kp increases speed but can lead to overshoot.
- Increasing Ki increases accuracy but reduces stability.
- Increasing Kd increases damping but raises noise.

Such parameters can generally be defined through manual practices, conventional methods, or state-of-the-art optimization tools such as Genetic Algorithms, Particle Swarm Optimization, or Differential Evolution. [14], [24]

1.2.3 Challenges associated with adjusting it manually

Manual tuning of a PID (Proportional–Integral–Derivative) controller is a traditional and ubiquitous method used in industrial control. While it is extremely simple to employ and does not require advanced analytic tools, it meets several challenges, especially when dealing with complex systems or systems with time-varying dynamics. The most important among them are shown below.

1. **Excessive Relying on Experience and System Awareness**
- Tuning by hand relies on deep familiarity with the nature of the target system: response, type (linear or nonlinear), and dynamic characteristics.
- It is typically unknown most of the time, so engineers are left resorting to simulation or field testing.
- This leaves the way open for human error, especially in sensitive systems that will react chaotically to slight parameter adjustments.

2. **Significant Time and Resource Consumption**
- Manual tuning is typically done with "trial and error," a procedure that can take considerable time to arrive at an acceptable range of parameters.
- Each tuning attempt entails a complete retest of the system response and results analysis, an additional overhead in industrial manufacturing environments where time and resources are optimized.
- In hard-to-shut-down or hard-to-change-operating-condition systems, experimentation is virtually impossible.

3. **Inability to Repeatably and Reliably Duplicate Results**
- The result of manual tuning depends on the person doing it, i.e., Engineer A may achieve completely different performance from Engineer B for the same system.

- There is no set mathematical algorithm to obtain optimum response; success depends on experience and personal judgment.
- This creates issues in documentation, calibration, and reproducibility of performance in different industrial settings or production lines.

4. Performance Sensitivity to Operating Environment Changes

- Industrial systems are constantly exposed to variations in conditions such as load, temperature, and external pressures.
- Hand tuning conducted under a specific environment may not work when the conditions change.
- The inability of the controller to react to such changes results in loss of performance or even stability loss.

5. Inability to Manage Nonlinear or Multivariable Systems

- In complex systems such as multiple-input/output (MIMO) systems or nonlinear dynamics systems, hand-tuning is become almost impossible.
- Interactions among variables and control within coupled loops require stringent mathematical manipulation that is hard to do manually.

6. The Effect of Noise and Time Delay

- The controller becomes extremely sensitive to abrupt changes in the signal, especially if there is noise or measurement fluctuations when applying the derivative coefficient kd.
- Manual tuning of the kd coefficient without the use of proper filters may amplify random signals rather than improve the response of the system.

The system delay also reverses the effect of manual tuning and leads to instability.

7. No Quantitative Verification Tools

Manual tuning is not based on quantitative criteria such as settling time, overshoot, cumulative error rate, and other performance metrics. Without them, it is impossible to measure the quality or efficiency of a controller, or compare with alternatives.

Conclusion

Even though the use of manual tuning will suffice in simple cases or low-order variables, considering its array of complexities it cannot easily fit systems of higher sophistication, sensitive systems or dynamic ones. This is the reason most of the studies and applications in practice are evolving to auto-tuning methods or algorithm-based tunings of intellect type like:

- Genetic Algorithms
- Particle Swarm Optimization
- Differential Evolution

These methods provide more precise, quicker, and responsive automated optimization to handle complex variations in system behavior. [14], [24]

1.2.4 Introduction to optimization :

Optimization is arguably the most important field of applied mathematics and engineering. Its basic goal is to find the best solution to a problem given a set of constraints and parameters. In control systems, optimization is used to find the best values of controller parameters—such as kp, ki, and kd in a PID controller—to improve system performance according to some criteria, such as settling time, overshoot, or steady-state error. [10]

General Definition:

Optimization is a mathematical process of maximization or minimization of an objective function under constraints imposed on variables. An objective function may be applied to describe an expense to be minimized (e.g., energy consumption), a performance to be optimized (e.g., accuracy of a steering system), or an error to be minimized. [10]

I. Types of Optimization Problems:

A. Single-objective optimization:

Single-objective optimization is a type of optimization problem that seeks to find the best possible value for a single objective function through either maximization or minimization of the function. It is commonly used in applications where a single key performance indicator (KPI) can be defined to represent the quality or efficiency to be optimized.

In the context of controller tuning—a PID controller, say—single-objective optimization is used to determine the values of Kp , Ki , and Kd

that reduce a certain error function and maximize the system's performance.

Mathematical Formulation:

A single-objective optimization problem can be formulated as:

Find: $x^* \in R^n$ such that $f(x^*) \leq f(x\ ) \quad \forall x \in \Omega$

Where:

- x : is the vector of decision variables (e.g., PID gains).
- $f(x\ )$: is the objective function to be minimized or maximized.
- Ω: is the feasible search space.
- The goal is to find the optimal point x^* that yields the best (minimum or maximum) value of f.
- **Common Objective Functions in Control Systems**

When optimizing a PID controller, the objective function may represent a performance metric of the system such as:

- **ISE (Integral of Squared Error):**

$$ISE = \int_0^\infty e^2(t)dt$$

- **IAE (Integral of Absolute Error):**

$$IAE = \int_0^\infty |\ e(t)\ |\ dt$$

Provides moderate penalization of errors.

- **ITAE (Integral of Time-weighted Absolute Error):**

$$\mathrm{ITAE} = \int_0^\infty t \cdot |\ \mathrm{e(t)}\ |\ \mathrm{dt}$$

Penalizes errors that persist over time.

- **ITSE (Integral of Time-weighted Squared Error):**

$$\mathrm{ITSE} = \int_0^\infty t \cdot \mathrm{e}^2(\mathrm{t})\mathrm{dt}$$

Gives importance to late large errors.

➢ Advantages:

- Easy to create and to analyze.
- Focused Performance: Conscious optimization of a specific goal.

- Comparability: It is possible to order solutions using a single figure.

➢ Limitations

- Overlooks other objectives: Might overlook other important system performance areas.
- Potential trade-offs: Optimizing for one parameter makes others worse off (for example, reducing rise time makes overshoot worse).
- Application to PID Tuning
- In the current case, single-objective optimization tries to determine the PID gains that maximize or minimize a given performance criterion (for example, ITAE). Optimization techniques like:
- Differential Evolution (DE)
- Genetic algorithms (GA)
- Particle Swarm Optimization (PSO) are generally employed to calculate the optimum solution that minimizes the performance index chosen. [24]

B. Multi-objective optimization:

Multi-objective optimization (**MOO**) deals with the simultaneous optimization of two or more conflicting objective functions. Unlike in the single-objective optimization case in which a single optimal value exists, **MOO** produces a Pareto-optimal set of good trade-off solutions in which the improvement in one objective results in the worsening of at least one of the objectives.

Mathematical Formulation:

$$Minimize\colon\ F(x) = [f1(x), f2(x), \ldots, fm(x)] subject\ to\colon\ x \in \Omega$$

Where:

- $F(x)$: Vector of objective functions.
- Ω: Feasible solution space.
- x: Vector of decision variables (e.g., PID gains).

Representation in Control Systems:

When designing a PID controller, one might want to:

- Minimize settling time.
- Minimize overshoot.
- Minimize steady-state error.

These objectives may conflict. For example, decreasing settling time might increase overshoot.

Pareto Optimality:

Since no single solution can optimize all objectives simultaneously, multi-objective optimization seeks the Pareto front [27], defined as:

- A group of solutions which cannot be improved in terms of one objective without detrimentally affecting another.
- Designers choose the final solution based on preferences or limitations concerning the application.

➢ Advantages:

- **Increases flexibility:** Allows the consideration of different performance dimensions.
- **Equitable trade-offs:** Encourages holistic approaches
- **Critical to complex systems:** Helpful in situations when there are conflicting goals.

➢ Challenges

- **Obstacles Higher complexity:** More difficult to solve and analyze in comparison to single-objective problems.
- **Key tools needed:** Requires visualization and decision-making tools.
- **Lack of a single solution:** Produces a range of equally viable options.

➢ Common Algorithms

Some of the most used algorithms in MOO include:

- **NSGA-II** (Non-dominated Sorting Genetic Algorithm II)
- **MOPSO** (Multi-Objective Particle Swarm Optimization)
- **MO-DE** (Multi-Objective Differential Evolution)

The algorithms are designed to explore the space of solutions and accurately estimate the Pareto front.

➢ PID controller tuning using multi-objective optimization

Multi-objective optimization finds application in setting the parameters of the PID controller to balance different aspects of the system response for:

- Velocity
- Accuracy
- Stability Resil This approach provides flexible control system designs with high performance that are well-suited to complex or nonlinear environments.

C. Constrained vs. Unconstrained Optimization

1. ● Unconstrained Optimization:

Unconstrained optimization deals with situations in which the decision variables have no restrictions or boundaries in their possible values. It includes maximizing or minimizing the objective function across the entire range of the search space without constraints.

$$f(x)\frac{min}{x}$$

- x: Vector of decision variables.
- $f(x)$: Objective function.

Example: Minimizing the error of a PID controller without imposing any limits on the gain values.

2. Constrained Optimization:

In contrast, constrained optimization involves **limits or conditions** (constraints) that must be satisfied by the solution. These constraints can be in the form of:

- **Equality constraints:** $hi(x) = 0$
- **Inequality constraints:** $g_j(x) \leq 0$

$$f(x): \begin{cases} j = 1, \ldots, m, g_j(x) \leq 0 \\ i = 1, \ldots, p, hi(x) = 0 \end{cases}$$

Example: Tuning PID gains under physical or system constraints, such as limiting the maximum overshoot to 10% or restricting controller gains to positive values only.

3. Summary:

Type	Definition	Example
Unconstrained	No constraints on variables	Minimize ITAE for PID freely
Constrained	With equality/inequality constraints	PID tuning with overshoot limit

Table 5: Classification of Optimization Problems with Examples

D. Linear vs. Nonlinear Optimization

1. Linear Optimization:

Linear optimization (also called **Linear Programming**) deals with objective functions and constraints that are **linear functions** of the decision variables.

$$c^T x \quad subject\ to\ Ax \leq b \underset{x}{min}$$

Where:

- $c^T x$:Linear objective.
- $Ax \leq b$: Linear constraints.

Example: Resource allocation or scheduling problems in industry where relationships are linear.

1. Nonlinear Optimization:

In nonlinear optimization, **either the objective function or at least one constraint is nonlinear**. These problems are typically harder to solve and often require iterative or metaheuristic methods.

$$f(x): \{ \begin{matrix} g_j(x) \leq 0 \\ hi(x) = 0 \end{matrix}$$

Example: PID tuning with performance indices such as ITAE, ISE, which are nonlinear with respect to the gains

Type	Characteristics	Solving Methods
Linear	Objective and constraints are linear	Linear programming (e.g., Simplex)
Nonlinear	At least one function is nonlinear	Gradient methods, metaheuristics

Table 6: Characteristics and Solution Methods of Linear vs. Nonlinear Optimizatio

Application to PID Controller Design:

- Most PID tuning problems are nonlinear and constrained, due to the nonlinear behavior of control systems and physical limits.
- Thus, nonlinearly constrained optimization techniques including Differential Evolution, Genetic Algorithms, and Particle Swarm Optimization are widely used.[24], [27]

II. Optimization in PID Controller Tuning

The problem is formulated as an optimization one in tuning the PID controller parameters so that the overall performance of the system can be determined by a cost function such as:

- Integral of Time-weighted Absolute Error **(ITAE)**

- Integral of Squared Error **(ISE)**
- Integral of Time-weighted Squared Error **(ITSE)**

Optimization finds optimal values of Kp, Ki, and Kd that will reduce the error function and make the system more responsive and stable.

Significance of Optimization in Control Systems:

- Improving the dynamic performance of the system.
- Reducing the need for frequent manual tuning.
- Access to nonlinear or complex systems.
- Satisfying robustness and reliability requirements.

Optimization Algorithms:

Optimization software utilized in this field are:

- Traditional algorithms (e.g., gradient methods).
- Metaheuristic algorithms, such as:
  - Genetic algorithms
  - Particle swarm optimization
  - Differential evolution

Intelligent algorithms are characterized by their ability to search the solution space exhaustively and not be stuck in local suboptimal solutions.

III. Improvement application areas:

1. Engineering:

Design electrical and mechanical systems intelligently.

Improving the performance of control systems such as PID.

Reducing energy consumption or lightening buildings.

2. Factory Operations

Production and supply chain planning.

Reduction of waste in raw materials.

Shifting production lines or controlling chemical processes.

3. Comprehending Economics and Financial Management:

- Selecting the optimal mix of investments to obtain additional money with minimum risk.
- Improving resource allocation.
- Reducing expenses and boosting revenue.

4. Smart computers (AI) and information-based learning:

- Improving neural network weights.
- Reconfiguring the settings in education programs
- Improving prediction models to minimize errors.

5. Moving and Learning:

Enhancing the transportation and delivery systems. Reducing the cost or expediting delivery.

1.3 The difference between linear, nonlinear, mono, and multi-goal optimization

1. Linear Optimization

Linear optimization is an optimizing problem in which:

• The objective function,

• All constraints are totally linear in the decision variables. That is, they contain no powers, products, or nonlinear functions (such as sine, log, etc.).

This kind of optimization is very common as it is simple and effective, and can be solved by techniques such as the Simplex algorithm or Linear Programming (LP).[10]

Example:
Minimize the cost:

$Z = 3x + 5yZ$
Subject to:
$x + 2y \leq 10 \quad , \quad x \geq 0 \quad , \quad y \geq 0$

Application Areas: resource allocation, production planning, budget management.

2. Nonlinear Optimization:

In this type, either the objective function or one of the constraints is **nonlinear**. This includes:

- Quadratic, exponential, or trigonometric functions,
- Or interaction terms like x^2, xy, etc.

Nonlinear optimization is typically **more complex** and requires advanced techniques such as:

- **Evolutionary algorithms** (e.g., Differential Evolution),
- **Numerical methods** (e.g., Newton-Raphson),
- **Stochastic algorithms** (e.g., Simulated Annealing).

Example:
Minimize:

$f(x) = x^2 + sin(x), x \in [0,10]$

Application Areas: tuning PID controllers, smart system design, control of complex systems.

---

3. Single-Objective Optimization

Single-objective optimization focuses on **optimizing only one performance metric**. The objective might be:

- Minimizing energy consumption,
- Maximizing speed,
- Minimizing tracking error,
- Or improving any single performance index.

This type is commonly used when the design goal is clear and does not involve trade-offs.

Example in control systems:
Minimize the Integrated Square Error (ISE):

$$J = \int_0^T e^2\,(t)\,dt$$

Advantages:

•Simple to implement and analyze.

• Applicable to single or simple-goal systems.

Limitations:

• Does not consider other significant performance factors like overshoot or response time. [14]

4. Multi-Objective Optimization

In multi-objective optimization, we want to optimize two or more objectives at the same time. These objectives are usually contradictory to one another (e.g., speed and accuracy), and thus the solution is typically a set of trade-off solutions called the Pareto Front.

Example in PID control:

Optimize for:

• Minimum Integrated Square Error (ISE),

• Minimal Overshoot,

• Minimal Settling Time.

Advantages:

• Offers a wider perspective of system performance.

• Allows flexible decision-making according to application priorities.

Challenges:

Harder to calculate and analyze and does not provide a single "best" solution, but a family of equally good compromises. [27]

Summary Comparison Table:

Optimization Type	Definition	Example	Common Techniques
Linear Optimization	Linear objective and linear constraints	Cost minimization in production	Simplex, Linear Programming (LP)
Nonlinear Optimization	At least one nonlinear objective or constraint	Nonlinear PID tuning	DE, GA, PSO, Gradient methods

Single-Objective	Optimize one performance metric	Minimize ISE only	Any standard optimization method
Multi-Objective	Optimize multiple conflicting metrics simultaneously	Minimize ISE, Overshoot, and Settling Time	NSGA-II, MOGA, Pareto-based DE

Table 7:Types of Optimization in Control Systems and Their Characteristics

1.4 The importance of using optimization in adjusting controllers

In control systems engineering, the performance of a controller—especially a PID controller—greatly depends on the correct selection of its parameters: Kp (proportional gain), Ki (integral gain), and Kd (derivative gain). These used to be tuned manually before, based on expert intuition and trial-and-error procedures. As systems became more sophisticated and demanded increased performance specifications, however, optimization techniques became a natural part of the tuning procedure.

Why Use Optimization?

1. **Improved Performance and Accuracy**

Optimization finds sets of parameters minimizing control errors (for example, Integral Square Error, Integral of Absolute Error) as well as reducing response time, overshoot, and increasing system stability.

2. **Less Human Effort and Subjectivity**

Manual tuning is tedious and variable. Optimization algorithms mechanize this process and create consistent, reproducible solutions without requiring profound manual intervention.

3. **Management of Complex, Nonlinear Systems**

Most real-world systems are nonlinear, time-varying, or multivariable. Optimization techniques—particularly intelligent and evolutionary algorithms—are able to search large solution spaces and deal with such complexities.

4. **Support for Multi-Objective Trade-offs**

Optimization can also be used to balance mutually exclusive objectives, like minimizing rise time and minimizing control energy or actuator wear, to produce a more effective and balanced control solution.

5. **Flexibility and Scalability**

Optimization frameworks can be tailored to various system models, constraints, and objectives, rendering them suitable for a wide range of control applications—including industrial automation, robotics, and energy systems.

6. **Integration with Design and Simulation Tools**

Newer programs such as MATLAB/Simulink have the ability to directly place optimization routines within dynamic system simulations, and thus the process of tuning becomes efficient and smooth.

Example Applications:

- PID auto-tuning for process control in industry: Optimization of temperature, flow, or pressure loop.
- **Autonomous systems:** Adjusting navigation and stability controllers.
- **Mechatronics and robotics:** Providing accurate position and velocity control. Power systems: Improvement of load frequency and voltage regulation control.

Conclusion:

The application of optimization to controller tuning is not just a current requirement but a quantum leap compared to manual techniques. Optimization enables more accurate, consistent, and effective control, particularly in complicated and sensitive systems. Regardless of whether the objective is to enhance a single performance measure or balance several conflicting ones, optimization methods provide a methodical, intelligent approach to attaining optimum controller performance. [14], [24], [27]

1.5 Justifications for using intelligent algorithms in this context

Intelligent optimization algorithms have been made a part of numerous engineering and research applications due to their capability to deal with intricate systems that are either impossible or hard to solve with conventional methods. Optimizing a PID controller with the differential evolution (DE) algorithm or any other intelligent algorithm, they are an apt choice for a number of reasons:

1. **Handling nonlinear and complicated systems:**

Actual systems tend to be nonlinear and complicated, and therefore it is hard to determine optimal solutions utilizing conventional mathematical techniques. Artificial intelligence techniques like the differential evolution (DE) algorithm are well suited to deal with such systems since they are able to determine good approximate solutions even when the input-output mapping is nonlinear.

2. Flexibility in working with different constraints and specifications:

There is often the need in the majority of engineering problems to include a set of different constraints in the optimization process, e.g., of time response, stability, or power consumption. Artificial intelligence techniques such as DE provide flexible solutions that can deal with these constraints efficiently without needing a precise model of every detail in the system.

3. Multi-objective optimization:

In most applications, one does not aim at optimizing a single aspect independently (e.g., response error), but at optimizing several aspects simultaneously (e.g., responsiveness, robustness, and energy conservation). Multi-objective optimization is efficiently performed by intelligent optimization algorithms like DE because these are able to search in several dimensions concurrently in order to reach compromise solutions that meet the different objectives.

4. No background knowledge of the mathematical model is needed:

In some cases, a complete and accurate mathematical model of the system may not be available, or it may be difficult to derive. AI algorithms do not necessarily require these precise models and can operate solely on inputs and outputs that are available through experimentation or simulation. This makes them a perfect option for the optimization of complex or completely unknown systems.

5. Capacity to avoid local solutions:

Smart optimization algorithms, like differential evolution (DE), have the ability to circumvent being trapped into local solutions (which can be suboptimal) due to their global search nature. This feature allows them to converge to better solutions irrespective of the complexity of the system.

6. Adaptation to Environmental Changes or Parameters:

System conditions or parameters can change over time in certain applications (e.g., changing environmental conditions or variable loads). The AI algorithms can accommodate these changes

easily, and hence they offer greater flexibility for applications with changing and dynamic environments.

7. Delivering Maximum Performance in Real Time:

In certain systems that demand quick response, like real-time control systems, AI algorithms are able to produce quick and efficient optimizations for delivering optimum performance without sacrificing system stability or speed.

8. Decreasing the Demand for Human Skill:

With intelligent optimization algorithms, system design heuristics and human experimentation can be minimized. The algorithms are able to perform optimizations on their own with no direct intervention or constant human tuning, saving effort and time.

Conclusion:

The application of intelligent optimization algorithms, as represented by the differential evolution (DE) algorithm, is the most appropriate strategic option for the enhancement of PID control systems because of their higher capability in dealing with nonlinear and complicated systems. The algorithms provide flexible and powerful solutions for finding optimal parameters under the framework of various constraints and multi-objective criteria, including response optimization, robustness, and efficiency. Algorithms of artificial intelligence have the ability to adapt to changes in environmental circumstances and changing system parameters, making them suitable for use in complex and dynamic systems. In addition, the algorithms avoid localized solutions and provide optimized results that achieve peak performance in a short period of time, hence reducing the need for large amounts of human expertise and trial-and-error experimentation. Finally, intelligent algorithms improve the overall effectiveness of the system in a way that is both efficient and safe, while providing flexibility and scalability in their application to various fields of engineering.

2 CHapter Two

Metadata Methods and the DE Algorithm

2.1 Introduction

2.1.1 General definition of approximate optimization.

In many complex real-world problems, it is often impossible or impracticable to achieve the best possible solution to the problem to any high level of accuracy due to the problem's complexity in relation to the objective function, the large search space, or the prevalence of mathematical constraints ruling out the use of conventional optimization methods using derivation and analytical methods. As such, approximative optimization presents itself as one possible alternative whose goal is to find near-optimal solutions within temporal constraints and minimizing excessive computational costs.

Approximate optimization is to be understood as a methodology adopted to address optimization problems in situations where it is extremely difficult, if not impossible, to achieve the best possible solution with accuracy using analytical methods. This methodology uses algorithms which utilize intelligent or stochastic search techniques to examine the solution space. It is not the goal of such algorithms to guarantee the achievement of the best possible solution but to balance the solution quality against the efficiency of the process utilized to obtain the solution. [24], [27], [38]

Approximate optimization methods are differentiated by the flexibility to deal with

- Non-differentiable or discontinuous functions.
- High-dimensional or complex search spaces.
- Problems concerning many goals or nonlinear constraints.

Approximate optimization is relevant in many important areas such as the adjustment of parameters of control systems (for example, PID), scheduling within manufacturing systems, design methods in engineering, network performance optimization, and resource allocation.

The optimization methods pertaining to approximation most widely discussed in scholarly literature include:

- Genetic algorithms
- Particle Swarm Optimization

- Simulated Annealing
- Differential Evolution

The algorithms mentioned above belong to a broader group known as metaheuristics, which will be explained in the next part of the chapter.

2.1.2 The importance of using meta-algorithms in control problems.

Control system design is beset with significant challenges in dealing with the complex, nonlinear, and uncertain model typical of real-world systems. One key challenge in this area is controller parameter adjustment, specifically for proportional-integral-derivative (PID) controllers. This tuning has immediate implications for system stability, response time, tracking quality, and adaptivity to different working conditions.

Traditional controller optimization design methods are mainly based on linear descriptions, such as the Ziegler-Nichols rule or frequency response approach. Such methods assume that there is an accurate system model and that it is appropriate to assess performance-based functions. However, in most real-world applications, these assumptions are rarely met because most real-world systems have nonlinear, time-varying, or time-delayed characteristics, which reduce the efficacy of conventional methods.

Metaheuristics have been critical in helping to solve control problems for many important reasons:

- **No need for precise modeling**:

Metaheuristic methods don't require a precise mathematical description of the system under consideration. Instead, they rely on the evaluation of performance (frequently represented by an objective function) through simulation or experimental means, making them suitable for complex or completely unknown systems.

- **Operating in a nonlinear and nonconvex search space:**

In many control-related issues, the performance criterion usually consists of multiple nonlinear and non-convex goals, making analytical algorithms inefficient in searching for the best solutions. On the other hand, metaheuristic methods have the ability to thoroughly scan the entire solution space, ultimately resulting in solutions approaching the optimum solution.

- **Dealing with multi-objective control problems:**

Metaheuristics, such as DE or NSGA-II, allow for addressing multiple objectives simultaneously, such as minimizing the time taken to reach a reference value, minimizing overshoot, and increasing robustness to disturbances or system changes.

- **Ability to deal with suboptimal conditions:**

Meta-algorithms dispense with the requirement for explicit definition of the objective function and have the potential to cope with noise and numerical inaccuracies occurring during simulations. Their use is therefore appropriate in suboptimal conditions, as seen through the case studies concerning MATLAB/Simulink applications to test controller performance.

- **Flexibility and the ability to grow:**

Meta-algorithms have the ability to be easily incorporated into any methodology, such as adaptive control, machine learning, or local optimization heuristics, thus making them valuable tools for the creation of intelligent and adaptive control systems.

- **Enabled execution and software interoperability:**

The ease of use of meta-algorithms, along with compatibility with the Simulink environment, obviates the need for domain-specific software. Hence, implementation of the algorithms is comparatively simple, and they may easily be put to use to optimize controllers. [27]

Conclusion:

Meta-algorithms have shown to be effective in optimizing controllers, especially where analytical models are difficult to apply or where multiple-objective response is desirable. Their existing utilization is indicative of an overall tactic for the development of intelligent and adaptive control schemes which attain maximum performance in less than ideal and complicated environments. [14], [24]

2.1.3 Illustrate the focus of this chapter on the DE algorithm.

Within the wider context of metamethods, this chapter is looking at the differential evolution (DE) algorithm, which is one of the most efficient stochastic population techniques employed in approximate optimization. There are scientific as well as practical reasons underpinning the choice of discussing the DE algorithm within this chapter, all of which make it well-suited to addressing controller tuning problems, not least with regards to PID controllers.

The Differential Evolution algorithm is distinguishable by its simple architecture and optimization method, which is basically based on differences amongst the individuals in the evolutionary population instead of using complicated mathematical formulations and objective functions. As such, this characteristic makes it uniquely beneficial in situations where mathematical modeling of the system is difficult or in the context where simulation-based evaluation is necessary. [24]

Reason for focusing on the Differential Evolution algorithm in this section:

1. ***Effectiveness in fine-tuning the control system parameters:***

Experiments show that the DE algorithm exhibits a robust capability to find accurate and effective solutions to PID tuning problems in comparison to many other meta-algorithms, especially in systems with variable or nonlinear parameters.

2. ***Ease of implementation and adaptability to simulation conditions:***

DE relies on a minimal set of parameters, such as the shift factor and the rate of hybridization. This is one method, albeit not the only one, which is easily implementable in MATLAB and readily integratable into the Simulink environment, enabling its direct use for controller tuning within a simulation environment.

3. ***The ability to balance exploration and exploitation:***

to achieve equilibrium The processes of hybridization and mutation in DE allow for a deep exploration of the solution space without losing focus on promising solutions, which is very important for the optimization of complex performance functions of control problems.

4. ***Extensibility for multi-objective problems:***

The Differential Evolution algorithm may be modified for multi-objective optimization, known as Multi-Objective DE, to extend its applicability to control problems requiring simultaneous optimization of different criteria like minimal settling time, reduced overshoot, and increased robustness.

5. ***Supported by current research:***

Several studies have demonstrated that DE outperforms other algorithms, such as PSO and GA, in control optimization problems in terms of accuracy, speed of convergence, and the number of evaluations required for the objective function. Building upon those advantages, this chapter will discuss the DE algorithm with reference to its basic principles, implementation stages, properties, and uses in control systems. It will also enable its utilization in subsequent chapters targeting the optimization of single- and multi-objective PID controllers. [24], [27], [35]

2.2 Metaheuristic Algorithms

1. **General definition:**

Metaheuristics are a class of algorithms aimed at solving complex optimization problems that tend to be difficult or impossible to solve using traditional methods. Metaheuristics are known for their broad applicability and flexibility and utilize advanced methodology for searching through the solution space. Their main goal is to obtain satisfactory suboptimal solutions within specific time limits, as opposed to seeking the exact optimal solution in large searching spaces.

Metaheuristics are characterized by:

- They do not require strict mathematical formulations but instead focus on the importance of empirical measurement or simulation.
- They have the capability to solve a wide variety of optimization problems, including nonlinear ones, multi-objective problems, and problems with nonstandard constraints.
- They have the capability to examine very large and complex searching environments which might include thousands to potentially millions of valid solutions.
- They utilize stochastic processes or approaches derived from natural or social processes, such as but not limited to natural selection and bird flocking behavior, to guide the search process with the goal of finding near-optimal or optimal solutions. [35], [24]

Characteristics of Meta-Methods:

- ***Research and Application***

Methodological algorithms balance exploration of new regions in the solution space with exploitation of known effective solutions.

- **Flexibility**

These algorithms have the ability to adapt to the nature of the problem and alter their methods of searching while finding the solution.

- ***Cognitive Random Inquiry***

Utilizes stochastic exploration along with methods that methodically steer the search to the most beneficial solutions.

- ***Ability to Tackle Multi-Objective Problems***

Methodological algorithms may be tailored to solve problems with many conflicting criteria, thus enabling the optimization of multiple criteria at the same time. Use of Meta-Methods in Control:

Utilizing meta Methodological:

algorithms have shown significant improvement in the effectiveness of control schemes, specifically in nonlinear or complex situations. A common example of these uses is the optimization of controllers' tuning parameters, such as the proportional-integral-derivative (PID) controller. Algorithmic optimization allows one to optimize parameters in a multidimensional objective space, where different optimizations might include error reduction, the speed of convergence to the setpoint, and the suppression of overshoot.

2. **Advantages:**

- ***Increased flexibility***

Meta-methods can be applied to a broad variety of optimization problems regardless of whether the objective function is linear or nonlinear and whether the problem is constrained or unconstrained.

- ***Lack of an Essential Derived Objective Function***

They depend upon derivative information about the objective function, which makes it possible for them to be applied in situations where mathematical modeling is difficult to achieve.

- ***Affective Stochastic Process***

Employes a stochastic searching process but with focused and tactical implementation to achieve near-optimal solutions in minimal time and in efficient manner.

- ***Dealing with Multi-Objective Problems:***

There are many meta-algorithms that can be altered to solve multi-objective optimization problems, such as minimizing error at the same time as gaining quick responses.

- ***Ability to Handle Large and Complex Search Domains***

These methods have the capability to traverse very large and diverse search spaces, which is an important consideration in solving problems with many variables or complex constraints.

- ***Adaptive to Problem Changes:***

Search strategies can be changed during optimization due to changes in the environment or particular characteristics of the problem being addressed.

- ***Finding good approximate solutions:***

It allows one to reach adequate solutions within a limited time frame, even if the optimum solution is unobtainable due to the complexity of the system.

3. ***Common examples:***

The meta-algorithm class consists of several methods that have been successful in addressing nonlinear, multivariate, and large search space optimization problems. The most popular of these algorithms are:

Genetic Algorithms (GA):

- ***Evolutionary algorithms:***

Algorithms based on the process of natural evolution and inheritance, such as selection, mating, and mutation. They represent every potential solution as an individual in a "population" that is refined over generations. They are used widely in those applications requiring exhaustive search in a nonlinear space, e.g., control system parameter optimization and system design optimization.

- ***Particle Swarm Optimization (PSO):***

Inspired by the social behavior of living organisms, more specifically birds' and fish's swarms, this algorithm moves each particle in the solution space based on its past experience and the experience of the best particles in the population, achieving a good compromise between exploration and exploitation. It is especially well-suited for fine-tuning medium to high dimensional problems.

- ***Artificial Bee Colony (ABC) Algorithm:***

It is based on a model of the foraging behaviour of honeybees in searching for food sources. The search is shared between "explorer bees" and "exploiter bees" in order to enhance the ability to search the search space effectively and to avoid local solutions. It is a very easy and quick algorithm to code, and is used in the optimization of multi-constrained systems and complicated systems.

- ***Differential Evolution (DE) Algorithm:***

It is based on the strategy of generating new solutions by vector differences between individuals of an evolutionary population. It is one of the most effective algorithms for high-dimensional problem optimization. It is characterized by structural simplicity and ease of

implementation, and it works well in controller tuning issues, especially in PID control systems, where it has been proven to achieve decisive performance and accuracy superiority over a variety of other algorithms. [24], [27], [35]

2.3 Differential Evolution- DE algorithm

Overview:

Differential evolution algorithm (DE) is the most powerful and popular meta-algorithm to be employed in continuous and nonlinear systems optimization. The algorithm is a population-based algorithm where a set of possible solutions (known as individuals or vectors) evolves over time towards converging at the optimum solution.

DE algorithm is based on the idea of generating new solutions based on the variations in between the members of an evolutionary population. Instead of using the usual hybridization and mutation operators for genetic algorithms, DE works a simple mathematical process based on the difference in between solution vectors to generate a new vector close to the optimal solution.

The DE algorithm has some properties which make it fit for being employed in applications in control systems, especially the optimal tuning of optimal parameters of control systems like PIDs. Ease of programming, the ability to work within multi-dimensional space, and improved stability in the determination of precise solutions without a requirement for derivative information of the target function are among such attributes. [35] , [23], [22]

2.3.1 Working Mechanism of the Differential Evolution Algorithm

The Differential Evolution (DE) algorithm operates through a simple yet powerful iterative process that involves four main steps: initialization, mutation, crossover, and selection. These steps are repeated over multiple generations until a stopping criterion is met. [22], [23], [24]**1. Initialization:**

The algorithm begins by generating an initial population of N candidate solutions (individuals), where each individual is a D-dimensional vector representing a potential solution to the optimization problem. The initial values are typically generated randomly within specified bounds:

$$x_{i,j} = x_j^{max} + rand(0,1) \cdot (x_j^{max} - x_j^{min})$$

Here, $x_{i,j}$,j is the j-th component of the i-th individual, and $rand(0,1)$ is a uniformly distributed random number between 0 and 1. [22]

2. Mutation:

For each individual x_i, a mutant vector u_i is generated using the differential mutation strategy:

$$u_i = x_{r1} + F \cdot (x_{r2} - x_{r3})$$

where:

- x_{r1}, x_{r2}, x_{r3} are three distinct individuals randomly selected from the population and different from x_i
- F is the mutation scaling factor, typically in the range $[0.4,1.0]$. [23]

3. Crossover:

To increase diversity, a trial vector u_i is created by combining the target vector x_i and the mutant vector v_i. This is usually done using binomial (uniform) crossover:

$$u_{i,j} = \begin{cases} v_{i,j}, & if\ rand(0,1) \leq CR\ or\ j = j_{rand} \\ x_{i,j}, & otherwise \end{cases}$$

Where:

- $CR \in [0,1]$ is the crossover rate, controlling the probability of inheriting components from the mutant vector,
- j_{rand} ensures that at least one component is taken from v_i.

4. Selection:

The fitness of the trial vector u_i is evaluated and compared with that of the original individual x_i. The one with the better fitness is selected for the next generation:

$$x_i^{(g+1)} = \begin{cases} u_i, & if\ f(u_i) \leq f(x_i) \\ x_i, & otherwise \end{cases}$$

This process continues for a fixed number of generations or until a predefined convergence criterion is satisfied. [22], [23]

2.3.2 Main Parameters in the Differential Evolution Algorithm

The convergence and behavior of the Differential Evolution (DE) algorithm towards the optimal solution depend significantly on a set of control parameters [22]-[23]. These parameters must be suitably adjusted for the successful working of the algorithm. The important parameters are discussed in detail as follows:

1. ***Population Size (NP):***

This is the population size (candidate solutions) for each generation. Each individual corresponds to a point in the search space. Larger population increases diversity, allowing the algorithm to explore more of the search space and improving its chances of converging to the global optimum. Larger population size also increases computational cost and runtime.

A standard suggestion is: $NP \geq 5 \times D$

where D is the number of dimensions of the objective function (i.e., variables). In high-dimensional problems, good tuning of NP is needed to avoid slowly converging or premature stagnation. [22]

2. **Mutation Factor (F):**

It is a scaling factor used during the mutation step that amplifies the difference between individually randomly selected beings. It describes the degree of variation introduced during each generation and is reflected in the formula of mutation:

$$u_i = x_{r1} + F \cdot (x_{r2} - x_{r3})$$

Values for F are usually within the range $[0.4,1.0]$.

- If F is very low (for instance, 0.1), the algorithm will probably develop very slow, with the likelihood of premature convergence.
- If F is very high (for instance, above 0.9), the search will become unstable, with potential overshooting of optimal regions.

Equilibrium F is therefore essential in order to attain a balance between exploration and stability of convergence. [23]

3. ***Crossover Rate (CR):***

This parameter determines the ratio of the trial vector that is inherited from the mutant vector and the target (original) vector. It controls diversity and is applied in the crossover process:

- A high CR (almost 1) involves more components from the mutant vector, which helps in exploration.
- A low CR (almost 0) makes the trial vector less variant from the target, promoting local optimum exploitation.

Common values are between 0.5 and 0.9 to balance both behaviors.

4. ***Dimensionality (D):***

   The number of decision variables of the optimization problem. The larger the dimensionality, the more complex the search space gets—this effect is known as the curse of dimensionality.

   Dimensionalitiy directly affects the required population size, the amount of iterations, and the computational expenses in general. Therefore, values of other parameters such as NP, F, and CR must be scaled in ratio to provide good performance in high-dimensional spaces. [22]

2.4 Steps to make the DE algorithm

The Differential Evolution algorithm is among the simple and highly effective evolutionary algorithms utilized in solving intricate nonlinear optimization problems. It is grounded on a population-based optimization paradigm (Ogata, 2010), in which the population of candidate solutions evolves incrementally through a series of operations with the aim of improved solution quality with every generation [2]. The overall process of DE consists of a series of successive steps explained below:

2.4.1 Population Initialization

In this initial step, a population of size NP is created randomly. Each member of the population, denoted by a vector xi_i, is a candidate solution in a

D-dimensional search space (D being the dimension of variables to be optimized). The elements of the vectors are randomly initialized in given lower and upper bounds problem-specific.

Initialization of each element is typically with:

$$x_{i,j} = x_j^{max} + rand(0,1) \cdot (x_j^{max} - x_j^{min})$$

where:

- $x_{i,j}$ is the j-th component of individual i,

- x_j^{min} and x_j^{max} are the minimum and maximum bounds of variable j,
- $rand(0,1)$ is a uniformly distributed random number in the range $[0,1]$. [22]

Step 2 gives a heterogeneous starting population, and so the algorithm can explore lots of regions of the solution space.

2.4.2 Fitness Evaluation

Once the initial population has been created, the objective function (fitness function) is evaluated for each individual x_i.This function quantifies the quality of the solution represented by each individual.

For **minimization problems**, lower values of the objective function are better; for **maximization problems**, higher values are preferred. The fitness values serve as the basis for selection in later stages. [22]

2.4.3 Mutation

Mutation is the core operation of DE. A mutant vector vi is formed from every target vector xi using three mutually distinct, randomly selected vectors x_{r1}, x_{r2}, x_{r3}

of the population at hand:

$$u_i = x_{r1} + F \cdot (x_{r2} - x_{r3})$$

where:

- F is the **mutation factor**, a real-valued parameter typically between 0.4 and 1.0 that controls the amplification of the differential variation.

The purpose of mutation is to **introduce new genetic material** into the population and encourage exploration of the search space.

2.4.4 Crossover

After mutation, a **trial vector** $u_i$is created by recombining the mutant vector viv_ivi with the original target vector x_i. The **crossover operation** uses a crossover rate$CR \in [0,1]$, which determines the probability of taking components from the mutant vector.

$$u_{i,j} = \begin{cases} v_{i,j}, & if\ rand(0,1) \le CR\ or\ j = j_{rand} \\ x_{i,j}, & otherwise \end{cases}$$

- j_{rand} is a randomly chosen index to ensure that at least one component is inherited from the mutant vector.

Crossover enhances **diversity** and balances **exploration vs. exploitation** in the search process.[24]

2.4.5 Selection

After generating the trial vector u_i, the algorithm compares it with the original target vector x_i. The one with the better fitness value is selected to survive in the next generation:

$$x_i^{(g+1)} = \begin{cases} u_i, & if\ f(u_i) \leq f(x_i) \\ x_i, & otherwise \end{cases}$$

This ensures that the population either **improves or maintains** its quality from one generation to the next. [24]

2.4.6 Iteration and Termination

The process from fitness evaluation to selection is repeated for a predefined number of generations or until a **stopping criterion** is met. Common stopping criteria include:

- Reaching a maximum number of iterations,
- Achieving a predefined level of solution quality,
- Lack of significant improvement over a number of generations.

At the end of the optimization process, the **best solution found** is returned as the output of the algorithm. [24]

2.5 Characteristics and advantages of DE

Differential Evolution (DE) is among the most popular and effective metaheuristic algorithms, particularly to optimize complex systems that are nonlinear, multi-objective, or hard to model analytically. The following is a cautious description of the most significant characteristics making DE a robust and sought-after tool in many optimization problems:

1. **Straightforward Structure and Simple Implementation**

DE is also renowned for its easy algorithmic structure based on clearly defined steps: initialization, evaluation, mutation, crossover, and selection. These steps are not founded on advanced mathematical calculations or complex logic, so the algorithm is very easy and can be easily written in any programming environment such as MATLAB, Python, or C++.

Example: DE can be implemented in MATLAB through simple loops and matrix manipulations, which significantly reduces prototyping and experimentation time with models. [24]

2. Derivative-Free Optimization

Unlike classical optimization methods such as Gradient Descent or Newton-Raphson, DE does not use derivative information regarding the objective function. It relies solely on function evaluations. This makes it especially appropriate for cases where:

- The objective function is non-differentiable (e.g., step functions or absolute values).
- The function is not analytically specified (e.g., simulation-based or experimental models).
- The optimization landscape has noise, discontinuities, or abrupt transitions. [22], [23]

3. High Explorative Ability and Prevention of Local Minima

DE's mutation scheme is focused on the difference between three different members of the population and not random perturbation alone. This setup provides:

- Good exploration of multiple areas of the search space.
- Good avoidance of local minima, a common weakness in conventional methods.
- Dynamic search direction, which adapts based on the population diversity.

Furthermore, the crossover rate (CR) offers fine control over the balance between inherited traits from mutated vectors and current individuals, enhancing the algorithm's balance between exploration and exploitation. [23], [24]

4. Efficient Performance in High-Dimensional Spaces

DE performs with consistent and robust behavior even if applied to problems involving huge variable counts. Compared to some of the other methods of optimization that perform badly concerning scalability, DE remains robust owing to:

- Vector difference mutation scales with problem size.
- Random sampling strategies avoiding early convergence.
- Population diversity maintaining spread over the solution space with sufficient pressure over worthwhile regions. [24]

Note: High-dimensional optimization is susceptible to DE parameter tuning or using hybrid methods for optimal results.

5. Customization and Flexibility

DE has one of the benefits that it is flexible. The algorithm is easy to adapt to fit the needs of a particular problem by using:

- Variations of the mutation strategy (e.g., DE/rand/1, DE/best/1, DE/current-to-best/1).
- Hybridization with other metaheuristics to obtain hybrid algorithms.
- Custom termination criterion and choice function.
- Problem-specific constraint inclusion, either in the objective function or evaluation process. [23]

6. Minimum and Intuitive Set of Parameters

Unlike other metaheuristics, which have numerous parameters, DE is very minimal and has the following parameters:

- **NP:** Population size
- **F:** Mutation factor or scaling factor
- **CR:** Crossover rate
- **D:** Dimension of the problem

These limited and intuitive set of parameters ease the setup and deployment of DE across a vast range of applications. [24]

2.6 Constraints of the Differential Evolution Algorithm

The Differential Evolution algorithm exhibits great effectiveness in solving complicated optimization problems; nonetheless, it is also prone to numerous constraints and challenges common to many other metaheuristic methods. Identification of these limitations is important to the algorithm's proper implementation, calling for the consideration of appropriate modifications or hybrids where appropriate. [22], [24]

1. Sensitivity to initial parameter seFngs

DE depends on many critical control parameters, which are as follows:

- Population Size (NP)
- Mutation factor (F)
- Crossover rate (CR)

These parameters greatly influence the algorithm's performance, and:

- Poorly chosen values can cause prolonged periods of convergence or suboptimal results.
- Lack of commonly agreed-upon standards for the choice of best parameters for different problems often requires manual tuning or trial-and-error experimentation.

2. Improved Convergence under Some Conditions

Even though DE is known for its vast potential for global exploration, this same forte sometimes leads to:

- Slow convergence, especially in smooth or unimodal functions.
- A requirement for a large number of generations to achieve satisfactory results.
- This is even more evident while handling high-dimensional or simple problems, where techniques of local search may better be applied.

3. Premature Convergence Risk

Differential evolution sometimes shows vulnerability to premature convergence, mostly due to declines in population diversity. This behavior may cause

- Differentiation of A Local Optimum from A Global Optimum.
- Reduced ability to examine new regions within the search space.

This problem typically arises when a lower crossover rate (CR) is used or when the individuals are too similar during the evolution process. [23], [24]

4. Substantial Cost Involved in Objective Function Evaluations

Since Differential Evolution inherently relies on the evaluation of the objective function without gradient information, it often requires:

- A large amount of evaluation (NP times generations), potentially at very high computational expense.
- Execution times can be considerably longer when the objective function involves complex simulations or requires real-time processing.

This deficiency is specifically true in case of computational fluid dynamics, structural modeling, or time-domain simulation methods.

5. Poor Administration of

Standard Differential Evolution is not inherently efficient in trying to resolve constrained optimization problems. Those constraints include:

- Variable bounds.
- Geometry or physical constraints.
- Logical or operational rules.

Therefore, it is critical to integrate supplementary techniques, such as penalty methods, repair methods, or constraint-handling techniques, which might add complexity to the algorithm or affect its convergence behavior. [23], [24]

➢ *Section Summary*

Despite these disadvantages, DE remains one of the most powerful and flexible optimization tools available. Most of its limitations can be mitigated through enhancements such as:

- Adaptive parameter tuning,
- Hybridization with other algorithms,
- Incorporation of problem-specific knowledge or constraint-handling techniques.

These developments help ensure the competitiveness of DE both in industrial practice and in scholarly research. [24]

2.7 Applications of the Differential Evolution Algorithm (DE)

Due to its ease of use, versatility, and high efficiency in optimizing nonlinear and complicated solution spaces, the Differential Evolution (DE) algorithm has become one of the most widely used optimization tools in an extensive variety of application domains in engineering and technology [22]-[24]. DE was found to be extremely effective at optimizing analytically difficult-to-solve or conventionally difficult-to-solve problems.

These are some major fields where the DE algorithm has performed outstandingly:

1) **PID Controller Parameter Tuning:**

One of the most common applications of DE is PID controller parameter tuning. The algorithm is used to search for the best values of the three gains:

- Proportional gain Kp.
- Integral gain Ki.
- Derivative gain Kd. [3], [24]

Its purpose is to maximize system performance by:

- Minimizing tracking error
- Minimizing settling time
- Avoiding overshoot
- Adding stability to perturbations

DE is a superior alternative to classical methods like Ziegler–Nichols or trial-and-error tuning, which takes much less effort and time. [2], [4]

2) **Multi-Objective Optimization:**

We need to optimize a collection of conflicting goals at the same time in most real-world systems, e.g.,

- Minimizing error without compromising response stability
- Enhancing performance and lowering power usage

DE can be generalized to multi-objective versions (e.g., MODE: Multi-objective DE) to:

- Form Pareto fronts to represent trade-offs among competing objectives
- Provide designers with the best current set of solutions to choose from by option. [27],[36]

Example: Minimizing rise time and steady-state error simultaneously in a PID-controlled system.

3) **Modeling of Complex Systems:**

DE is also being applied extensively in system identification, wherein model parameters of unknown parameters are determined in dynamic systems. The dynamic systems can be:

- Nonlinear electrical or mechanical systems
- Thermal or chemical processes involving multiple variables DE tunes model parameters so that the model output closely approximates measured data. [19], [49]

Benefit: Can be utilized even when there is no analytical system description

4) **Resource and Energy Allocation:**

In domains such as communications, network management, and energy management:

- DE optimizes the power dispatch among the generation units
- It is also used to schedule events with limited resources, such as time or energy

Example: Minimizing energy usage in wireless sensor networks or optimizing HVAC control systems in smart buildings.

5) **Optimization of Device and Antenna Design:**

In engineering design processes:

- DE is applied to determine antenna shapes for optimal radiation efficiency with shape, size, and material
- constraints It can also be utilized for mechanical or thermal design, i.e., minimum weight, maximum heat, or optimal distribution of stress

Example: Satellite antenna radiation pattern design using DE for uniform coverage. [49]

Conclusion:

The Differential Evolution algorithm is a very efficient and general-purpose optimization technique in a very wide range of applications—from traditional PID control to emerging smart systems. Its simplicity, generality, and acceptable global search capability make it an excellent addition to the contemporary engineering optimization toolbox.

Conclusion

This chapter provided a theoretical background to put into perspective the place of metaheuristic algorithms in tackling difficult optimization problems—especially those that

exhibit nonlinear, multi-objective, and constraint-rich behavior. Through the paradigm of approximate optimization and the features that define metaheuristics, it was discussed how the limitations of classical derivative-based approaches are overcome by these techniques.

Particular emphasis was placed on the Differential Evolution (DE) algorithm, which works by population-based operations by utilizing differential vectors to create new candidate solutions. This allows DE to effectively and systematically search the space. Its major advantages are ease of implementation, low parameter tuning, independence from derivative information, and ease of generalizing it to constrained and multi-objective problems.

On the basis of these strengths, DE will be used as the main optimization tool in the subsequent chapters. DE will be employed to optimize the PID controller parameters in an effort to enhance system response time, reduce error indices, and enhance robustness across changing conditions. The algorithm should be superior to conventional tuning techniques in terms of accuracy as well as computational complexity.

3 Chapter Three

Robustness and Multi-Objective Optimization

3.1 Introduction

In the development of contemporary control systems, it is not sufficient that the system is capable of satisfying optimal performance specifications; it should also be capable of resisting changes and disturbances that tend to alter its behavior during real operation. Robustness then becomes vitally essential as a key feature of very reliable control systems.

Physical systems tend to be exposed to non-ideal situations, i.e., parameter variations, external disturbances, or even approximated mathematical models at the design stage. In such a case, the system will never be able to sustain its performance unless designed with some tolerance against uncertainty. It has thus become necessary to encompass performance enhancement beyond one objective alone (e.g., error minimization), but to encompass performance and robustness in the design and optimization procedure.

PID controller is the most popular control device in industrial uses because of its simplicity and effectiveness. Nevertheless, it is highly sensitive to system or environmental changes. Thus, the introduction of the robustness principle in the PID parameter tuning process tries to make the system capable of coping with dynamic changes without loss of stability or performance degradation.

To this end, multi-objective optimization techniques are used, which allow for evaluation of the compromise between different objectives, such as error minimization and disturbance robustness, for example. This approach is very much at the core of successful design in complex real-world environments, a topic well covered in this chapter. [1], [2], [27]

3.2 The concept of durability in control systems

3.2.1 Behavioral and Mathematical Definitions of Robustness

Robustness is one of the main attributes of control system design today. Robustness is associated with the ability of a system to maintain satisfactory performance in the face of uncertainties, disturbances, or modeling inaccuracies. In behavioral terms, robustness means a system remains functionally healthy even when the external or internal conditions drift away from nominal assumptions. Mathematically, a control system is robust if it meets performance and stability requirements over a specified set of disturbances and model variations.

Suppose a system is described by a set of parameters θ. The system is robust if it meets the design requirements for all values of θ in a bounded uncertainty set: $\theta \in \Theta$

Θ, represents the uncertainty set, which captures all probable variations due to model simplifications, physical variations, or operational perturbations.

3.2.2 Robustness vs. Stability

We must distinguish between robustness and stability because, although related, they differ:

Stability is the intrinsic ability of a system to regain equilibrium after a perturbation. It is normally analyzed under nominal conditions. [1]

On the other hand, robustness refers to a system's capacity for surviving deviations from nominal conditions, including parameter variations, noise, and unmodeled dynamics.

Robustness therefore signifies stability. A system may be stable under idealized conditions and break down with real-world uncertainties, and is therefore not robust. [1] ,[2]

3.2.3 Sources of Uncertainty in Control Systems

Practically, control systems are often subjected to a variety of uncertainties that can degrade performance or even destabilize the system. They are:

- Parameter variations: Variations in gains, masses, or time constants due to temperature, aging, or operating load.
- Modeling errors: Arising from linearity, neglect of nonlinearity, or simplification in system modeling.
- External disturbances: Sensor noise, mechanical vibrations, or power fluctuation.
- Time delays: Ubiquitous in networked control systems, causing phase lag and performance deterioration.
- Nonlinear effects: Like actuator saturation, backscatter, dead zones, or hysteresis.

Robust control design aims to ensure the guarantee of acceptable system performance amidst these uncertainties. [3]

3.2.4 Measures of Robustness

Several measures have been developed in control theory for measuring the robustness of a control system, e.g.,:

- Stability margins: Expressed by the gain margin (GM) and phase margin (PM), they indicate how much gain or phase shift a system can tolerate before becoming unstable.

- Sensitivity function (S(s)): E xpressed by:

$$S(s) = \frac{1}{1+G(s)C(s)}$$

where $G(s)$ is the plant and $C(s)$ is the controller. A lower magnitude of $|S(s)|$ across the frequency domain implies **greater robustness** to disturbances and model uncertainties.

- Worst-case Performance Analysis: This method examines the system under worst-case or extrema conditions, usually through Monte Carlo analysis or structured uncertainty modeling, to ascertain degradation in performance under realistic disturbances.

Robustness is therefore a primary issue in control system design, particularly for safety-critical or mission-critical applications. A robust design for a system ensures stability as well as acceptable performance, even in the presence of model uncertainty and environmental disturbances. [33]

3.3 The importance of durability in PID controller design

The PID controller remains a industrial process and automation control standard due to its easy structure, easy application, and well-documented success across many systems. Nonetheless, in realistic operating environments where uncertainties, nonlinearities, and disturbances cannot be avoided, standard PID tuning methods cannot assure consistent behavior. This uncovers an inherent flaw: the lack of inherent robustness in traditionally tuned PID controllers.

Robustness in PID design is not just desirable—it is mandatory. It keeps the control system from being disrupted by parameter variations, structural model discrepancies, and external disturbances, but still remain stable and deliver acceptable control quality. As systems are trending towards higher levels of autonomy and complexity, the incorporation of robustness in PID control design has become a necessity and no longer a choice. [34]

3.3.1 Impact of Parameter Uncertainty on PID Performance

PID controllers are usually tuned on the basis of nominal models based on fixed plant dynamics. Real systems, in actuality, go through changes in plant parameters such as process gain, delay time, and time constants—due to wear, temperature fluctuations, aging, or load variations. Even minute changes in these parameters can lead to:

- Decreased transient response, e.g., increased overshoot, rise time, or settling time.
- Change in system poles, affecting stability margins.

- Amplification of measurement noise, especially in high-gain derivative actions.
- Instability, particularly in systems with large feedback delay or time-varying dynamics.

This sensitivity underlines the significance of creating controllers that perform reasonably well not merely at a single operating point but across an ensemble of possible system behaviors. [39]

3.3.2 Role of PID in Ensuring Robust Stability and Performance

A stable PID controller is one that exhibits satisfactory performance even when the system deviates from its nominal model. To accomplish this, the controller must be:

- Stability-preserving under bounded parameter variations and model uncertainties.
- Low on sensitivity, especially to high-frequency noise and model mismatch.
- Robust to unmodeled dynamics, e.g., actuator limits, nonlinearities, or time delays.

Advanced PID design techniques incorporate robust performance criteria in an explicit manner, such as:

Minimization of the H∞ norm of the sensitivity function $\| S(s) \|_\infty$,Maximization of gain and phase margins,Formulation of worst-case scenarios in simulation-based tuning procedures. [33]

3.3.3 Comparative Analysis: Conventional vs. Robust PID Tuning

Criterion	**Classical PID Tuning**	**Robust PID Tuning**
Design Focus	Nominal model-based tuning	Parametric uncertainty and worst-case performance
Sensitivity to disturbances	High	Controlled and minimized
Adaptability to plant change	Poor to moderate	High
Stability Margins	Not guaranteed	Explicitly considered in design
Methods	Ziegler–Nichols, Cohen-Coon, manual trial-and-error	Evolutionary algorithms, multi-objective optimization
Use in safety-critical systems	Limited	Strongly recommended

Table 8:Comparison Between Classical and Robust PID Tuning Approaches

Eventually, robust PID design is a middle ground compared to typical tuning, thanks to its ability to correct inadequacies resulting from nominal-model assumptions. Robust design represents a new paradigm in transitioning from merely meeting performance specifications to providing assured and dependable control over numerous operating conditions. Future control-system implementations, particularly those in safety-critical and autonomous systems, will depend heavily on such techniques aimed at achieving robustness.

3.4 3.4 Multi-Objective Optimization

3.4.1 Concept and Motivation

In real-world engineering designs—especially in control system design—improving one measure of performance alone is often insufficient. Instead, designers must balance a number of typically competing goals. Such design problems come under the umbrella of multi-objective optimization (MOO). As opposed to single-objective optimization, which attempts to find one global optimum, MOO attempts to find a collection of optimal trade-off solutions in which all objectives are simultaneously satisfied to the maximum extent possible.

Formally, a multi-objective optimization problem can be expressed as:

$$min_{x \in \boldsymbol{\Omega}} \, F(x) = [f1(x), f2(x), \dots, fk(x)]$$

Where:

- $x \; is$ the decision vector in the feasible region Ω^2
- $f_i(x)$ are the objective functions (e.g., error, energy, overshoot),
- $k \geq 2k$ is the number of objectives.

3.4.2 4.2 Examples of Control System Conflicting Objectives

In control engineering, it is generally the case that optimizing one system performance metric will come at the expense of another. Some typical conflicting objectives are:

Settling Time vs. Overshoot:

Minimizing settling time can lead to increased overshoot and instability.

Control Effort vs. Accuracy:

Increased tracking accuracy can entail increased control input, leading to increased energy consumption or actuator wear.

Robustness vs. Speed:

Making robustness (e.g., gain/phase margins) better usually involves conservative tuning that depresses response speed.

Sensitivity vs. Disturbance Rejection:

Sensitivity reduction to parameter variations tends to lower the system's ability to reject disturbances quickly.

Achieving compromise solutions best describing the system designer and application-specific requirements priorities is the challenge. [27]

3.4.3 The Pareto Front Concept

One of the central ideas of MOO is the Pareto front, the collection of non-dominated solutions. A solution is Pareto optimal if there is no other feasible solution that will make one objective better without making at least one other worse.

Definition (Pareto Optimality)

A decision vector x_1 **dominates** x_2 if:

- for all $i = 1,2, \dots, ki$ and $f_i(x_2) \geq f_i(x_1)$
- There exists at least one j such that $f_j(x_1) < f_j(x_2)$

The Pareto front thus consists of all the non-dominated solutions in the objective space. These solutions present designers with a variety of options to choose among, depending on which trade-offs are preferable for an application. [35]

As an example, a control system may require a balance between low settling time and moderate control effort. The Pareto front provides all such balanced configurations.

3.5 Multi-objective optimization techniques

Multi-objective optimization (MOO) problems are more difficult to solve than single-objective optimization because of conflicting objectives that cannot be optimized at the same time. There are special algorithms required that can handle the trade-offs between objectives and provide a good set of Pareto-optimal solutions.

Some of the most popular and most used Multi-Objective Evolutionary Algorithms (MOEAs) include the Non-dominated Sorting Genetic Algorithm II (NSGA-II) and the Multi-objective Differential Evolution (MODE) algorithm. Both algorithms represent different yet effective ways of searching and approximating Pareto fronts, particularly in control system applications like PID controller tuning.

3.5.1 NSGA-II

Non-dominated Sorting Genetic Algorithm II

NSGA-II, developed by Deb et al. in 2002, is a second-generation evolutionary algorithm that tries to eliminate the weaknesses of the earlier MOEAs, such as the absence of elitism, poor diversity handling, and excessive computational complexity.

Major Elements of NSGA-II:

1. **Non-dominated Sorting**
- The population is divided into several Pareto fronts.
- Individuals achieve dominance ranks by the number of subjects dominating them.

2. **Crowding Distance Calculation:**
- Measures the solution density surrounding a candidate.
- Fosters diversity through prioritizing members that are based in remote parts of the front.

3. **Choosing Method:**

Uses a mix of elitism and crowding distance to help choose solutions for the next generation.

4. **Genetic Operators:**

Employs crossover and mutation (as in conventional genetic algorithms) to alter the population.

Advantages of NSGA-II:

- Ensures solution diversity distribution over the Pareto front.
- Offers excellent performance on multiple assessments and practical problems.
- Reduced computational complexity: $O(MN^2)$, where :

M = number of goals and N is the number of individuals.

Used mostly for network enhancement, multi-objective controller design, resource allocation, and in embedded systems. [36]

3.5.2 MODE

(Multi-objective Differential Evolution)

MODE is a straightforward extension of the classical Differential Evolution (DE) algorithm to multi-objective problems. It employs the simple and efficient components of DE and introduces mechanisms for selecting solutions according to Pareto concepts and maintaining diversity.

- **Central Structure of MODE:**

Differential Mutation:

New candidate solutions are generated by applying the scaled difference between random individuals:

$$vi = xr_1 + F \cdot (xr_2 - xr_3)$$

Crossover:

- Mutated vector is added to the current solution to create a trial vector.

Pareto-based Selection:

- Agents are selected based on non-domination rank.
- Preference is given to those which are closer to the ideal Pareto front.

Diversity Mechanisms:

Techniques such as crowding distance or external archives are utilized to provide spread of solutions.

Strengths of MODE:

Retains the low parameter tuning and simplicity of DE.

Very powerful for continuous and nonlinear optimization.

Easily customizable for any constraints or robustness requirements.

Particularly helpful for control problems with continuous control, e.g., multi-objective tuning of PID controllers, where goals such as error reduction, energy efficiency, and robustness must be balanced. [36]

3.5.3 3.5.3 Analytical Comparison: NSGA-II vs. MODE

Aspect	NSGA-II	MODE
Algorithm Type	Genetic Evolutionary Algorithm	Differential Evolution-based
Variation Mechanism	Crossover and mutation	Scaled difference vectors
Diversity Maintenance	Crowding distance	Archive management or distance-based methods
Performance on Linear Systems	Excellent	Excellent
Performance on Nonlinear Systems	Very good	Outstanding (especially with continuous parameters)
Implementation Complexity	Moderate to high	Low to moderate
Customizability	High	Very high

Table 9:Comparison Between NSGA-II and MODE in Multi-Objective Optimization

3.5.4 Managing Trade-offs Between Objectives

In multi-objective optimization, managing trade-offs between competitive objectives is a primary difficulty. In most real-world applications, optimization of all objectives simultaneously is not feasible since the optimization of one criterion tends to degrade another. This necessitates a strategic decision-making process to decide on the most tolerable compromise solution or balanced solution based on system requirements or designer preference.

- Trade-off examples in real-world applications:
- Minimizing settling time may come with the compromise of greater overshoot and reduced system stability.
- Making accuracy of tracking better might involve more control effort, higher energy usage or actuator stress.
- Making robustness generally results in conservative tuning, and this might decrease the responsiveness of the system.

- Decreasing parameter sensitivity might restrict the system to reject disturbances quickly. [27]

What MOEAs do with trade-offs

Multi-objective evolutionary algorithms (MOEAs) such as NSGA-II and MODE are not designed to find a single "optimal" solution. Instead, they are designed to find a set of equally optimal solutions, i.e., the Pareto front, where:

- No solution in the front is strictly better than another for all objectives.
- Each solution is one of a different compromise between mutually conflicting goals.
- The designer can choose the most appropriate solution based on contextual needs or operational constraints.

Mechanisms for enhancing trade-off representation

Advanced MOEAs utilize a number of significant mechanisms in an attempt to produce a high-quality and well-spread Pareto front.

1. **Non-dominated Sorting:**

Sorts solutions into layers based on Pareto dominance relationships.

2. **Crowding Distance:**

Calculates crowding density around solutions in objective space to promote diversity and prevent clustering.

3. **Preference-based Ranking:**

Allows designer preferences to be utilized by assigning weights to specific objectives or employing desirability functions.

4. **External Archives:**

non-dominated solutions from generation to generation and help keep the population diverse during the optimization process.

These mechanisms make not only mathematically sound but also useful in practice for well-informed decision-making, the Pareto front.

Selecting a solution from the Pareto front

After a Pareto front has been generated, the designer will still have to choose a final solution to be implemented. Popular choices are:

- **Heuristic selection:** Subjective choice of a point that, in an intuitive sense, is a good compromise.
- **Weighted sum method:** Aggregating many goals into a single scalar function of weighted sum via pre-specified weights.
- **Utility-based decision-making:** Bringing different goals under the assessment of their relative desirability using utility or cost functions.

The compromise-oriented framework sees multi-objective optimization especially naturally well-suited to real engineering systems where no single goal can serve all performance measures in all their complexity.

3.5.5 Summary

NSGA-II and MODE are among the most powerful frameworks for multi-objective optimization.

- **NSGA-II** works outstandingly by establishing well-shaped and diverse Pareto fronts together with mature ranking and selection mechanisms.
- **MODE** is particularly ideal for engineering control problems through offering a trade-off between exploration effectiveness and ease of integration with performance-based constraints.

MODE will be used as the main multi-objective optimization method for **PID controller tuning** throughout this project with a view to reducing control error, energy usage, and enhancing system robustness all at the same time.

3.6 The relationship between robustness and multi-goal optimization

In real control system design, the minimization of performance measures such as minimum error or fast response time is not typically sufficient. Real systems operate in the presence of uncertainty, appearing in the form of parameter variations, modeling errors, external disturbances, and random environments. Robustness—the ability of a system to maintain adequate performance in the presence of such uncertainties—must therefore be considered as a central design objective.

Multi-objective optimization frameworks offer an ideal environment for incorporating robustness directly into the control design process. By making robustness an independent objective alongside conventional performance metrics, designers can obtain solutions that not only operate optimally under nominal conditions but remain reliable under adverse conditions as well.

3.6.1 Formulating Robustness as an Optimization Objective

To add robustness in a multi-objective optimization algorithm, it needs to be transformed into a measurable objective function. Several approaches are used for measuring robustness in optimization:

1. Sensitivity Function Minimization

Robustness is usually associated with the system sensitivity to disturbances or model uncertainty, which is characterized in terms of the sensitivity function:

$$S(s) = \frac{1}{1 + G(s)C(s)}$$

Where $G(s)$ is the plant and $C(s)$ the controller. The aim is to keep the peak value of

$| S(j\omega) |$ down over the frequency domain, making the system respond less to parameter variations or noise.

2. Worst-case Performance Evaluation

Here, the system is tested under a predetermined set of worst-case situations (e.g., ±20% variation in gain or delay), and the worst-case performance degradation (e.g., worst-case overshoot, settling time, or error) is considered as an objective to be optimized.

3. Stability Margins

Robustness can be quantified in terms of gain margin and phase margin that provide an estimate of how close the system is to instability. They either can be minimized as objectives or used as constraints to require a minimum degree of robustness.

4. Minimization of Performance Dispersion

Here, robustness is quantified in terms of measuring the degree to which the performance of the system varies over a given set of operating conditions. The objective is to minimize variation between best-case and worst-case performance, which indicates homogeneous behavior under uncertainty. [33]

3.6.2 Examples of Incorporating Robustness in Multi-Objective Formulations

Example 1: Error Minimization + Robustness (Sensitivity) Improvement

$$Minimize \begin{cases} ISE = \int_0^\infty e^2(t)dt \\ max_\omega |S(j\omega)| \end{cases}$$

The goal is to simultaneously reduce the integral of squared error and ensure that the system is not overly sensitive to modeling inaccuracies or disturbances.

Example 2: Disturbance-Tolerant Performance + Energy Minimization

$$Minimize \begin{cases} ISE = \int_0^\infty t|e^2(t)|dt \\ Control\ Energy = \int_0^\infty \mu^2(t)dt \end{cases}$$

This formulation targets robust performance under disturbances while limiting the control effort to avoid actuator saturation or energy waste.

Example 3: Minimizing Performance Variability Across Conditions

The system is tested under multiple operating models (nominal, under-parameterized, over-parameterized), and the objective is

$$Robustness\ Index = max(ISE) - min(ISE)$$

Here, the focus is on minimizing variability and accomplishing consistent control performance.

3.6.3 Conditions for Applying This Technique to Tune PID Controllers

Multi-objective evolutionary algorithms such as MODE and NSGA-II are commonly applied in the tuning of PID controllers for minimization of errors and improvement of transient responses. Incorporating robustness as an optimization objective involves relocation of design priority from maximizing performance to maximizing sustainable, reliable, and fault-tolerant performance.

The Pareto front obtained in such cases consists of solutions with different trade-offs between accuracy and robustness.

Designers can then select the most suitable controller configuration depending on application requirements—whether speed, reliability, or fault tolerance is vital. [36]

Summary:

Integration of robustness within a multi-objective optimization platform is a control system design paradigm change. Instead of optimizing performance separately, the system is optimized with consideration for actual-world uncertainties, resulting in solutions that are high-performing and also robust, reliable, and adaptive. This is crucial to designing PID controllers that have to operate in dynamic, uncertain, or safety-critical conditions.

Conclusion

This chapter broached the issue of how multi-objective optimization and robustness relate, two very important areas of control system design during the present era. Traditional control design tends to focus on optimizing certain objectives like minimizing error or maximizing rate of response. These goals, however, are not enough alone in actual applications where uncertainty, nonlinearity, and variability of parameters are the standard.

Robustness here is the capacity of the system to handle external problems, model inaccuracies, and parameter changes. The chapter emphasized how robustness should be a main goal in optimization rather than as an additional requirement or constraint. By making robustness a crisp number—through sensitivity functions, stability margins, or worst-case performance measures—it can be incorporated into a multi-objective optimization framework along with traditional performance measures like ISE (Integral of Squared Error), ITAE (Integral of Time-weighted Absolute Error), or control effort.

We have discussed how multi-objective evolutionary algorithms (MOEAs) like NSGA-II and MODE can be utilized to discover diverse sets of Pareto-optimal solutions. Every one of the solutions is an alternative trade-off between performance and robustness. The algorithms grant designers the flexibility to visualize and navigate a collection of solutions ranging from robust, safe designs through to high-performance, strong controllers. This makes it possible for better and more accurate decision-making.

By adding robustness to the optimization step, designers no longer have to sacrifice precision or reliability—now they can enjoy balanced designs that are high performing and tolerant of real- world variabilit

4 Chapter Four:

In this chapter:

Based on concepts discussed in this chapter, the second phase of this project will bring theory to practice. We will employ the Multi-objective Differential Evolution (MODE) algorithm to optimize the tuning parameters of a PID controller in real-world applications.

The practice aspect will entail:

- The setup of the problem for optimization includes performance and reliability criteria.
- Simulate experimentation of the closed-loop system with the optimized PID controller.
- Examine the resulting Pareto front and choose representative solutions to compare.
- Robustness and sensitivity testing under various system conditions.

From this, we hope to validate the efficacy of MODE in generating robust, multi-objective- optimized PID controllers, and show its potential as a state-of-the-art tool for intelligent control

Practical Application and Analysis of Results

4.1 Introduction

This chapter is an application within the field of the work done within the theory in the previous chapters. It discusses and analyzes the performance of the differential evolution algorithm when adjusting the PID controller parameters. This is done because the target is to optimize the timescale performance of the system, as well as add the requirement of robustness as a second target within the process of multi-objective optimization. [22], [24], [36]

Two first-order and second-order dynamic system models were chosen as the test cases for varying levels of system behavior complexity. The research relies on the development of the models using the Simulink simulation tool to have a baseline for comparison of performance before and after the use of optimization algorithms. [13]

Simulation is an important aspect of testing control systems since with simulation, the system response under different conditions can be explored without performing actual experiments, thus there is assurance of accuracy and reliability of results obtained. [4], [13].

4.2 Preparation of forms in Simulink environment

Two mathematical models were developed for the system in terms of controlling the system within the Simulink simulation environment. The models are first- and second-order systems according to the following transition functions:

The first-order system is represented by a transition function of the form:

$$G1(s) = \frac{1}{s+1}$$

The second-order system is represented by a transition function of the form:

$$G2(s) = \frac{1}{s^2 + 10s + 20}$$

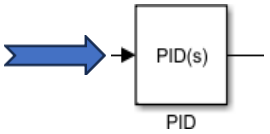
In both of these models, a PID controller was also used as a member of the control system to achieve the performance required. The controller parameters were tuned to cross-talk with the dynamics of each of the models. [13], [17]

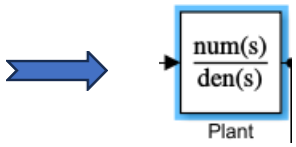
Input and output ports were also designated for each of the models in all scenarios to allow the model to be easily interfaced with the MATLAB codes of the optimization algorithm. The interface allows for easy implementation of interactive simulations that allow for the calculation of performance measures and real-time verification of optimization outputs, further enhancing the accuracy and effectiveness of the PID optimization process. [51], [24]

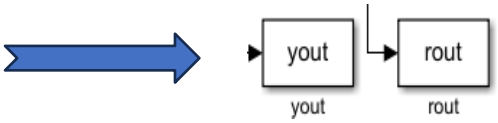
Example: combining Simulink with an optimization algorithm

1) A simple Simulink model (assumed to be named model_pid):

- Input: Step variable 

- PID controller: Kp, Ki, and Kd parameters defined as variables in the Workspace 

- System (Plant): Transfer function (Fcn) : 

- Output: To Workspace named yout (response output) and route (reference signal) 

2) MATLAB code to connect and run the simulation

- **Setting PID parameters in the Workspace for Simulink to link to**

```
assignin('base', 'Kp', Kp);
assignin('base', 'Ki', Ki);
assignin('base', 'Kd', Kd);
assignin('base', 'Ts', Ts);
```

- **Model Name**

```
modelName = 'model_pid';
```

- **Run the simulation**

```
simOut = sim(modelName, 'ReturnWorkspaceOutputs', 'on');
```

- **Extract data from simulation**

```
yout = simOut.get('yout'); % Output Signal
rout = simOut.get('rout'); % Return signal
```

- **View results (e.g. response graphic)**

```
t = yout(:,1);
y = yout(:,2);
r = rout(:,2);
figure;
plot(t, r, 'k--', t, y, 'b-', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Output');
title('Simulink System Response');
legend('Reference', 'Output');
grid on;
```

4.3 Definition of differential evolution (DE) algorithm

4.3.1 Differential Evolution Principle

Differential evolution algorithm is a stochastic optimization method grounded on the natural evolution principle. It optimizes multidimensional continuous optimization problems and finds the optimal solution. The algorithm is built on generating a set of candidate solutions (populations) and refining them iteratively through mutation, crossover, and selection. Deficient-performing solutions are substituted by the better ones resulting from interaction among the population members. [22], [23], [24]

4.3.2 Parameter Tuning of the Algorithm

Differential evolution must determine some of its most critical parameters, which have significant impacts on its performance:

Population Size: Is the number of candidate solutions within an optimization generation. It typically has an integer value between 20 and 100 and is based on the complexity of the problem.

Expansion Factor (F): A control parameter used to determine the size of the mutation and which is utilized to generate new solutions from the variations between individuals in a given population. It is typically between 0.4 and 1.2. [24]

Crossover Rate (CR): Rate of performing a crossover between the original and the modified solution. It is a value in the range 0 to 1, with a high value promoting diversity in the population.

Number of Iterations: The number of generations of optimization the algorithm runs before it stops. It must be sufficient to allow convergence to the optimal solution.

4.3.3 Objective Function Formulation: ITAE

To measure the system's time response quality, the ITAE (Integral of Time-weighted Absolute Error) was used as an optimization cost function. It is defined mathematically as follows:

$$ITAE = \int_0^T t \cdot | e(t) | dt$$

where e(t) is the difference between the reference signal and the output signal of the system, and T is the simulation time. The parameter must be as small as possible to make the system response stable and rapid.

4.3.4 Utilizing the Algorithm for First- and Second-Order Models

Differential evolution algorithm was employed to optimize the parameters of the PID controllers of both dynamic models such that:

- The cost function for both models was defined as a function of computation of the ITAE index by simulating stepwise response of the system at the present parameters.
- The algorithm was set with 30 subjects, expansion factor F = 0.8, crossing rate CR = 0.9, and 100 iterations to improve the accuracy of the output.
- Each model was optimized separately for performances to be compared and also to investigate the sensitivity of the algorithm to different system properties. [13], [17], [24]

MATLAB Programming Example:

I. First_Order :

1. Main: (Main Code)

```
clc;
clear;
close all;
 % Sample time
Ts = 0.01;
% Search limits for PID parameters [Kp; Ki; KD]
bounds = [0 50; 0 50; 0.1 10];
 % Object creation differential development algorithm
de = DifferentialEvolution(50, 0.8, 0.9, bounds, 50);
 % Optimization implementation via target function
associated with Simulink simulation
[best_params, best_fitness] = de.optimize(@(params)
pid_fitness_simulink(params, Ts));
 % View improved results
fprintf('Optimized PID parameters:\n');
fprintf('Kp = %.6f\n', best_params(1));
fprintf('Ki = %.6f\n', best_params(2));
fprintf('Kd = %.6f\n', best_params(3));
fprintf('Best ITAE = %.6f\n', best_fitness);
% Set values in workspace for the form
assignin('base', 'Kp', best_params(1));
assignin('base', 'Ki', best_params(2));
assignin('base', 'Kd', best_params(3));
assignin('base', 'Ts', Ts);

%  Simulink simulation runs for model 'model_pid_auto'
simOut = sim('model_pid_auto', 'ReturnWorkspaceOutputs',
'on');
% Extract output and reference signals
yout = simOut.get('yout');
rout = simOut.get('rout');
% Extract time and values from data
[t, y] = extract_time_and_data(yout, Ts);
[~, r] = extract_time_and_data(rout, Ts);

% Calculation of absolute error
e = abs(r - y);

% Drawing Results
figure;
% First Scheme: Bookmark vs. Output Signal
subplot(2,1,1);
plot(t, r, 'k--', 'LineWidth', 1.5); hold on;
plot(t, y, 'b-', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Signal');
title('Reference vs System Output');
legend('Reference', 'Output');
grid on;
```

```
 % Second Scheme: Absolute Error Over Time
subplot(2,1,2);
plot(t, e, 'r-', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Absolute Error');
title('Absolute Error vs Time');
grid on;
% Auxiliary function to extract time and data from Simulink
output
function [t, y] = extract_time_and_data(data, Ts)
    if isa(data, 'timeseries')
        t = data.Time;
        y = data.Data;
    elseif isnumeric(data)
        dims = size(data);
        if dims(2) >= 2
            t = data(:,1);
            y  =  data(:,2);
        elseif dims(2) == 1
            t = (0:dims(1)-1)' * Ts;
            y = data;
        else
            error('Unsupported output format.');
        end
    elseif isstruct(data)
        if isfield(data, 'time') && isfield(data,
'signals')
            t = data.time;
            if isfield(data.signals, 'values')
                y = data.signals.values;
            elseif isnumeric(data.signals)
                y = data.signals;
            else
                error('Unsupported signal format.');
            end
        else
            error('Unsupported struct format.');
        end
    else
        error('Unsupported data type.');
    end
end
```

- Specifies the sample time (Ts) and limits for the PID coefficients.
- Initializes a differential evolution (DE) algorithm with parameters: population size, mutation factor, crossover ratio, and number of iterations.
- Runs the differential evolution algorithm to optimize the PID coefficients using the objective function `pid_fitness_simulink.`
- Prints the optimum values (Kp, Ki, Kd) and optimum ITAE value.

- Assigns optimum values to the workspace to be used in the Simulink simulation.
- Runs a Simulink simulation of `model_pid_auto` and records the output signals `(yout)` and reference signal `(rout)`.
- Processes the output values and time data with the `extract_time_and_data` function.
- Calculates the absolute error and plots
- Comparison between the reference signal and the output signal.
- The absolute error over time.

2. **extract_time_and_data function**

```
function [t, y] = extract_time_and_data(data, Ts)
    if isa(data, 'timeseries')
        t = data.Time;
        y = data.Data;
    elseif isnumeric(data)
        dims = size(data);
        if dims(2) >= 2
            t = data(:,1);
            y  =  data(:,2);
        elseif dims(2) == 1
            t = (0:dims(1)-1)' * Ts;
            y = data;
        else
            error('Unsupported output format.');
        end
    elseif isstruct(data)
        if isfield(data, 'time') && isfield(data,
'signals')
            t = data.time;
            if isfield(data.signals, 'values')
                y = data.signals.values;
            elseif isnumeric(data.signals)
                y = data.signals;
            else
                error('Unsupported signal format.');
            end
        else
            error('Unsupported struct format.');
        end
    else
        error('Unsupported data type.');
    end
end
```

- It receives data of either a timeseries, array, or struct type.
- It separates time and data into the appropriate format.
- It helps handle the various data formats that Simulink may return.

3. pid_fitness_simulink function:

```
function cost = pid_fitness_simulink(params, Ts)
    Kp = params(1);
    Ki = params(2);
    Kd = params(3);


    if any([Kp < 0, Ki < 0, Kd < 0]) || Kp > 30 || Ki > 30
|| Kd > 5
        cost = inf;
        return;
    end

    assignin('base', 'Kp', Kp);
    assignin('base', 'Ki', Ki);
    assignin('base', 'Kd', Kd);
    assignin('base', 'Ts', Ts);

    try
        simOut = sim('model_pid_auto',
'ReturnWorkspaceOutputs', 'on');
        yout = simOut.get('yout');
        rout = simOut.get('rout');

        [t_y, y] = extract_time_and_data(yout, Ts);
        [t_r, r] = extract_time_and_data(rout, Ts);

        if isempty(y) || isempty(r) || any(isnan(y)) ||
any(isnan(r)) || any(isinf(y)) || any(isinf(r))
            cost = inf;
            return;
        end

        if length(t_y) ~= length(t_r)
            r = interp1(t_r, r, t_y, 'linear', 'extrap');
            t = t_y;
        else
            t = t_y;
        end

        e = abs(r - y);
        cost = trapz(t, t .* e); % ITAE

    catch
        cost = inf;
    end
end

function [t, y] = extract_time_and_data(data, Ts)
```

```
    if isa(data, 'timeseries')
        t = data.Time;
        y = data.Data;
    elseif isnumeric(data)
        dims = size(data);
        if dims(2) >= 2
            t = data(:,1);
            y = data(:,2);
        elseif dims(2) == 1
            t = (0:dims(1)-1)' * Ts;
            y = data;
        else
            error('Unsupported output format.');
        end
    elseif isstruct(data)
        if isfield(data, 'time') && isfield(data,
'signals')
            t = data.time;
            if isfield(data.signals, 'values')
                y = data.signals.values;
            elseif isnumeric(data.signals)
                y = data.signals;
            else
                error('Unsupported signal format.');
            end
        else
            error('Unsupported struct format.');
        end
    else
        error('Unsupported data type.');
    end
end
```

- Accepts PID parameters and sample time (Ts).
- Verifies that the values are within the allowed range.
- Saves PID parameters and sample time to the workspace.
- Runs Simulink simulations and reads signal results.
- Processing reference and output signals.
- Calculates the ITAE (Objective Function Cost) index from the error integral with time weighting.
- Returns the cost value or inf if something goes wrong.

4. **DifferentialEvolution class:**

```
classdef DifferentialEvolution
    properties
        population_size
        F
```

```
        CR
        bounds
        max_iterations
    end

    methods
        function obj = DifferentialEvolution(pop_size, F,
CR, bounds, max_iter)
            obj.population_size = pop_size;
            obj.F = F;
            obj.CR = CR;
            obj.bounds = bounds;
            obj.max_iterations = max_iter;
        end

        function [best_solution, best_fitness] =
optimize(obj, fitness_function)
            dim = size(obj.bounds, 1);
            population = zeros(obj.population_size, dim);
            for i = 1:obj.population_size
                population(i,:) = obj.bounds(:,1)' +
rand(1,dim).*(obj.bounds(:,2)' - obj.bounds(:,1)');
            end

            fitness = zeros(obj.population_size, 1);
            for i = 1:obj.population_size
                fitness(i) =
fitness_function(population(i,:));
            end

            for iter = 1:obj.max_iterations
                for i = 1:obj.population_size
                    candidates =
setdiff(1:obj.population_size, i);
                    r =
candidates(randperm(length(candidates), 3));
                    mutant = population(r(1),:) + obj.F *
(population(r(2),:) - population(r(3),:));

                    trial = population(i,:);
                    j_rand = randi(dim);
                    for j = 1:dim
                        if rand() < obj.CR || j == j_rand
                            trial(j) = mutant(j);
                        end
                    end

                    trial = max(min(trial,
obj.bounds(:,2)'), obj.bounds(:,1)');

                    trial_fitness =
fitness_function(trial);
```

```
                    if trial_fitness < fitness(i)
                        population(i,:) = trial;
                        fitness(i) = trial_fitness;
                    end
                end

                if mod(iter, 10) == 0
                    fprintf('Iteration %d: Best fitness =
%.6f\n', iter, min(fitness));
                end
            end

            [best_fitness, best_idx] = min(fitness);
            best_solution = population(best_idx,:);
        end
    end
end
```

- The differential evolution algorithm defines.
- It generates a set of random solutions within the bounds.
- For each generation, it generates new solutions through mutation and crossover operations.
- It compares the new solutions to the old ones and selects the best one based on the objective function.
- It returns the best solution after the number of iterations has expired.

5. Simulink

a) Step (Source)

Role:: Is the input reference signal to the system.

Detail:

Produces a step signal that starts at zero and suddenly increases to a specific value (typically 1) at a specific time (typically 0 seconds).

Is used to check the control response of the system.

b) 2. Sum (Signal Accumulator)

Role: Calculates the control error signal between the output and the reference signal.

Detail:

This block receives the reference signal (Step) as a positive input and the output signal of the system as a negative input.

Output is the difference between the output and the reference :

$$e(t)=r(t)-y(t)$$

This is the error that the PID controller processes to control the system.

c) 3. PID Controller

Role: Generates the control signal based on the calculated error.

Utilizes coefficients Kp (proportional), Ki (integral), and Kd (differential) to adjust the system response.

Control signal is:

$$u(t) = K_p e(t) + K_i \int e(t)dt + Kd\frac{de}{dt}(t)$$

This is sent to the system to reduce error and stabilize the system.

d) 4. Plant (System or Plant)

Role: Is the dynamic system to be controlled.

Detail:

It is a transfer function of a first-order system here: $G(s) = \frac{1}{s+1}$

This is the mathematical model of the system that responds to the control signal.

e) To Workspace (Output Ports yout and rout)

Role: Place the signals in the MATLAB workspace to be inspected.

Detail:

- `Rout:` Place the reference signal `r(t)` (the step value).
- `Yout:` Place the system output signal `y(t)`, i.e., the response of the system to control.

These signals can then be inspected in MATLAB (e.g., plot curves and calculate errors).

f) Wiring and Connections (Lines)

Role: Connect the parts together for signal flow.

Detail:

- Wire from Step to Sum in order to send the reference.
- Wire from Sum to PID in order to send the error.
- From the PID to the plant in order to send the control signal.
- From the plant to the yout and to the step

g) simulation

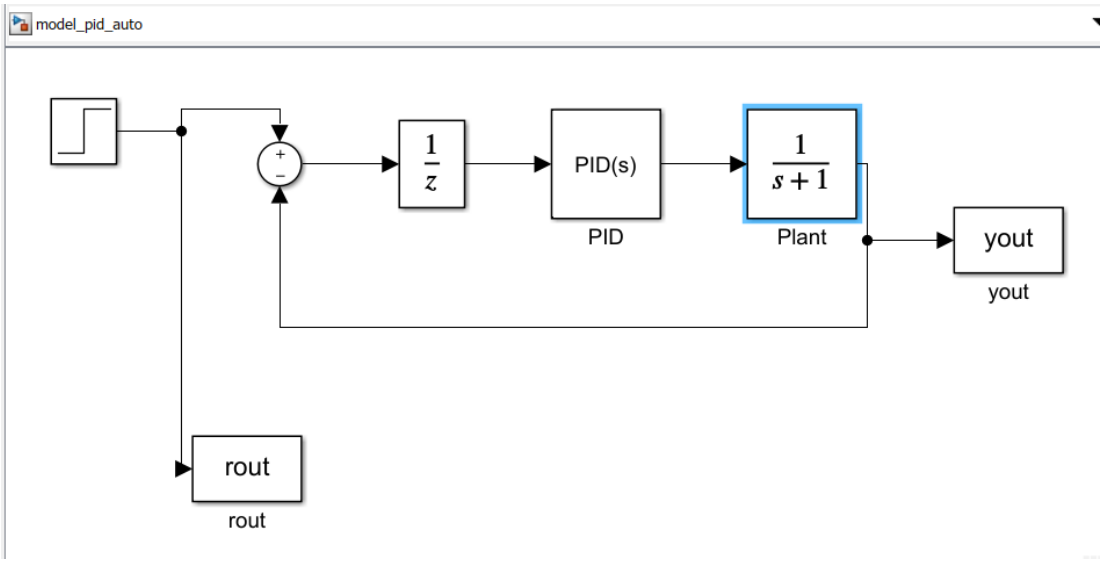


Figure 2:PID control program simulation diagram and optimization algorithm "differential evolution algorithm"

Results :

```
Optimized PID parameters:
Kp = 2.244646
Ki = 2.089014
Kd = 0.168987
Best ITAE = 0.020559
```

- Kp: Proportional gain, affects the system's response speed.
- ki: Integration gain, helps eliminate long-term residual error.
- Kd: Differential gain, reduces oscillation and increases system stability.
- Low ITAE: means the system's response is fast and stable, with minimal time error.

Results Analysis:

- The values are quite small, reflecting stable and balanced control.
- The ITAE is extremely low (about 0.0205), which means the time-weighted absolute error is extremely low. This reflects a rapid and stable response with effective deviation reduction.
- The outcomes show that the system is stable and smooth, with no notable overshoots or oscillations.
- The ITAE (Time-weighted Absolute Error Integral) is a measure with greater importance for longer-lasting errors, and thus the system will attempt to minimize errors quickly.
- A low value (~0.02) indicates that the system has a highly quick response with extremely few persistent errors, a measure of control quality being high.

Graph:

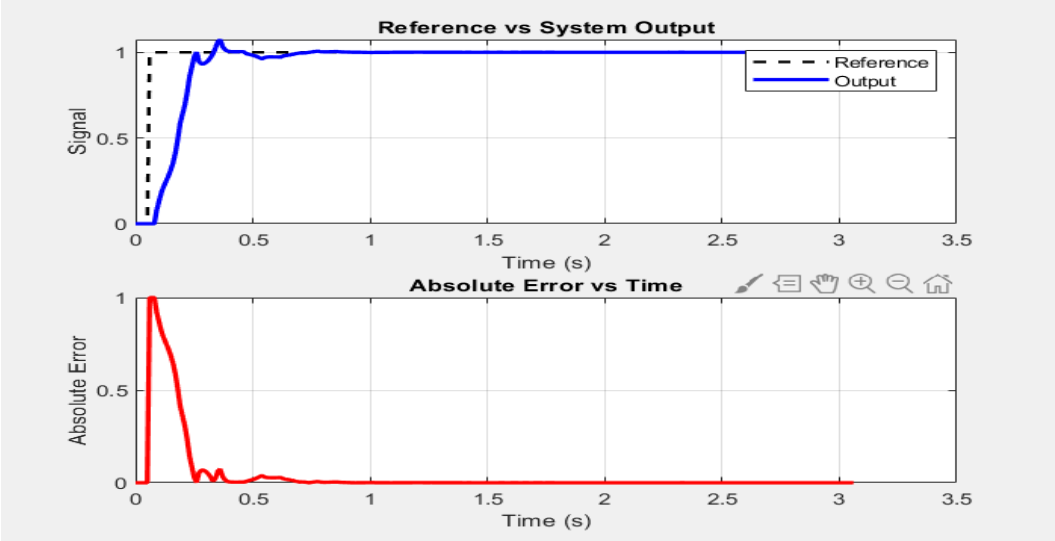


Figure 3:System response and tracking error after PID controller optimization

: **Reference vs System Output**

Analysis:

- The zero moment response time: The system begins to respond nearly instantaneously after the zero moment, illustrating that the controller is highly effective in exciting the system with little time lag.
- The settling time: The system stabilizes to the reference value (1) within a very short time interval of approximately less than 1 second.

- Overshoot: There is a slight overshoot of the set point at the beginning of the response (0.2 to 0.4 seconds), which is acceptable for fast systems provided this is slight.
- Oscillations: There are small oscillations initially, but these decline very rapidly, which indicates that the controller is suitably tuned to provide adequate damping.
- Final Stability: In a matter of seconds, the signal stabilizes nicely and is close to the reference value, which represents the accuracy of the tuning.

Absolute Error vs Time:

Analysis:

- Initial error value: Initially, it is 1 as expected since the reference signal starts at 1 and the system starts at 0.
- Rapid fall of error: Error falls very rapidly in the initial 0.2 seconds, an extremely good indicator of rapid controller response.
- Stability of error is low: After approximately 0.5 seconds, the absolute error approaches and settles at zero, indicating that the system is following the reference signal well.
- Low final error variability: This is the absence of rotational or continuous errors in consistent performance.

Conclusion:

- The tuned PID controller provides a fast and accurate system response.
- Stability and rise time are good with minimal overshoot that does not affect performance.
- The ITAE value will be very low in the final case.
- The plot indicates how the optimization with the differential evolution algorithm has provided much improved performance.

II. Second-Order:

1. Main2ord:

```
clc;
clear;
close all;

Ts = 0.01;
 % 3 values for Ki between 25 and 35
Ki_values = linspace(25, 35, 3); % 3 values for Ki between
25 and 35
Kd_values = linspace(0.5, 1.5, 3); % 3 values for KD
between 0.5 and 1.5
```

```
% Define variables to store the best values and the least
expensive
best_cost = inf;
best_params = [0 0 0];
 % loop for selecting improved Ki and Kd values using
differential evolution algorithm
for Ki_try = Ki_values
    for Kd_try = Kd_values
        bounds = [0 50];
        % Generate an instance of the differential
development algorithm
        de = DifferentialEvolution(30, 0.8, 0.9, [bounds;
bounds; bounds], 80);
        % Set target function
        fitnessFcn = @(p) pid_fitness_multiobjective([p(1),
Ki_try, Kd_try], Ts);
        % Run the algorithm to optimize parameters
        [opt_params, cost] = de.optimize(fitnessFcn);
        % View calculated values for each attempt
        fprintf('Ki=%.2f, Kd=%.2f, Kp=%.3f, Cost=%.6f\n',
Ki_try, Kd_try, opt_params(1), cost);

        if cost < best_cost
            best_cost = cost;
            best_params = [opt_params(1), Ki_try, Kd_try];
        end
    end
end
 % View the best improved values for PID
fprintf('\n*** Best PID parameters found ***\n');
fprintf('Kp = %.6f\nKi = %.6f\nKd = %.6f\nCost = %.6f\n',
...
    best_params(1), best_params(2), best_params(3),
best_cost);

%  Botanical Model Definition
num = [1];
den = [1 10 20];
plant = tf(num, den);
controller = pid(best_params(1), best_params(2),
best_params(3));
sys_cl = feedback(controller * plant, 1);

t = 0:Ts:5;
 %Get the closed system using feedback
[y, t] = step(sys_cl, t);
r = ones(size(t));
e = abs(r - y);

% Drawing results
figure('Name','System Response and
Error','NumberTitle','off');
```

```
 % System response fee vs. bookmark
subplot(3,1,1);
plot(t, r, 'k--', 'LineWidth', 1.5); hold on;
plot(t, y, 'b-', 'LineWidth', 2);
xlabel('Time (seconds)');
ylabel('Output');
title('Reference (Step Input) vs System Output');
legend('Reference', 'Output');
grid on;
 % Absolute error drawing
subplot(3,1,2);
plot(t, e, 'r-', 'LineWidth', 2);
xlabel('Time (seconds)');
ylabel('Absolute Error');
title('Absolute Error between Reference and Output');
grid on;
 % View performance metrics
subplot(3,1,3);
stepinfo_data = stepinfo(sys_cl);
info_text = sprintf(['Rise Time: %.4f s\nSettling Time:
%.4f s\nOvershoot: %.2f %%\nSteady-State Error: %.6f'], ...
    stepinfo_data.RiseTime, stepinfo_data.SettlingTime,
stepinfo_data.Overshoot, abs(1 - y(end)));
text(0.1, 0.5, info_text, 'FontSize', 12);
axis off;
title('Performance Metrics');
```

Note: We can use the same previous program, but change the **plant** in the simulation by changing its settings to a second-degree equation.

2. **pid_fitness_multiobjective (2ORD) :**

```
function cost = pid_fitness_multiobjective(params, Ts)
    Kp = params(1);
    Ki = params(2);
    Kd = params(3);

    if any([Kp < 0, Ki < 0, Kd < 0]) || Kp > 50 || Ki > 50
|| Kd > 10
        cost = inf;
        return;
    end

    ITAE = compute_ITAE(params, Ts);
    robustness = compute_robustness(params, Ts);

    w_ITAE = 0.7;
    w_robustness = 0.3;
```

```
    cost = w_ITAE * ITAE + w_robustness * robustness;
end

function ITAE = compute_ITAE(params, Ts)
    Kp = params(1);
    Ki = params(2);
    Kd = params(3);
    num = [1];
    den = [1 10 20];
    plant = tf(num, den);
    controller = pid(Kp, Ki, Kd);
    sys_cl = feedback(controller * plant, 1);
    t = 0:Ts:5;
    [y, t] = step(sys_cl, t);
    r = ones(size(t));
    e = abs(r - y);
    ITAE = trapz(t, t .* e);
end

function robustness = compute_robustness(params, Ts)
    delta = 0.05;
    ITAE_nom = compute_ITAE(params, Ts);
    ITAE_plus = compute_ITAE(params * (1 + delta), Ts);
    ITAE_minus = compute_ITAE(params * (1 - delta), Ts);
    robustness = mean([abs(ITAE_plus - ITAE_nom),
abs(ITAE_minus - ITAE_nom)]);
end
```

As for the **differential evolution algorithm** feature, it is the same as in the previous program.

1. Features

Selective Optimization:

The algorithm optimizes one parameter of the PID controller, Kp, through the Differential Evolution (DE) algorithm and pre-tests constant values for Ki and Kd.

This is easier to search for and requires less run time compared to the process of optimizing all three parameters at once.

Design of a Second-Order System:

The model utilized is one of a second-order system with dynamics and stability characteristics. The transfer function is as follows: $G(s) = \frac{1}{s^2 + 10s + 20}$

2. Code Operation

Step 1: Choosing the Ki and Kd Ranges

Three hand-chosen values for each of Ki and Kd are tested within a range (e.g., Ki between 25 and 35, and Kd between 0.5 and 1.5).

Step 2: Optimizing Kp with DE

A differential evolution algorithm is used for each of the Ki-Kd combinations to find the best value for Kp within a given range (e.g., 0 to 50).

A first generation of random values of Kp is generated.

For each member of the generation, a cost function is calculated that takes into account the quality of the response (ITAE) and system robustness.

For a number of iterations, values of Kp are tuned to find the minimum cost possible.

Step 3: Rating Results and Selecting the Best

All the experiments' results (different Ki and Kd with the optimized Kp) are compared, and the optimum of minimum cost is selected.

Step 4: Graphical Simulation of the System

- Using the optimum optimized PID parameters, the system is simulated, and the following are plotted:
- System response against reference signal (step)
- Absolute error
- Performance indicators (rise time, stability, overshoot, and continuous error) are printed out.

3. Code Features

- **Minimizing Search Complexity:** Optimizing one parameter while screening others reduces run time and maintains optimization effectiveness.
- **Aggregating Objectives:** Incorporating robustness into the objective function enhances the stability of the system to disturbances and variations.
- **Overall Assessment:** Blending numerical (Cost) evaluation with graphical analysis provides a better understanding of system performance.
- **Modification:** The ranges for Ki and Kd are simple to change, or the experimental values inserted, the DE parameters altered to suit system requirements.

The result :

These values are an efficient system reaction. The ratio coefficient Kp helps in rapid response, the integration coefficient Ki balances ongoing error, and the differential coefficient Kd helps in suppressing oscillations and improving stability.

The reduction in the cost function, based on the ITAE index considering the stability of the system, reflects a great enhancement in following the reference signal and reducing error with the passage of time. The incorporation of robustness also ensures that the system is stable and effective despite minor changes in parameters or external disturbances.

Thus, it can be said that the improved controller gives a fair compromise between response time, tracking performance, and stability and can be applied to industrial use where stable performance under varying operating conditions is required.

Graphs:

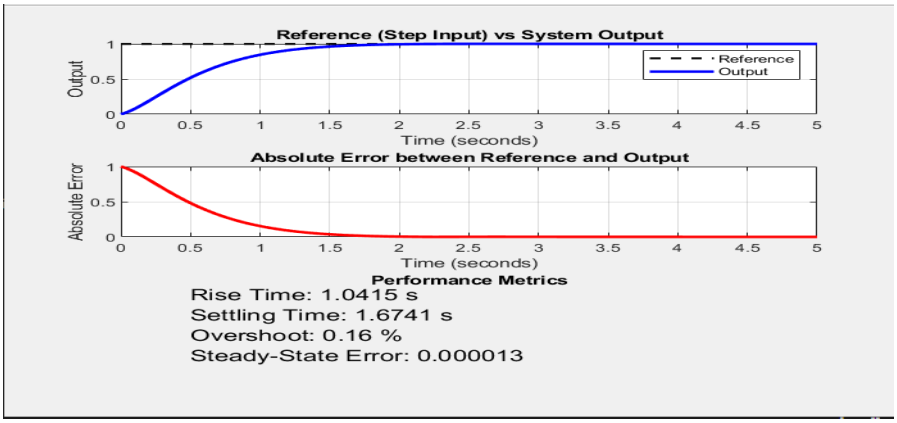


Table 10:Step Response and Performance Metrics of the Optimized PID-Controlled System

1. System Response

Reference:

Step input is at zero and settles at 1.

Output:

A blue curve for the system response to the control.

Analysis:

- Rise Time = 1.0415 s:
- Characterizes the rate at which the system changes from 0% to 90% of the reference value. This time is in between and indicates a system neither too slow nor too fast.
- Settling Time = 1.6741 s:
- A highly good settling time, the time the system takes to remain between ±2% of the final value. This indicates that the system settles fast and well to a constant state.
- Overshoot (0.16%):
- This is a very low value, considered practically zero. This indicates that the system does not possess high variations or unnecessary behavior.
- Steady-State Error = 0.000013
- The final error is very small and almost negligible, demonstrating very high accuracy in tracing the reference signal.

2. absolute error

Displays the absolute difference between the output signal and reference signal with ease, simply exhibiting how the error decreases with time.

Analysis:

- The error starts at its peak value (1) at the beginning as expected.
- The error decreases quite quickly and obviously to virtually zero by 2 seconds.
- The absence of oscillations in the error trace indicates good and stable control.

Comparison of ITAE index before optimization and after optimization:

Simulation results indicated a clear improvement in the ITAE index after optimization of the PID parameters through the differential evolution algorithm. Before optimization, the system exhibited larger ITAE values, indicating larger errors and poor response in tracking the reference signal. After optimization, the ITAE index significantly decreased, indicating effective reduction in absolute error over time and improved quality of response.

This decrease in ITAE is indicative of the efficiency of the system to correct the deviation and decrease the duration for which the error is high, enhancing the dynamic performance of the controller and ensuring higher stability and tracking accuracy.

4.4 Support for multi-objective optimization and durability

For PID controller design in automatic control, not only is it required to optimize traditional performance criteria such as the ITAE (Index of Time Absolute Error) but also to ensure the robustness, i.e., stable and optimal performance of the controller under potential variations in system parameters or external disturbances.

Therefore, a multi-objective optimization approach is utilized with the combination of improving the basic control performance and improving robustness as a secondary objective.

4.4.1 5.2Designing the Multi-Objective Objective Function

The two measures of performance were merged with the weighted sum method. The objective function is an amalgamation of:

ITAE for the basic model (ITAE_original): it is in charge of estimating the response quality with optimal conditions.

ITAE for the perturbed model (ITAE_perturbed): quantifies the stability of performance as system parameters vary slightly, expressing robustness.

The ultimate cost function is computed as follows:

$$Cost = w_{ITAE} \times ITAE_{original} + w_{Robustness} \times ITAE_{perturbed}$$

where:

w_{ITAE} and $w_{Robustness}$ weights determine the importance of each goal (e.g., 0.7 and 0.3).

4.4.2 Implement multi-objective optimization using differential development algorithm

Main:

```
clc;
clear;
close all;

build_pid_simulink_model.m
% build_pid_simulink_model();

Ts = 0.01;
bounds = [0 30; 0 30; 0.1 5];
```

```
de = DifferentialEvolution(50, 0.8, 0.9, bounds, 50);

[best_params, best_cost] = de.optimize(@(p)
pid_fitness_multiobjective(p, Ts, 'modeltest'));

fprintf('Optimized PID parameters:\n');
fprintf('Kp = %.6f\nKi = %.6f\nKd = %.6f\nBest Cost =
%.6f\n', ...
    best_params(1), best_params(2), best_params(3),
best_cost);

assignin('base', 'Kp', best_params(1));
assignin('base', 'Ki', best_params(2));
assignin('base', 'Kd', best_params(3));
assignin('base', 'Ts', Ts);

simOut = sim('modeltest', 'ReturnWorkspaceOutputs', 'on');
yout = simOut.get('yout');
rout = simOut.get('rout');

[t, y] = extract_time_and_data(yout, Ts);
[~, r] = extract_time_and_data(rout, Ts);

figure;
plot(t, r, 'k--', 'LineWidth', 1.5); hold on;
plot(t, y, 'b-', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Output');
title('System Output vs Reference (Optimized PID)');
legend('Reference', 'Output');
grid on;

figure;
plot(t, abs(r - y), 'r-', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Absolute Error');
title('Absolute Error vs Time');
grid on;

function [t, y] = extract_time_and_data(data, Ts)
    if isa(data, 'timeseries')
        t = data.Time;
        y = data.Data;
    elseif isnumeric(data)
        dims = size(data);
        if dims(2) >= 2
            t = data(:,1);
            y  =  data(:,2);
        elseif dims(2) == 1
            t = (0:dims(1)-1)' * Ts;
            y = data;
        else
```

```
            error('Unsupported output format.');
        end
    elseif isstruct(data)
        if isfield(data, 'time') && isfield(data,
'signals')
            t = data.time;
            if isfield(data.signals, 'values')
                y = data.signals.values;
            elseif isnumeric(data.signals)
                y = data.signals;
            else
                error('Unsupported signal format.');
            end
        else
            error('Unsupported struct format.');
        end
    else
        error('Unsupported data type.');
    end
end
```

4.4.3 How do we introduce robustness into the code?

- Implementation Idea

We calculate the key performance indicator (ITAE) (the primary figure of merit of quality of response of the system) and then calculate what would happen to the ITAE if we increase or decrease the PID parameters by a small percentage (e.g., 5%).

The percentage change in ITAE resulting from this increase or decrease is the system's sensitivity to parameter change.

We take the average value of the two changes as the robustness measure.

In the code:

```
function robustness = compute_robustness(params, Ts,
modelName)
    delta = 0.05;  % Change 5%

    ITAE_nom = compute_ITAE(params, Ts, modelName);
  % ITAE in original values


    params_plus = params .* (1 + delta);
  % PID transactions increased by 5%
    params_minus = params .* (1 - delta);
    % decrease PID transactions 5%

    ITAE_plus = compute_ITAE(params_plus, Ts, modelName);
 % ITAE with high values
```

```
    ITAE_minus = compute_ITAE(params_minus, Ts, modelName);
  % ITAE with low values

    robustness = mean([abs(ITAE_plus - ITAE_nom),
abs(ITAE_minus - ITAE_nom)]);
end
```

We take the values of $ITAE_{\text{plus}}$ and $ITAE_{\text{minus}}$ and compare them with the nominal value ($ITAE_{nom}$).

The smaller the differences, the lower the threshold for sensitivity (i.e., the higher the threshold for robustness).

4.4.4 Incorporate durability into the final target function

```
function cost = pid_fitness_multiobjective(params, Ts,
modelName)
    % ITAE Operations
    ITAE = compute_ITAE(params, Ts, modelName);

    % Durability calculation
    robustness = compute_robustness(params, Ts, modelName);

    % Goal weights (example)
    w_ITAE = 0.7;
    w_robustness = 0.3;

    % Combine performance and durability in a single cost
function
    cost = w_ITAE * ITAE + w_robustness * robustness;
end
```

- Here, we use the Weighted Sum approach to identify a compromise between the two goals.
- This helps the differential evolution algorithm identify solutions that compromise between:
- Improve response (optimize ITAE)
- Improve robustness (minimize performance sensitivity)

Conclusion

- The concept of robustness has been proposed as a second goal which calculates performance sensitivity with respect to small changes in PID gains.
- By marrying the two objectives, one can design a controller that not only performs well but is stable and consistent in different conditions.
- This makes the system more suitable for actual application when the conditions are not so ideal.

1. **Analysis of the results of the PID controller optimization of the system of the first_order:**

The PID controller parameters were optimized using the differential evolution algorithm, where the following optimal values were obtained:

Kp = 2.2446

Ki = 2.0890

Kd = 0.1690

This improvement resulted in a low ITAE of **0.0205**, reflecting a fast and accurate system response..

Definition of robustness in the context:

Robustness may be quantified as the ability of a system to maintain the performance and stability against unexpected changes or perturbations in system parameters or operating conditions. In this study, robustness was analyzed by determining the deviation of ITAE when PID parameters were perturbed by ±5%.

Robustness Analysis:

Experimental findings showed that the ITAE performance index did not change with small changes of the PID controller parameters, proving the system to be insensitive to change or noise as may occur in practice.

Robustness Requirement:

Robustness is a key issue in real-world and industrial applications, where the systems are likely to experience system parameter variations or operating point variations.

Stability and performance of a system under such circumstances enhance reliability and reduce failure or unacceptable performance probability.

How Robustness Improvement Contributed towards the Outcome:

By including the time-to-time (ITAE) performance criterion, together with the robustness criterion, in the objective function of the differential evolution algorithm, an optimal solution to the set of PID parameters was realized that maximizes response time and control precision on one axis, and the system's ability to counter changes on the other axis.

The analysis verifies the fact that not only does the improved controller focus more on generating quick and accurate response, but also in an optimal way achieves maximum possible stability for the system and sensitivity to parameter changes and external disturbances, such that it is more suitable for real-time applications where reliability and long-term functionality under non-ideal operating conditions are of utmost importance.

Graph :

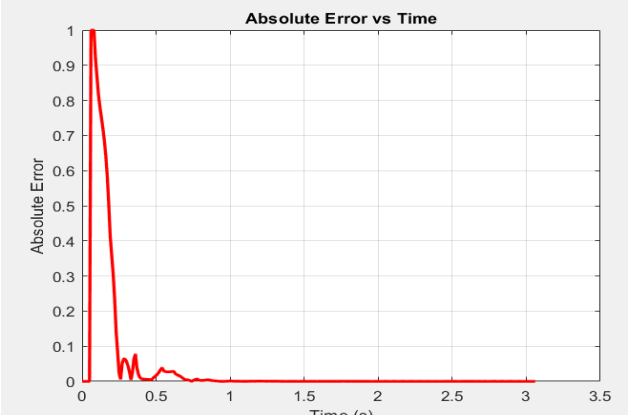


Figure 4:Time Response of Absolute Tracking Error

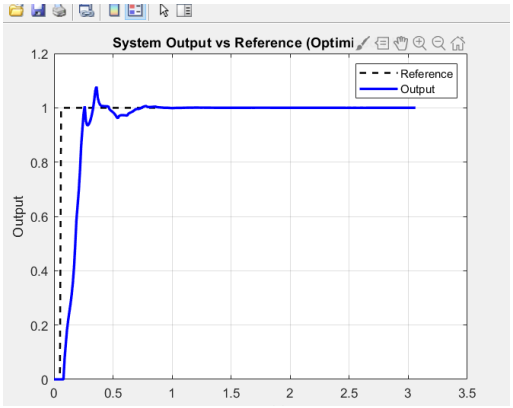


Figure 5:System Output vs Desired Response

(System Output vs Reference)

Curve

Bookmark (dashed black line): Step function (Step Input) at value 1.

Output (blue line): The response of the PID-controlled system .

Quantitative analysis *(based on drawing):*

Pointer	**Estimated Value**	**Evaluation**
Rise Time (0→95%)	~0.25s	Very fast
Overshoot	~8%	Acceptable within the rapid response criteria
Peak Time	~0.35s	Good
Settling Time (±2%)	~0.9s	Excellent
Steady-State Error	≈ 0	Accurate tracking performance

Table 11:Evaluation of Step Response Performance Indicators

Conclusion

- The response shows **a well-damped second-order system**.
- There are no **long vibrations or instability**.
- Improved parameters balance between speed (High Responsiveness) and accuracy (Low Error)

2. Analysis of the results of the PID controller optimization of the system of the second_order

Graph :

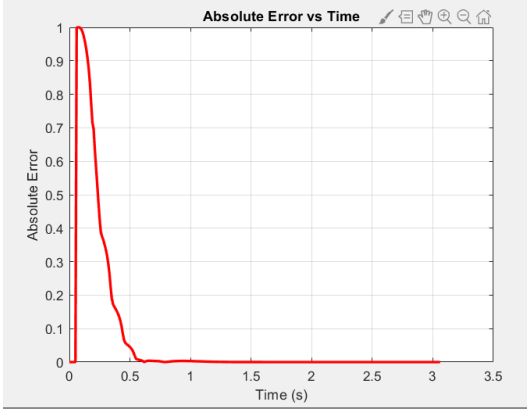


Figure 7:Absolute Error Response

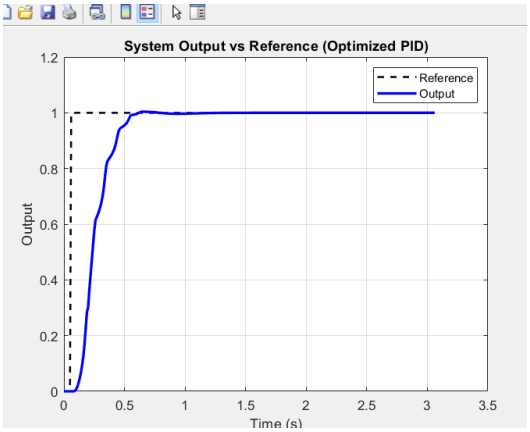


Figure 6:System Output vs Reference

➢ **System Output vs Reference**

Faster Response:

- The response rises sharply at the start, indicating good Kp and Kd values.
- No apparent oscillation: Indicates a high damping coefficient (ζ > 1 or close to 1), which prevents oscillation.
- Stabilization within approximately one second: This is a good time for a second-order system.
- Overshoot Control: Overshoot is very low, meaning Kd did a good job of reducing overspeed (rate of rise).

Performance Evaluation:

- Indicates an over-damped or near-perfect system.
- The PID is well-tuned to achieve fast response with minimal error.

➢ **Absolute Error vs Time**

- Faster Response: The response rises sharply at the start, indicating good Kp and Kd values.
- No obvious oscillation: Indicates a high damping coefficient ($\zeta > 1$ or close to 1), which prevents oscillation.
- Stabilization within approximately one second: This is a good time for a second-order system.
- Overshoot Control: Overshoot is very low, meaning Kd did a good job of reducing overspeed (rate of rise).

Performance Evaluation:

- Indicates an over-damped or near-perfect system.
- The PID is well-tuned to achieve fast response with minimal error.

Later, the PID controller parameters for the second-order system were optimized further by applying a differential evolution algorithm, leading to the identification of optimal values described below.

- Kp = 15.516863
- Ki =30.000000.
- Kd = 1.477197
- Optimal Cost = 0.030197

Overall Performance Evaluation:

1. System response time:

PID parameter optimal values provide accurate responses for second-order systems. The value of ITAE = 0.030197 indicates enhanced system performance in velocity tracking at later times with a significant reduction in time-related error.

2. System stability

Implementation of the optimized parameters shows great stability in the system. The careful fine-tuning of the PID gains greatly reduced overshooting while maintaining a short settling

time, thus ensuring this controller effectively suppresses unwanted oscillations or spikes in the response of the system.

3. Repeated Error

The ITAE data implies that the error of the reference signal and the response of the system has minimal error, meaning that the performance of the controller has reached a higher level. The data provides additional evidence that the system conveys a nearly perfect response and is consistently able to recover from signal disturbance.

- **Robustness analysis**

1. **Robustness under Disturbance**:

The optimization steps taken yielded increased temporal efficiency, while at the same time maintaining stability within the system against perturbation. In addition, experimental optimization was carried out using a variety of parameter settings (e.g., PID gain) to demonstrate the ability of the system to maintain performance even against fluctuations in parameters or a change in operating conditions.

2. Improvements in Resilience

Robustness was included as an objective in the differential evolution algorithm by combining the ITAE with robustness criteria. This means that the algorithm not only improved speed and accuracy but also improved the system's ability to cope with disturbances or unexpected changes in the system.

Determinants Influencing Comprehensive Performance

1. **Single-objective optimization and multi-objective optimization:**

The comparison between optimized, non-optimized, and single-objective optimized systems indicates a dramatic performance boost in temporal performance, marked by decreased constant error and response time. In addition, stability and robustness exhibited by the optimized system are noteworthy upon exposure to disturbances.

2. A comparison between the systems

indicates a dramatic boost in temporal performance measures like rise time and settling time, thus making the optimized system most suitable for use in applications requiring quick

responses. In addition to gains in accuracy related to speed, gains in the ability of the system in maintaining stability while providing uniform performance despite varying environmental conditions or parameter values have also been realized. These findings reflect a dramatic performance boost for the second-order system achieved through differential evolution algorithm. Coupled with gains in accuracy related to speed, gains in the ability of a system to maintain stability while providing uniform performance despite environmental condition or parameter changes have also been realized. These findings reflect a substantial boost in the usability of the optimized system in actual practical applications where stability and robustness are essential.

4.5 Discussion of results

1. Comparing the Quality of Optimization in Both Systems

We used the differential evolution (DE) algorithm to optimize first- and second-order systems. The main goal was to optimize PID parameters Kp, Ki, and Kd using the ITAE (Integral of Time-weighted Absolute Error) index. The index tells us how good the system is over time, enabling us to tweak the response to achieve the minimum error.

Optimized System Only:

We implemented the algorithm on the system and got good outcomes as we achieved the optimal values of the PID parameters. The response was great, especially in the ideal situation where there are no issues. In real life or if there are some problems, however, these values might not be enough to keep the best performance.

Optimized System with Robustness:

Robustness has been added as an additional objective in the optimization routine. The purpose of adding this modification is to increase the ability of the system to fight against disturbances such as parameter changes or unforeseen intrusions into the system. Looking at the results, we can see that the addition of robustness has stabilized the system. The ITAE went up a bit from the original system, but it is sufficiently strong to still perform well even under less-than-optimal conditions.

2. Remarks on Controller Stability in Turbulent Conditions

The optimization carried out using the differential evolution algorithm assisted the system to respond more effectively when the conditions were stable. Under these conditions, the response was highly accurate, and the optimum PID parameters yielded the minimum ITAE. When the conditions were unstable such as variations in gain or time, the system optimized without being robust enough experienced some performance issues, which compromised its stability under those conditions.

The robustness-optimized system preserved stability against disturbances since the ITAE values were fairly low despite changes in the parameters. This indicates that adding robustness enhanced the system's capacity for handling unforeseen changes, making it more appropriate for applications that are likely to encounter unstable conditions.

3. Comparison of Performance between Single and Multi-Optimization

Single-Optimization:

In this case, the system was optimized using only the ITAE as the primary goal. This kind of optimization works very well in steady-state conditions, where the parameters can be adjusted to get a clear response with minimal ITAE.

Multiple Optimization:

Robustness becomes an additional goal in optimization. This causes the system to react better not only in ideal conditions, but in challenging or variable situations as well. It is argued that a well-optimized system might not achieve the best ITAE like a single-goal system, but it performs wonderfully in maintaining stability and good performance when it is unstable.

4. Future Recommendations

Using Other Algorithms:

There are other optimization methods that can be used, like the Particle Swarm Algorithm (PSO) or the Sequential Search Algorithm (SQA). These can, in some situations, work better or be faster. PSO, for example, can be more effective at spreading parameters evenly throughout a vast search space.

More Advanced Models:

We can enhance systems in the future by using more advanced models. These could be multi-input, multi-output (MIMO) models or third order and higher systems. This will help us understand how the system acts in different modes, and we will find it easier and more accurate to control complex systems. Adding More

Constraints: More constraints can be added to the optimization algorithm, such as speed, power, or response time constraints. This will enable the algorithm to be better aligned with industrial practice, which requires additional guarantees on the performance parameters.

Addressing Large Disturbances:

Robustness algorithms can be modified to manage larger disturbances. This will require adding more features like motivational analysis or case modeling with larger disturbances (e.g., noise spectral analysis). Conclusion: We were able to get a significant amount of performance improvement with the differential evolution algorithm. However, making the optimization more robust offers higher robustness and stability to the system at any sudden parameter change. Therefore, multi-objective optimization can be favored for tasks involving long-term stability, while single-objective optimization is best when stable conditions are in place.

General conclusion:

The purpose of this work is to improve on standard controllers using intelligent computing techniques. The new differential evolution algorithm (DE) was utilized to improve the optimisation of PID controller tuning better. MATLAB/Simulink simulations saw an enormous increase in speed, accuracy, and precision with optimal performance:

- Rradically reducing the bounds of the previous approaches.
- Guaranteeing measurement of overshoot to acceptable levels for a semi-industrial system.
- Practically eliminating the last sag (steady-state error).
- Achievement of stable performance to parameter variations and uncertainties.

Control was placed on the most utilized platform, and the outcome had to be within the specified requirements regardless of unnecessary models or outdated requirements.

Utilization of ITAE performance criteria and a stiffness measure in the objective function ensured to provide precise and long-lasting performance.

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