PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA
Ministry of Higher Education and Scientific Research
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Faculty of Exact Science
Department of material science

Module: Mechanic 2
Courses with Exercises and Laboratory works

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Level: L1 / S2 — physio-chemical

Year University: 2025-2026

preface

The study of materials science constitutes a foundational pillar in modern engineering and applied sciences. In order to understand and advance industrial materials, it is essential to investigate their intrinsic properties across multiple dimensions, including physical, chemical, electrical, thermal, and mechanical characteristics. Each of these categories provides a distinct perspective on how a material behaves under specific conditions and environments.

The present document focuses exclusively on **mechanical properties**, which represent one of the most critical categories for engineering design and structural analysis. Mechanical properties define how a material responds to applied forces and loads, and they encompass key parameters such as **hardness**, **elasticity**, and **brittleness**. Understanding these properties is indispensable for selecting appropriate materials in the design of components, structures, and industrial systems.

This document is developed within the framework of the course *Mechanics of Materials – Module 2*, which aims to provide students with a rigorous understanding of the resistance and deformation behavior of solid bodies subjected to external mechanical forces. The course covers the fundamental concepts governing how solid, deformable bodies respond to various loading conditions, with particular emphasis on practical engineering applications. Topics addressed include internal stresses, strains, deformation analysis, and failure criteria, all of which are essential for sound engineering judgment and decision-making.

This manuscript is intended for second-year Physio-Chemical Sciences students at Mohamed Khadar University of Biskra. It integrates fundamental concepts from physics and chemistry with applied mechanical engineering principles, providing students with the analytical skills required to assess and predict material behavior in industrial applications.

I would like to express my sincere gratitude to everyone who contributed ideas and support to the preparation of this manuscript.

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CHAPTER 1: GENERAL CONCEPTS ABOUT STRENGTH OF MATERIALS

1.1. Introduction:

Strength of Materials is a core discipline in mechanical engineering that provides the theoretical basis for understanding how engineering materials respond to different types of loading. Since mechanical components and structural systems are manufactured from a wide variety of materials, engineers must understand their mechanical behavior under service conditions. This course aims to introduce students to the principles of applied loads, stress and deformation, the resulting effects on materials and structures, and the methods used to ensure safe and reliable engineering design [1].

1.2. The main purpose:

Strength of Materials is concerned with the mechanical response of deformable solids subjected to external loads. Its primary objective is to determine appropriate dimensions for structural and mechanical components so that they can safely withstand the applied forces throughout their intended service life while satisfying the required strength and safety criteria [2].

1.3. Hypotheses of material:

The fundamental assumptions of Strength of Materials are mainly related to the characteristics of the material, the geometry of the structural member, and the nature of the applied loads. The principal assumptions include:

- **Homogeneity, isotropy, and continuity:** The material is assumed to possess uniform mechanical properties throughout its volume, exhibit identical behavior in all directions, and be free from significant internal defects or discontinuities at the macroscopic scale.
- **Linear elasticity:** The material is considered to behave elastically within the working range, meaning that stress is directly proportional to strain according to Hooke's law. Consequently, once the applied load is removed, the material fully recovers its original shape and dimensions without permanent deformation [2].

1.4. Assumptions about beams:

1.4.1. Definition:

A beam is a structural element characterized by one dimension, its length, being significantly greater than its cross-sectional dimensions. When subjected to external loads, the beam

primarily resists bending and undergoes deformation depending on the magnitude and distribution of the applied forces and the material properties [2].

1.4.2 Geometry:

A beam is generally a solid generated by a plane area (S) whose center of gravity (G) describes a curve (C). The plane of the area (S) remains normal to the curve (C), as shown in figure 1.

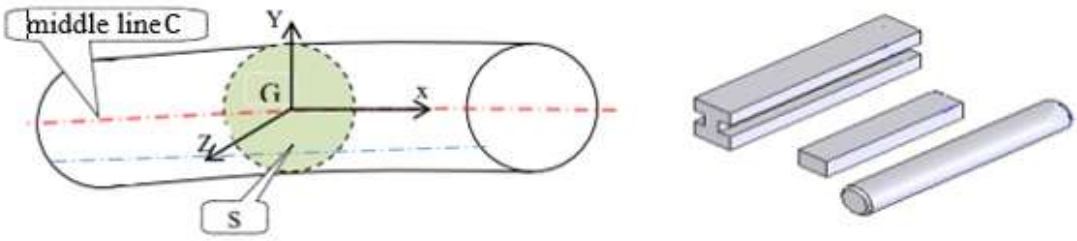


Fig. 1 Examples of beams.

- The area of the section (S) called the straight section or normal section of the beam.
- The curve (C) is called the neutral axis (or centroidal axis) of the beam.

1.5. Validity domain of material resistance:

The analysis of beams in Strength of Materials is commonly based on several simplifying assumptions that facilitate the mathematical formulation while providing sufficiently accurate results for engineering applications. These assumptions include:

- **Small deformation assumption:** The deformations produced by external loads are considered very small compared with the original dimensions of the structural member, allowing the geometry to remain essentially unchanged during analysis.
- **Navier–Bernoulli (plane sections) assumption:** Cross-sections that are initially plane and perpendicular to the beam's longitudinal axis remain plane and normal to the neutral axis after deformation, indicating that shear deformation is neglected.
- **Saint-Venant's principle:** At locations sufficiently distant from the points where loads are applied, the stress and strain distributions are governed only by the equivalent resultant force and bending moment, regardless of the detailed manner in which the loads are introduced.

Although these assumptions simplify the structural analysis, they generally provide reliable predictions of the mechanical behavior of beams and other structural elements subjected to

various loading conditions, making them widely applicable in engineering design and analysis [2–7].

1.6. Kinds of charge and liaisons:

1.6.1 Liaisons: There are three types of liaisons between beam and supports are following:

- **Moving -articulation support: (sphere or cylinder or roller)**

  The simple support (fig 2.) introduces a single reaction unknown into the study of the beam.

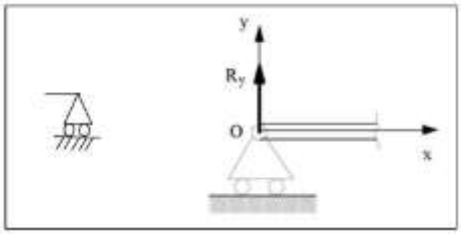


Fig 2. Simple support.

- **fixed -articulation support:**

  The articulation allows rotation at the end of the beam and introduces two reaction unknowns, by projection on two directions of the mean plane as shown in figure 3.

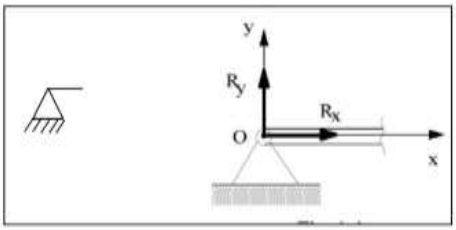


Fig 3. Fixed articulation support.

- **fixed support:**

- This support contacting the beam to a part considered as fixed, this type of support therefore introduces three reactions unknowns, the two projections of R on two axes of the mean plane and the intensity of the Moment M perpendicular to the mean plane. such as in figure 4 [3-4].

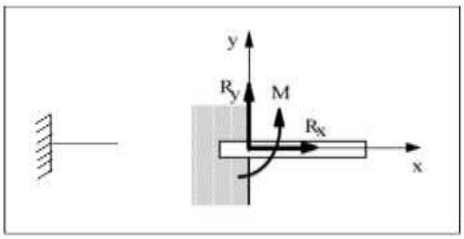


Fig 4. Fixed support

1.7. Types of charges (loads):

External loads acting on structural and mechanical components can be classified according to different criteria, including the duration of application, the manner in which the load is distributed, and the mode of action on the structural member.

- **Classification According to the Duration of Application**

Based on the time over which they act, loads are generally divided into the following categories:

1. **Static load:** A load applied gradually until the structure reaches equilibrium without inducing significant dynamic effects.
2. **Dead (permanent) load:** A constant load that remains unchanged during the service life of the structure, such as the self-weight of structural members and permanently attached equipment.
3. **Live load:** A variable load whose magnitude or location changes with time, such as pedestrians, vehicles, or movable machinery.
4. **Impact load:** A rapidly applied load that generates dynamic stresses because of its sudden application.
5. **Repeated (cyclic) load:** A load that is applied repeatedly with changing magnitude, direction, or both, and may eventually cause fatigue damage under long-term operation [5].

- **Classification According to Load Distribution**

Depending on how the load is applied to the structural member, it may be classified as:

1. **Concentrated (point) load:** A load acting over a very small area relative to the dimensions of the member and therefore idealized as acting at a single point.
2. **Distributed load:** A load spread continuously over a length or surface. Distributed loads may be:
    - **Uniformly distributed**, where the load intensity remains constant.
    - **Non-uniformly distributed**, where the load intensity varies along the member either linearly or nonlinearly.

- **1.7.3. Classification According to the Mode of Application**

According to the direction and nature of the applied force, loads can be categorized as follows:

1. **Axial load:** A force acting parallel to the longitudinal axis of the member. When its line of action passes through the centroid of the cross-section, it is referred to as a **centric load**; otherwise, it is an **eccentric load**.
2. **Direct shear load:** A load acting parallel to the resisting cross-section, producing shear stresses within the structural element.
3. **Torsional load:** A twisting load generated by applied moments that rotate a member, such as a shaft, about its longitudinal axis.
4. **Bending (flexural) load:** A transverse force or bending moment that causes curvature and deflection of the structural member.
5. **Combined load:** A loading condition in which two or more types of loads, such as axial, bending, shear, or torsion, act simultaneously on the same component.

These classifications provide a fundamental basis for evaluating the mechanical response of structural elements and selecting the appropriate analysis methods used in engineering design and strength calculations [5].

The Loads applied to the beam may consist of a concentrated load (load applied at a point as in fig 5.a), uniform load (fig 5.b), uniformly varying load (fig 5.b), or an applied couple or moment (fig 5.d). These loads shown in the following figures 5.

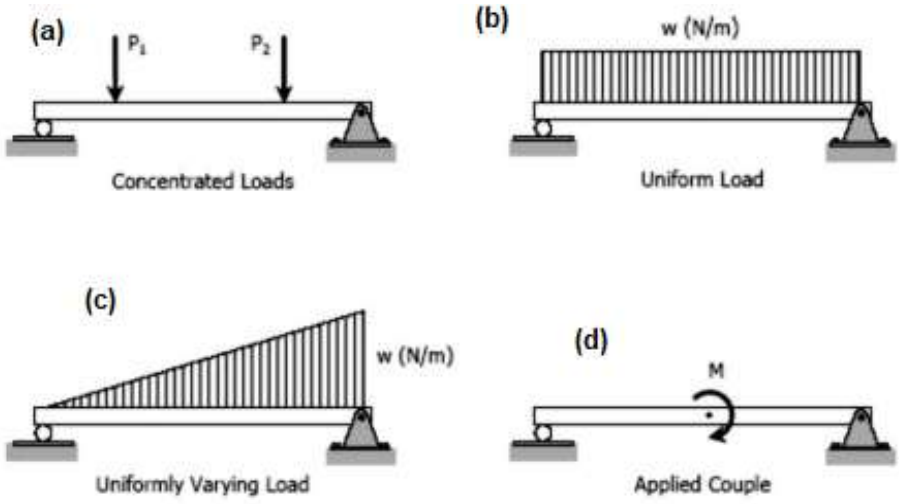


Fig 5. Kinds of loads of beam

1.1.8. Conditions for Equilibrium system:

The first condition for equilibrium is that the forces along each coordinate axis add to zero. From Newton's law, we know that the following relations apply as:

$$\sum \vec{F} = \vec{0} \Rightarrow \begin{cases} \sum \mathrm{Fx} = 0 \\ \sum Fy = 0 \\ \sum Fz = 0 \end{cases}$$

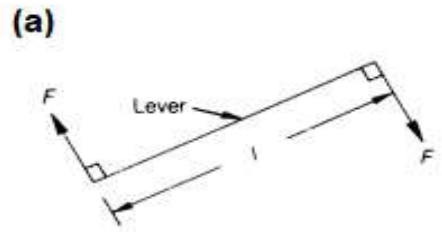
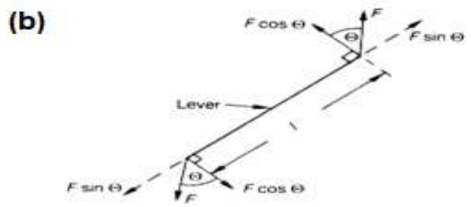
The second condition of equilibrium is that the resultant of all applied moments, including bending and twisting moments must be zero.

$$\sum \vec{M}_p(\vec{F}) = \vec{0}$$

The two equilibrium conditions used to determine support reactions and internal forces on cross-sections of structural members [2].

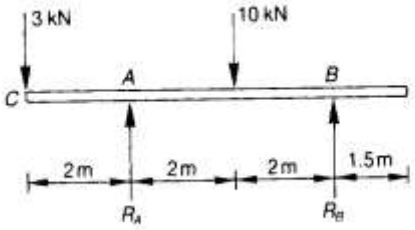
- **Couples**

A couple can be described as the moment produced by two equal and opposite forces acting together, as shown in Figure 6 where,

F = force (N) L = lever length (m) the moment at the couple (clockwise-couple) **M = F x L** (N.m)	 Fig (6.a): couple with vertical force.
For the counter-clockwise couple of M = F COS θ x 1 The components of force F sin θ will simply place the lever in tension, and will not cause a moment.	 Fig (6.b): couple with not vertical force.

Example:

Determine the values of the reactions of R_A, and R_B, for the simply supported beam shown.



Solution

Taking moments about B:

$$\sum \vec{M}_B\left(\vec{F}\right) = \vec{0}$$

Clockwise couples = counter-clockwise couples

R_A x 4 = 3 x 6+10 x 2 =18 + 20

⇒ R_A = 9.5 kN

Resolving forces vertically:

$$\sum Fy = 0$$

R_A+R_B = 3+10

or

R_B = 13 - 9.5 = 3.5 kN

1.9. Engineering Materials:

Engineering materials can be broadly categorized according to their composition and mechanical characteristics. The major groups include:

1. **Ferrous metals and alloys**, which contain iron as the principal constituent, including steel, cast steel, alloy steel, and cast iron.
2. **Non-ferrous metals and alloys**, such as aluminum, copper, nickel, zinc, and their respective alloys, including brass and bronze.
3. **Non-metallic materials**, including natural materials such as wood, leather, and ivory.
4. **Polymers**, which comprise synthetic materials such as polyethylene, epoxy resins, and phenolic compounds.
5. **Ceramic materials**, including glass, bricks, porcelain, and similar brittle materials with high hardness and thermal resistance.

6. **Composite materials**, which are formed by combining two or more constituent materials to achieve enhanced properties. Common examples include reinforced concrete, plywood, and fiberglass.

The selection of a suitable engineering material is a fundamental aspect of the design process. Design engineers must choose materials with appropriate mechanical properties and dimensions to ensure adequate strength, reliability, and cost-effectiveness under the intended service conditions. In addition to mechanical performance, practical considerations such as structural weight, component size, manufacturability, durability, and aesthetic appearance should also be taken into account during material selection [4].

1.10. Stress

Stress is the intensity of internal forces developed within a cross-section of a solid body, in reaction to externally applied forces (loads, moments). It is expressed as force per unit area (N/m^2 or Pa), and at any point on a section it can be split into two components:

- **Normal stress (σ)** — perpendicular to the cross-section, associated with the normal force (tension/compression).
- **Shear stress (τ)** — lying in the plane of the cross-section, associated with the transverse (shear) force.

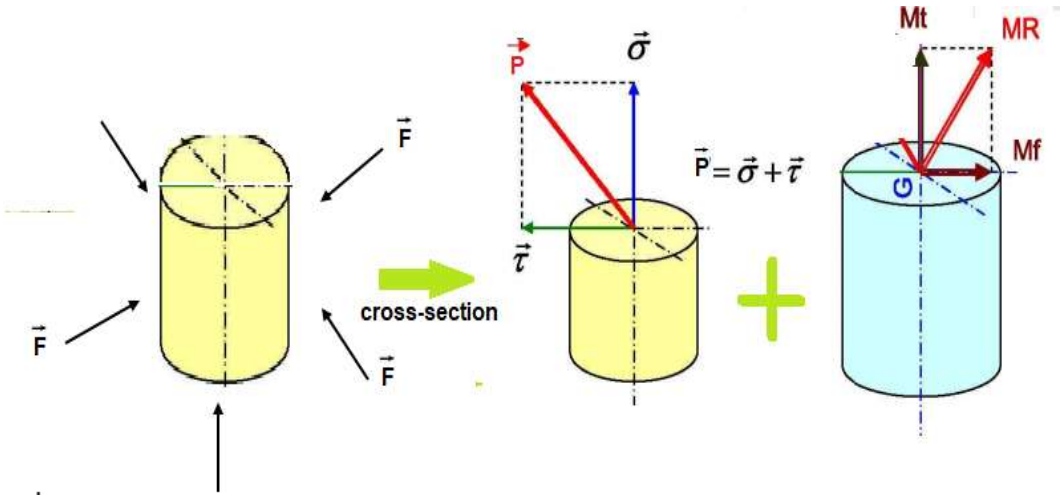


Fig 7: types of forces

The figure 7 present in left a cylindrical bar subjected to external forces F applied in different directions (general loading case). Middle: by making an imaginary cut (cross-section), the total stress vector P at a point on that section is revealed. This vector decomposes into: σ (vertical, normal to the section) τ (horizontal, tangential to the section) giving the relation: $\vec{P} = \vec{\sigma} + \vec{\tau}$

In Right: the same cut, but now showing the resultant internal forces (rather than point stresses) reduced to the centroid G of the section. The resultant force R and resultant moment M_R decompose into:

M_f (bending moment, horizontal — linked to σ).

Mt (torsional moment, vertical — linked to τ)

So overall, the figure traces the progression from external loading → to local stresses (σ, τ) at a point → to the resultant internal actions (N, T, Mf, Mt) over the whole section, which are the quantities actually used in practice to design and check structural/machine elements in Strength of Materials.

1.11. Strain:

• Strain is a measure of the material's response to stress and is expressed as a ratio to the change in length to the original length:

$$\boldsymbol{\varepsilon} = \frac{\boldsymbol{\Delta L}}{\boldsymbol{L_0}}$$

Where elongation results in a positive strain, compression results in a negative strain. [2]
Shear strain is based on the rotation of the object, measured in radians:

$$\gamma = \text{angle of deformation}$$

Modulus of Rigidity is similar to Young's Modulus but measures the ratio of shear stress to angle of deformation:

$$G = \boldsymbol{\tau} / \gamma$$

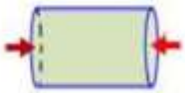
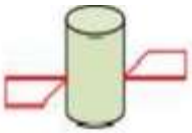
Young's Modulus defines the relationship between normal stress and lateral strain:

$$E = \sigma / \boldsymbol{\varepsilon}$$

1.12. Basics types of deformation:

We can brief all the simple (pure) deformations in this table:

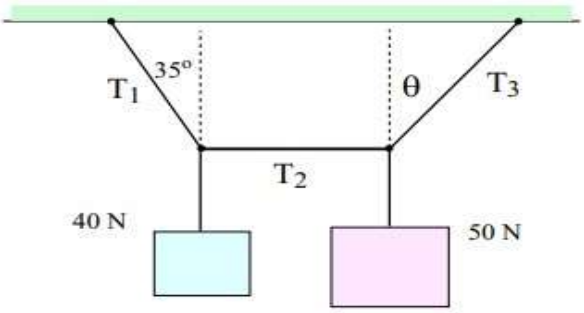
Table 1.1

Type of Force	Deformation	Description	N	T	Mt	Mf
Normal force, Axial force, Thrust		The member is being stretched by the axial force and is in *tension*. The deformation is characterized by axial *elongation*.	1	0	0	0
Normal force, Axial force, Thrust		The member is being compressed by the axial force and is in *compression*. The deformation is characterized by axial *shortening*.	1	0	0	0
Shear force		The member is being sheared. The deformation is characterized by *distorting* a rectangle into a parallelogram.	0	1	0	0
Torque, Twist moment		The member is being twisted and is in torsion. The deformation is characterized by *angle of twist*.	0	0	1	0
Bending moment		The member is being bent and the deformation is characterized by a *bent shape*.	0	0	0	1

1.13. Application Exercices (Equilibrium, reactions and tension)

Exercise 1 :

The system in Fig.1 is in equilibrium with the string in the center exactly horizontal. Find (a) tension T1, (b) tension T2, (c) tension T3 and (d) angle θ.



The system has weights of 40 N and 50 N. T_1 makes 35° with the vertical, T_2 is horizontal, and T_3 makes angle θ with the vertical.

At the left junction (T_1, T_2, and 40 N):

$\Sigma F_x = 0$: $T_2 = T_1 \sin(35°)$

$\Sigma F_y = 0$: $T_1 \cos(35°) = 40$ N

(a) Tension T_1:

$T_1 = 40 / \cos(35°) = 40 / 0.8192$

$T_1 = 48.83$ N

(b) Tension T_2:

$T_2 = T_1 \times \sin(35°) = 48.83 \times 0.5736$

$T_2 = 28.0$ N

(c) & (d) — At the right junction (T_2, T_3, and 50 N):

$\Sigma F_x = 0$: $T_3 \cos(\theta) = T_2 = 28.0$ N

$\Sigma F_y = 0$: $T_3 \sin(\theta) = 50$ N

$\tan(\theta) = 50 / 28.0 = 1.786$

$\theta = \arctan(1.786)$

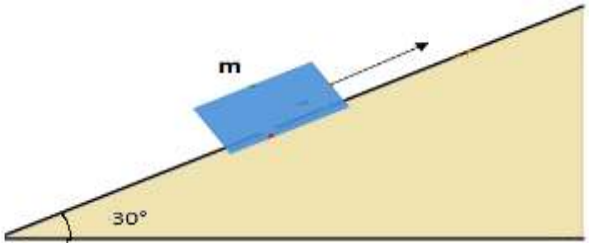
$T_3 = 50 / \sin(60.7°) = 50 / 0.873$

$\theta = 60.7°$

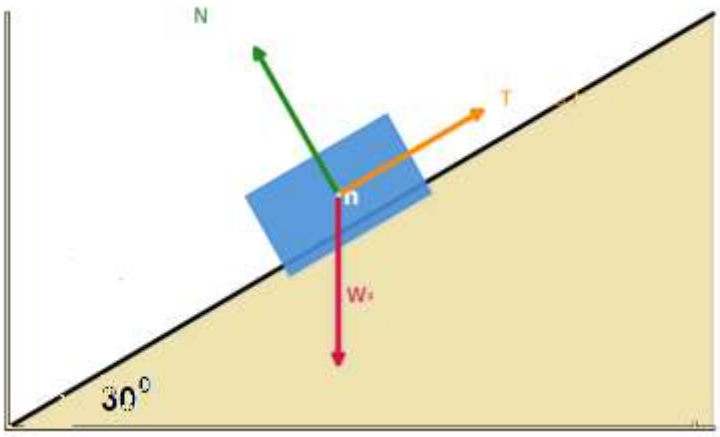
$T_3 = 57.3$ N

Exercise 2 :

A block of mass m = 25 kg rests on a smooth inclined surface at 30°. The block is held by a string parallel to the incline. Find the normal reaction N and the tension T.



Given:	m = 25 kg, θ = 30°, g = 9.81 m/s², frictionless surface, string parallel to incline
Find:	Normal force N and tension T



Solution

W = mg = 25 × 9.81 = 245.25 N

$W_x = W \sin 30° = 122.6$ N (along slope) $W_y = W \cos 30° = 212.4$ N (into surface)

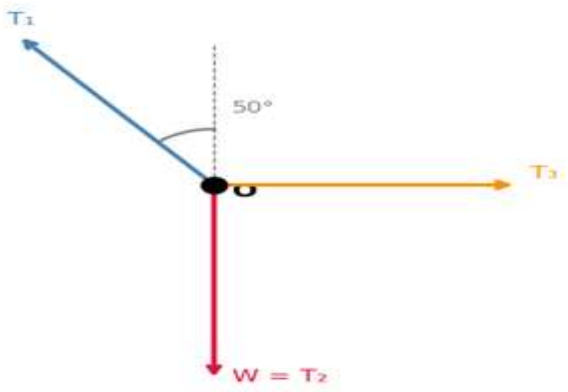
$\Sigma F_y = 0$ ($\perp$ incline): $N = W \cos 30° = 212.4$ N

$\Sigma F_x = 0$ ($\parallel$ incline): $T = W \sin 30° = 122.6$ N

$N \approx 212.4$ N | $T \approx 122.6$ N

Exercise 3 :

Three strings hold a knot O. String T3 is exactly horizontal. String T1 makes 50° with the vertical. Weight W = 3.7 kN hangs vertically from the knot. Find T1, T2, T3 and angle θ.



Solution

$\Sigma F_y = 0$: $T1 \cos 50° - T2 = 0 \rightarrow T1 = 3.7 / \cos 50° = 5.76$ kN

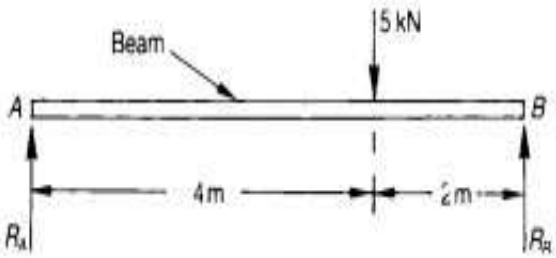
$\Sigma F_x = 0$: $T1 \sin 50° - T3 = 0 \rightarrow T3 = 5.76 \times \sin 50° = 4.41$ kN

Angle θ with ceiling: $\theta = 90° - 50° = 40°$

T1 = 5.76 kN | T2 = 3.70 kN | T3 = 4.41 kN | θ = 40°

Exercise 2:

Determine the values of the reactions R_A, and R_B, when a beam is simply supported at its ends and subjected to a downward force of 5 KN?



Beam Reaction

Calculations

A simply supported beam AB carries a downward point load of 5 kN. Determine the support reactions R_a and R_2.

Given Information

Parameter	Value
Total beam length	6 m
Applied load	5 kN (downward)
Distance A to load	4 m
Distance load to B	2 m
Support type	Simply supported (A & B)

Step 1: Sum of Moments about A = 0

Taking moments about support A (clockwise positive):

The reaction R_2 acts upward at B (6 m from A).

The 5 kN load acts downward at 4 m from A.

$$\Sigma M(A) = 0$$

$$R_2 \times 6 = 5 \times 4$$

$$6R_2 = 20$$

$$R_2 = 20 \div 6$$

$$\mathbf{R_2 = 3.33\ kN}$$

Step 2: Sum of Vertical Forces = 0

For vertical equilibrium, upward forces = downward forces:

$$\Sigma F_v = 0$$

$$R_a + R_2 = 5\ kN$$

$$R_a + 3.33 = 5$$

$$\mathbf{R_a = 1.67\ kN}$$

Verification: Moments about B = 0

Taking moments about B (distances from B: load is 2 m, R_a is 6 m):

$$\Sigma M(B) = 0$$

$$R_a \times 6 = 5 \times 2$$

$$1.67 \times 6 = 10$$

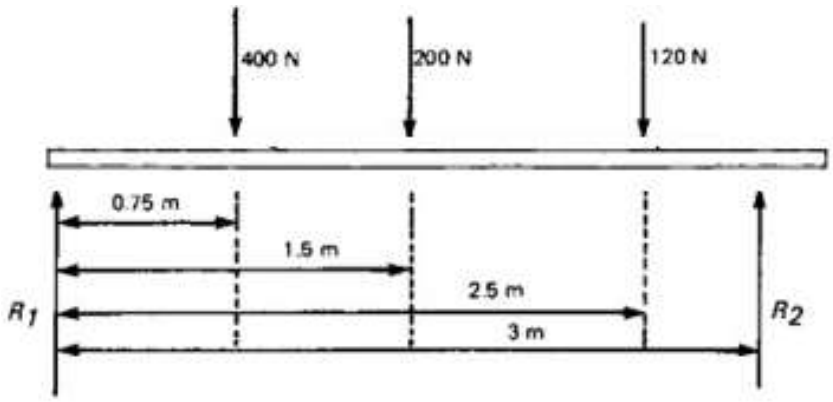
$$10 \approx 10\ \checkmark$$

Verification confirmed. The reactions are correct.

Exercise 4:

A beam 3 m long is simply supported at each end and carries loads as shown below.

Calculate the reactions.



Given Information

Beam length: 3 m

Support type: Simply supported at both ends

Applied Loads

Load	Magnitude	Position from R_1
Load 1	400 N	0.75 m
Load 2	200 N	1.5 m
Load 3	120 N	2.5 m

Step 1: Sum of Moments about $R_1 = 0$

Taking moments about R_1 (positive clockwise):

$$\Sigma M(R_1) = 0$$

$$R_2 \times 3 = (400 \times 0.75) + (200 \times 1.5) + (120 \times 2.5)$$

$$3R_2 = 300 + 300 + 300$$

$$3R_2 = 900$$

$$\mathbf{R_2 = 300\ N}$$

Step 2: Sum of Vertical Forces = 0

For equilibrium, the sum of upward forces equals sum of downward forces:

$$\Sigma F_v = 0$$

$$R_1 + R_2 = 400 + 200 + 120$$

$$R_1 + 300 = 720$$

$$\mathbf{R_1 = 420\ N}$$

Verification: Moments about $R_2 = 0$

Taking moments about R_2 to verify (distances from R_2: 400N at 2.25m, 200N at 1.5m, 120N at 0.5m):

$$R_1 \times 3 = (400 \times 2.25) + (200 \times 1.5) + (120 \times 0.5)$$

$$420 \times 3 = 900 + 300 + 60$$

$$1260 = 1260\ \checkmark$$

Verification confirmed. The reactions are correct.

ADDITIONAL EXERCISES

Exercise 1:

A simply supported beam AB of length L = 6 m carries a concentrated point load F = 24 kN at a distance a = 2 m from support A. Determine the reactions at A and B.

Exercise 2:

A simply supported beam AB is 8 m long. It carries a point load of 12 kN at 3 m from A.

- Find R_a and R_2.

Exercise 3:

A beam AC is simply supported at A (x = 0) and C (x = 8 m). It carries: a uniformly distributed load (UDL) of w = 10 kN/m over the entire span, and a point load P = 30 kN at B (x = 3 m from A). Find the support reactions.

CHAPTER 2: TENSION AND COMPRESSION

II.1 Tension:

II. 1.1 Definition:

When a load F acts perpendicular to the cross-sectional area of a bar (figure 8), the bar elongates under the effect of this external force.

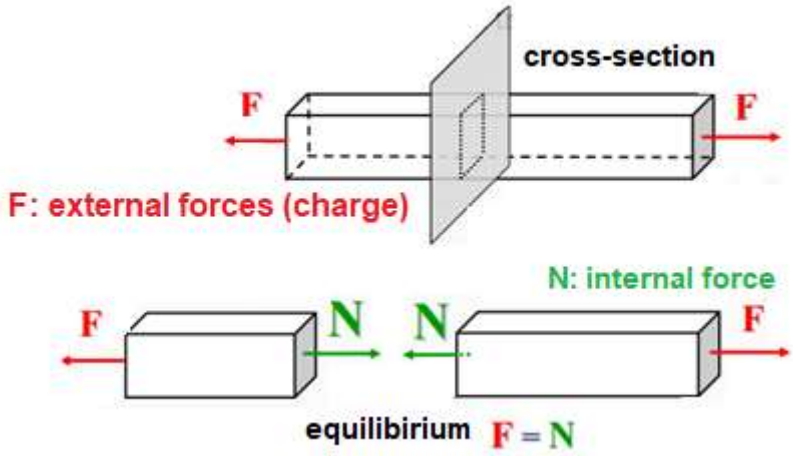


Fig. 8

The relationships of tension is:

$$Normal\ stress\ (\sigma) = \frac{F}{A}$$

$$tensile\ strain\ (\mathcal{E}) = \frac{\text{increase in length } (\Delta L)}{\text{original length L}}$$

Units:

The basic units of tensile stress in S.I units i.e. (International System) are N / m^2 (or Pa Pascal) MPa = 10^6 Pa, GPa = 10^9 Pa, KPa = 10^3 Pa

Sometimes (N/mm^2) units are also used because this is an equivalent to MPa [2-7].

II. 1.2 Stress-strain diagram

The shape of the stress-strain diagram depends on the material being tested; the example below corresponds to a medium-carbon structural steel. Metallic materials used in engineering are generally grouped into two families: ductile and brittle.

the initial portion of the stress-strain curve is a straight line. Robert Hooke observed this proportional relationship between elongation and applied axial force as early as 1678, and the resulting law — stating that stress is proportional to strain within the elastic (proportional) limit — now bears his name[7-8]:

$$\sigma = E\,\boldsymbol{\varepsilon}$$

On a stress-strain diagram, this relationship appears as the slope of the curve (rise over run), though it only applies within the elastic region. Young's Modulus varies from one material to another and standard values are available in reference tables

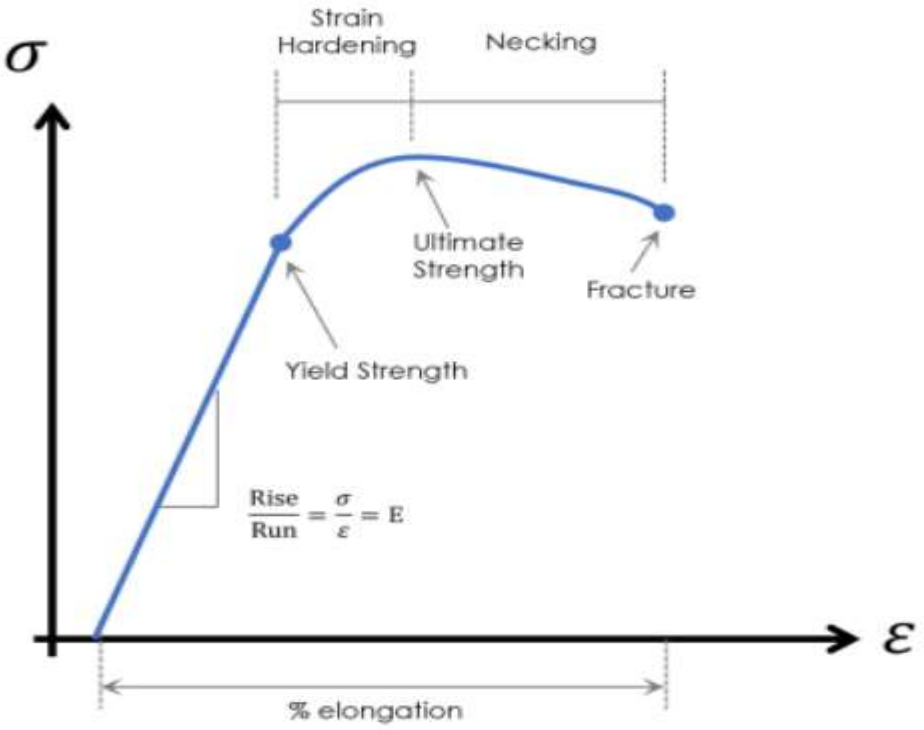


Fig. 9

Elastic Region)	Elastic Region: the range over which an applied stress produces a temporary strain; once the load is removed, the material recovers its original dimensions and the elongation returns to zero
Plastic Region	Region in which the applied stress causes permanent, irreversible strain. Once unloaded, the material no longer returns to zero elongation but instead retains the deformation corresponding to the yield point, without re-entering the elastic range.
Yield Strength	The boundary separating elastic from plastic behavior, beyond which deformation can no longer be fully recovered. Once the stress applied exceeds the value at this point, the material is driven permanently into the plastic range.
Brittle Material	A material that fails after only a very small amount of elongation.
Ductile Material	A material capable of considerable elongation, including visible necking, before it fails; it typically spends an extended period in the plastic range before reaching the fracture point.

Ultimate Strength	The highest stress level reached by the material during the test. In ductile materials, stress tends to drop once necking begins; in brittle materials, the fracture point essentially coincides with this maximum value.
Fracture Point	The stage at which the specimen actually separates. For low-ductility materials this coincides almost exactly with the ultimate stress, whereas for highly ductile materials it occurs noticeably further along the curve.

Poisson's Ratio expresses how lateral strain relates to axial strain, indicating the tendency of a specimen to narrow (or "neck") as it is stretched [2][8]:

$v = -lateral\ strain/\ axial\ strain = -\varepsilon_y/\ \varepsilon_x = -\varepsilon_z/\varepsilon_x\ ;\ \varepsilon_y = \varepsilon_z = -v\sigma_x/\ E$

II.1.3 Resistance condition:

For security reasons related to the use of the devices, it is required that:

$$\sigma_{max} \leq \sigma_p$$

Where: $\sigma_p = \frac{\sigma_e}{s}$

σ_p, σ_e, s : are applied stress, limit stress and factor of safety respectively.

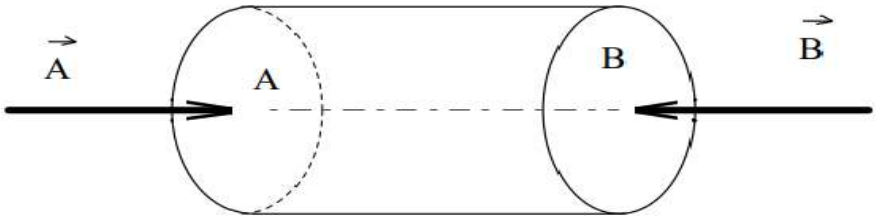
$s = \frac{\sigma_e}{\sigma_p}$ is ratio of this strength (ultimate or yield strength) to allowable strength is called the factor of safety, it is takes the values according for different materials and varies between $2 \leq s \leq 10$

- Elongation, Z % = $\frac{L_{f-L_0}}{L_0}\ x\ 100$, is chiefly influenced by uniform elongation, which is dependent on the strain-hardening capacity of the material.

II.2 COMPRESSION

II.2.1 Definition

Simple compression occurs in a beam when it is acted on by two equal and opposite forces acting through the centers of the end sections, tending to shorten the member.



The elements of reduction in G of the cohesion efforts are expressed by:

$$\{Cohesion\} = \begin{Bmatrix} N & 0 \\ 0 & 0 \\ 0 & 0 \end{Bmatrix}_{(\vec{x},\vec{y},\vec{z})} \quad \text{avec } N < 0$$

II.2.2 Compression test

Since the specimen does not return to its original length, the reduction in cross-sectional area is instead computed using the following relation [2][7]:

$$A\% = \frac{A_0 - A_f}{A_0}.100$$

II.2.3. Elastic deformations

This behavior allows the same relationship to be established across different materials:

$$\frac{F}{S} = -E\frac{\Delta l}{l} \qquad \text{with} \qquad \Delta l < 0$$

- Normal stress

We define the normal stress σ in the straight section (S) by the relation:

$$\sigma = \frac{N}{S} \qquad \text{with : } \sigma < 0 \text{ because } N < 0$$

- **HookE's law**

  We have already seen

$$\sigma = \frac{N}{S} \quad \text{and} \quad \frac{F}{S} = E\frac{\Delta l}{l}$$

  We can conclude that:

$$\sigma = E\frac{\Delta I}{I} = E.\varepsilon$$

II.2.4 Resistance conditions

For safety reasons, the normal stress σ must remain lower than a limit value called practical extension stress:

$$\sigma_{pe} = \frac{\sigma_e}{s}$$

s is a safety coefficient which varies from 1.1 to 10 depending on the of domains application. The resistance condition simply reflects the fact that the real stress must not exceed the previous threshold, i.e.:

$$|\sigma_{real}| = \frac{|N|}{s} < \sigma_{pe}$$

For security reasons related to the use of the devices, it is required that:

$$\sigma_{max} \leq \sigma_p$$

Where: $\sigma_p = \frac{\sigma_e}{s}$

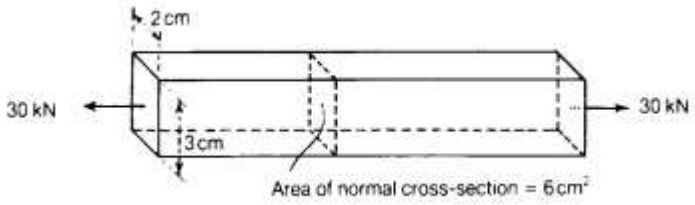
σ_p, σ_e , s : are applied stress, limit stress and factor of safety respectively.

$s = \frac{\sigma_e}{\sigma_p}$ is ratio of this strength (ultimate or yield strength) to allowable strength is called the factor of safety, it is takes the values according for different materials and varies between $2 \leq s \leq 10$

II.2.5 Application Exercices (Tension System)

Exercise 1:

A steel bar of rectangular cross section, 3 cm by 2 cm, carries an axial load of 30 kN. Estimate the average tensile stress over a normal cross section of the bar.



Answers:

Given Data

Width	b = 3 cm
Height	h = 2 cm
Axial Load	F = 30 kN = 30,000 N

Step 1 — Cross-Sectional Area

The bar has a rectangular cross section. The area is calculated as:

$$\mathbf{A = b \times h = 3\ cm \times 2\ cm = 6\ cm^2}$$

Converting to SI units:

$$\mathbf{A = 6 \times 10^{-4}\ m^2}$$

Step 2 — Stress Formula

The average normal (tensile) stress over a cross section perpendicular to the bar axis is given by:

$$\mathbf{\sigma = F / A}$$

Where:

- F = axial tensile force (N)
- A = cross-sectional area (m^2)
- σ = average normal stress (Pa or MPa)

Step 3 — Calculation

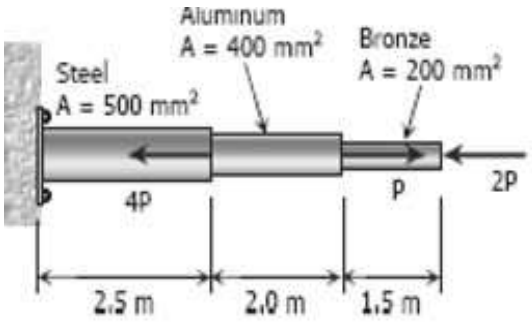
$$\mathbf{\sigma = 30{,}000\ N / (6 \times 10^{-4}\ m^2)}$$

$$\mathbf{\sigma = 50{,}000{,}000\ Pa}$$

Result: σ = 50 MPa (Tensile)

Exercise 2:

An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in Fig. P108. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa[3].



Answers:

1. Given Data

Steel segment	A_st = 500 mm², L = 2.5 m, σ_allow = 140 MPa
Aluminum segment	A_al = 400 mm², L = 2.0 m, σ_allow = 90 MPa
Bronze segment	A_br = 200 mm², L = 1.5 m, σ_allow = 100 MPa
Applied loads	4P (←) at steel/aluminum junction; P (←) at aluminum/bronze junction; 2P (→) at free end

---

2. Internal Forces by Section Method

Cut each segment and apply equilibrium (ΣFx = 0) from the FREE END leftward.

Bronze segment (cut between P and 2P, measure from free end):

ΣFx = 0 → F_br + 2P = 0 → F_br = −2P (compression)

Wait — check direction: 2P acts to the RIGHT (→), so it tends to pull the right face.

Taking rightward as positive: $F_{br} - 2P = 0 \rightarrow F_{br} = +2P$ (TENSION)

Aluminum segment (cut between 4P and P, measure from free end):

$\Sigma Fx = 0 \rightarrow F_{al} - 2P + P = 0 \rightarrow F_{al} - P = 0$

Loads to right of cut: 2P(→) and P(←) → net = $+2P - P = +P$

$F_{al} = +P$ (TENSION)

Steel segment (cut to the left of 4P, measure from free end):

Loads to right of cut: 2P(→), P(←), 4P(←)

Net rightward = $+2P - P - 4P = -3P$

$F_{st} = -3P$ (COMPRESSION)

Summary table:

Segment	**Area (mm²)**	**Internal Force**	**Nature**
Steel	500	$F_{st} = 3P$ (C)	Compression
Aluminum	400	$F_{al} = P$ (T)	Tension
Bronze	200	$F_{br} = 2P$ (T)	Tension

---

3. Stress Inequalities ($\sigma = F / A \leq \sigma_{allow}$)

For each segment, the average normal stress must not exceed the allowable value:

Segment	**Internal Force**	**Area (mm²)**	**Stress (MPa)**	**Allowable (MPa)**
Steel	$\lvert F_{st}\rvert = 3P$	500	$\sigma_{st} = 3P/500$	≤ 140 MPa
Aluminum	$\lvert F_{al}\rvert = P$	400	$\sigma_{al} = P/400$	≤ 90 MPa

Bronze	$\|F_{br}\| = 2P$	200	$\sigma_{br} = 2P/200 = P/100$	≤ 100 MPa

---

4. Solving for Maximum P

From the Steel condition:

$3P / 500 \leq 140 \rightarrow P \leq (140 \times 500) / 3 \rightarrow P \leq 70{,}000 / 3 \rightarrow P \leq 23{,}333$ N

From the Aluminum condition:

$P / 400 \leq 90 \rightarrow P \leq 90 \times 400 \rightarrow P \leq 36{,}000$ N

From the Bronze condition:

$2P / 200 \leq 100 \rightarrow P \leq (100 \times 200) / 2 \rightarrow P \leq 10{,}000$ N

The governing (most restrictive) condition is the one giving the SMALLEST P:

Steel →	$P \leq 23{,}333$ N $\approx$ 23.33 kN
Aluminum →	$P \leq 36{,}000$ N = 36.00 kN
Bronze →	$P \leq 10{,}000$ N = 10.00 kN ← GOVERNS

$P_{max} = 10{,}000$ N = 10 kN

(Governed by the Bronze segment)

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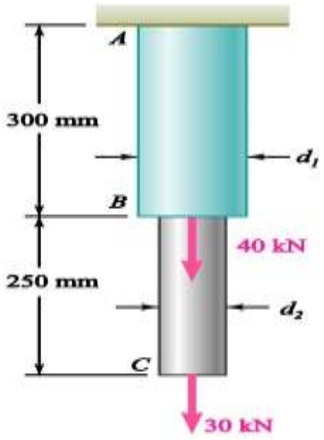
5. Verification at P = 10 kN

Segment	Internal Force	Area (mm^2)	Stress (MPa)	Allowable (MPa)
Steel	3 × 10,000 = 30,000 N	500	30,000/500 = 60 MPa	140 MPa ✓
Aluminum	1 × 10,000 = 10,000 N	400	10,000/400 = 25 MPa	90 MPa ✓
Bronze	2 × 10,000 = 20,000 N	200	20,000/200 = 100 MPa	100 MPa ✓ (exact)

All three segments satisfy their respective stress limits. The bronze segment is at exactly its allowable stress, confirming it governs the design.

Exercise 3:

Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that d_1=50 mm and d_2=30 mm, find the average normal stress at the midsection of (a) rod AB, (b) rod BC [3][9].



Answers:

1. Given Data

Rod AB	Diameter d_1 = 50 mm, Length = 300 mm
Rod BC	Diameter d_2 = 30 mm, Length = 250 mm
Load at B (down)	40 kN
Load at C (down)	30 kN
Support	Fixed wall at A (top)

---

2. Cross-Sectional Areas

Both rods are solid cylinders, so: $A = \pi d^2 / 4$

Area of rod AB	$A_{AB} = \pi \times (50)^2 / 4 = \pi \times 2500 / 4 = 1963.50$ mm²
Area of rod BC	$A_{BC} = \pi \times (30)^2 / 4 = \pi \times 900 / 4 = 706.86$ mm²

---

3. Internal Forces (Section Method)

A free-body diagram cut is made at the midsection of each rod.

Convention: forces acting downward on the exposed face are taken as compession.

(Fixed wall at A)

(a) Midsection of rod BC — cut between B and C:

Consider the portion BELOW the cut (free body with only load at C):

$\Sigma Fy = 0 \rightarrow F_{BC} = 30$ kN (Compression — rod is being pushed/pulled down by C)

(b) Midsection of rod AB — cut between A and B:

Consider the portion BELOW the cut (free body with loads at B and C):

$\Sigma Fy = 0 \rightarrow F_{AB} = 40$ kN + 30 kN = 70 kN (Compression)

Internal force — rod AB	F_AB = 70 kN = 70,000 N (Compression)
Internal force — rod BC	F_BC = 30 kN = 30,000 N (Compression)

---

4. Average Normal Stress ($\sigma = F / A$)

(a) Rod AB:

$\sigma_{AB} = F_{AB} / A_{AB} = 70{,}000 / 1963.50$

$\sigma_{AB} = 35.65$ MPa (Compressive)

(b) Rod BC:

$\sigma_{BC} = F_{BC} / A_{BC} = 30{,}000 / 706.86$

$\sigma_{BC} = 42.44$ MPa (Compressive)

(a) σ_{AB} = 35.7 MPa (Compressive)

(b) σ_{BC} = 42.4 MPa (Compressive)

---

5. Summary

Rod	d (mm)	A (mm^2)	F (kN)	σ (MPa)
AB	50	1963.5	70 (C)	35.7 (C)
BC	30	706.9	30 (C)	42.4 (C)

(C) = Compressive

ADDITIONAL EXERCISES

Exercise 1:

A steel rod of circular cross-section (diameter d = 20 mm) and length L = 2 m is subjected to an axial tensile force F = 60 kN. Using E = 200 GPa, determine:

(a) The normal stress σ in the rod.

(b) The elongation ΔL of the rod.

(c) The lateral strain if Poisson's ratio ν = 0.3.

(d) The change in diameter of the rod.

Exercise 2:

A composite bar is made of three materials. The bar carries a load of P = 100 kN.

Material properties:

- Aluminum: A = 900 mm², E = 70 GPa
- Steel: A = 600 mm², E = 200 GPa
- Bronze: A = 400 mm², E = 100 GPa

All three materials have the same length L = 500 mm.

Determine:

(a) The stress in each material,

(b) The total deformation.

Exercise 3:

A steel bolt of diameter 16 mm passes through an aluminum tube of inner diameter 18 mm and outer diameter 30 mm. Both have length L = 300 mm. A nut is tightened until the compressive stress in the aluminum is 30 MPa. Find:

(a) The tensile stress in the bolt.(b) The total deformation. Use E = 200 GPa, E = 70 GPa.

Exercise 4(Factor of Safety): A structural steel bar (σ = 250 MPa) carries an axial load of 120 kN with a required safety factor s e = 2.5. (a) What is the allowable stress σ ? pe

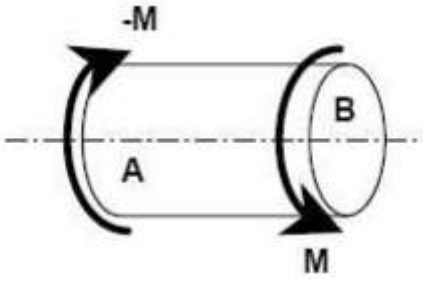
(b) What minimum circular cross-section diameter is required?

(c) If a rectangular section with b = 2h is used instead, find b and h.

CHAPTER 3: SIMPLE TORSION

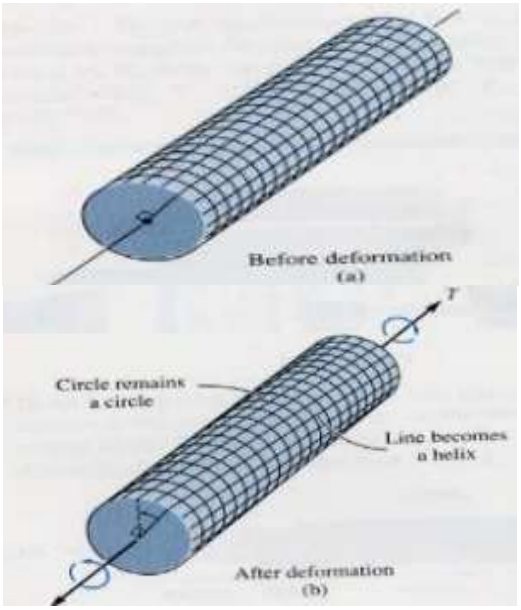
III.1 Definition:

A straight beam is stressed in torsion each time the actions at the ends (A and B) a re reduced to two equal and opposite couples M and –M.



- Torsion refers to the twisting of a structural member when, it is loaded by couples that produce rotation about the longitudinal axis.

- The couples that cause the tension are called Torques, Twisting Couples or Twisting Moments. [7][9]



III.2 Deformations – Torsion angle

III.2.1. Experimental findings

Straight sections before deformation remain straight after deformation (plane and perpendicular to the mean line).

The fibers or generators initially parallel to the mean line wind in helices around this axis. The length of the fibers remains substantially invariable or constant (small deformation hypothesis).

The following conditions used in the torsion of the circular shaft: [7]

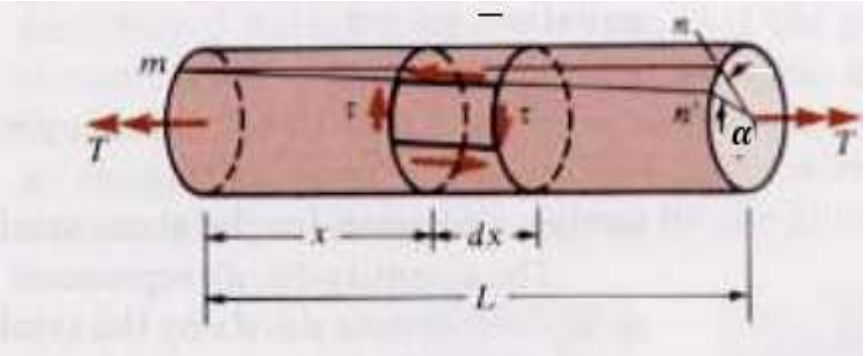
1. Sectional planes perpendicular to the axis of the shaft remain plane during torque application.

2. The shear strain varies linearly from a value of zero at the axis of the shaft to a maximum at the extreme radius.

3. For linearly elastic materials, Hook's Law is applicable and shear stress is linearly proportional to shear strain.

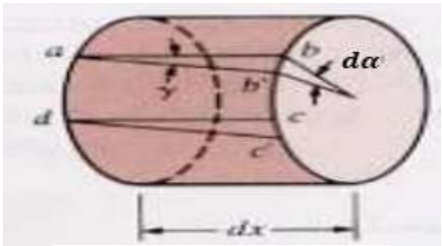
III. 2.2 Torsion of Circular Bars:

- If the right hand of the bar rotates through a small angle α
  α: Angle of twist.



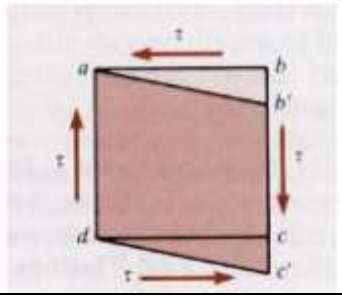
- Line will rotate to a new position

- The element **abcd** after applying torque



- The element **abcd** after applying torque

$\begin{cases} d \rightarrow d' \\ b \rightarrow b' \end{cases} \rightarrow$ Element in state of "pure shear"

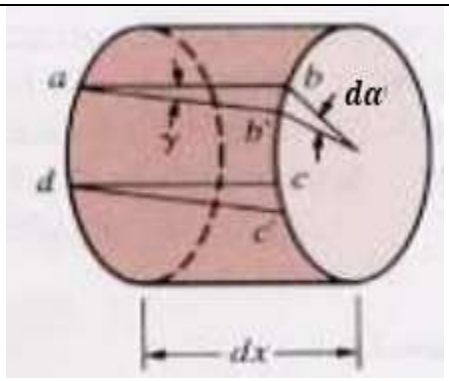


- During torsion, the right-hand cross section of the original

configuration of the element (**abcd**) rotates with respect to the opposite face and points b and c move to b' and c'.

- The lengths of the sides of the element do not change during this rotation, but the angles at the corners are no longer 90°. Thus, the element is undergoing pure shear and the magnitude of the shear strain is equal to the decrease in the angle bab'. This angle is :

$$\tan(\gamma) = \frac{bb'}{ab}$$



III.2.3 Shear Strain γ:

tan(γ) ≈ γ because under pure torsion the angle γ is small. So

$$\gamma = \frac{bb'}{ab} = \frac{r d\alpha}{dx}$$

Under pure torsion, the rate of change $\frac{d\alpha}{dx}$ of the angle of twist is constant along the length of the bar. This constant is equal to the angle of twist per unit length θ

rate of change of the angle of twist φ

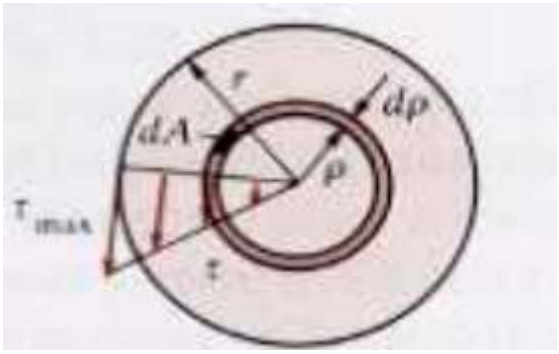
$\frac{d\alpha}{dx} \rightarrow$ Rate of change of the angle of twist φ = Θ

$\frac{d\alpha}{dx} = \theta \rightarrow$ Angle of twist per unit length.

$$\gamma = \frac{\rho\, d\alpha}{dx} = \rho\theta$$

•Since every cross section is subjected to the same torque so $\frac{d\alpha}{dx}$ is constant, the shear strain varies along the radial line γ = 0 at ρ = 0 (center of shaft).

γ = maximum at ρ = r (outer surface of shaft). $\frac{d\alpha}{dx} = \frac{\gamma}{\rho}$



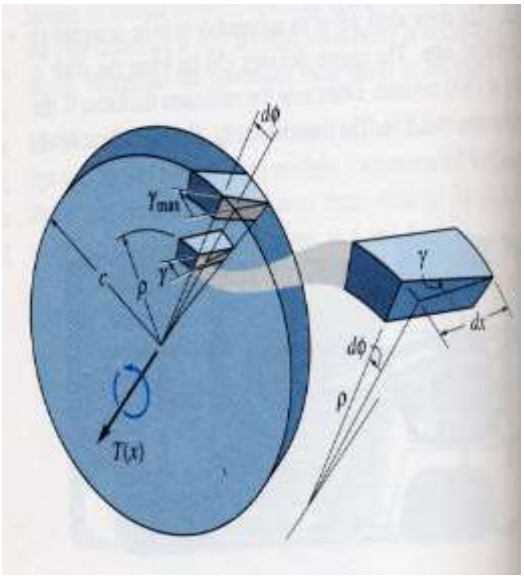
Since:

$\frac{d\alpha}{dx} = const$ → The shear strain within the shaft varies linearly along any radial line, from zero at the axis of the shaft to a maximum to at its outer surface

$$\frac{d\alpha}{dx} = \frac{\gamma_{max}}{r}$$

Hence

$$\frac{d\alpha}{dx} = \frac{\gamma}{\rho} = \frac{\gamma_{max}}{r} \Rightarrow \gamma = \rho\frac{\gamma_{max}}{r}$$



In the case of pure torsion, the rate of change $\frac{d\alpha}{dx}$ is constant along the length of the bar, because every cross section is subjected to the same torque. Therefore, we obtain $\theta = \frac{\alpha}{L}$, where L is the length of the shaft. Equation $\boldsymbol{\gamma = \rho\,\theta}$ becomes $\boldsymbol{\gamma = \rho\,\theta = \rho\,\frac{\alpha}{L}}$

If the external torque applied to shaft in equilibrium requires internal torque inside the shaft

Hook's Law of Torsion: $\boldsymbol{\tau = G\,\gamma = G\,\rho\,\theta}$

γ : Shear strain

G: shear modulus (modulus of rigidity)

τ : shear stress

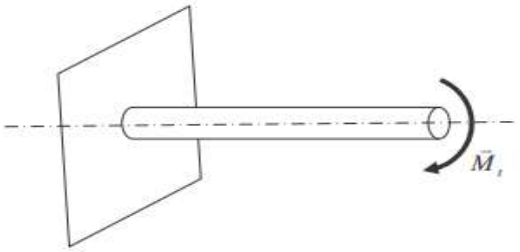
- **Unit angle of twist θ**

If we assume that, the straight sections all rotate together in the same way, then the angle of twist between any two straight sections is proportional to the distance between them. In other words:

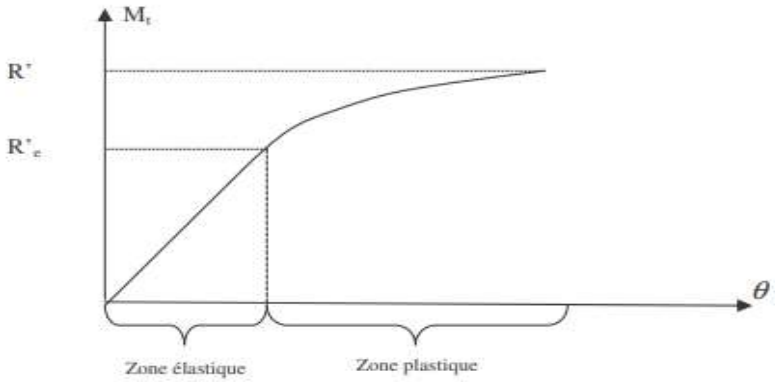
$$\theta = \frac{\alpha}{L}$$

III.3 Torsion test

The test is carried out on a cylindrical test piece of which one end is fixed, and on the other end a torque is exerted, the twisting moment Mt of which increases slowly.



We draw the diagram, which allows us to see how the nail θ of which rotates, one relative to the other, the extreme sections of the test piece varies, as a function of the torque, the diagram obtained is similar to that of the shear test [7]



Fi

The beam having not undergone any variation in length N = 0 → σ = 0.

On the diagram, we also find:

Re': Elastic limit.

R': Limit to rupture.

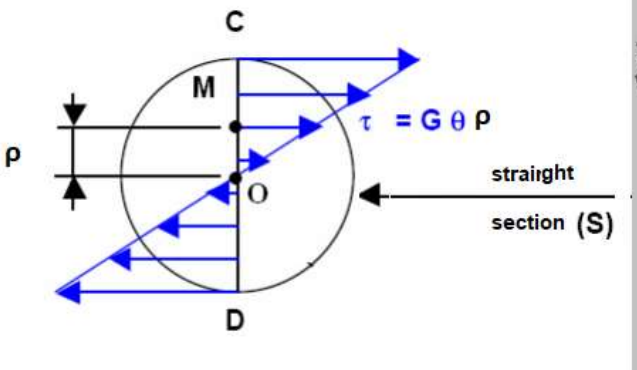
III.4. Tangential stress and twist angle

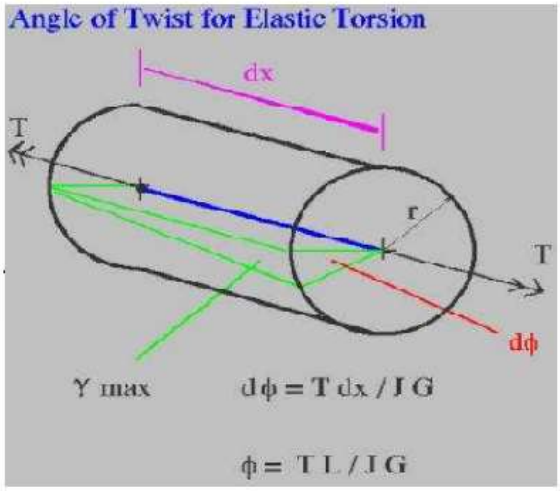
In the following, we assume the deformations are elastic and therefore very small. Sliding, which is only a torsional shear, characterized by the angle of deviation, and corresponds to a tangential stress.

$$\tau = G\,\gamma$$

$$\gamma = \rho\,\theta = \rho\,\frac{\alpha}{L}$$

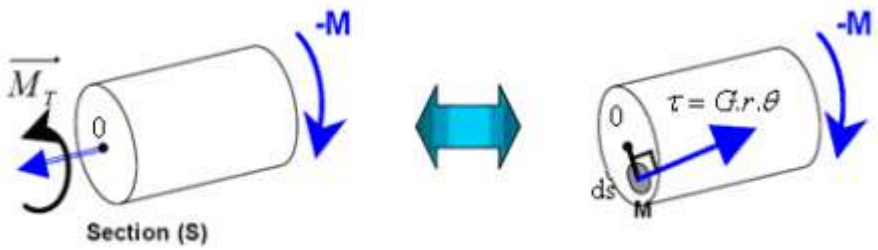
$$\tau = G\,\rho\,\theta$$





III.5 Tangential moment Mt and twist angle θ

In the hypothesis of straight sections, the tangential constraint and perpendicular at radius r. an equilibrium beam described in the torsion test is ensured if



$$\sum \vec{M}_{ex/0} = \vec{C} + \underbrace{\iint_S r.\vec{\tau}.ds}_{Mt} = \vec{0}$$

so:

$$Mt = \iint rds(Gr\theta) = G.\theta. \underbrace{\iint \rho^2 ds}_{moment\text{-}d'inertie/I_0}$$

Hence the angle of twist is written:

$$\theta = \frac{1}{G}.\frac{Mt}{I_0}$$

The tangential or sliding stress at a point of a straight section located at

a distance r from the neutral fiber, has the expression:

$$\tau = \rho \frac{Mt}{I_0}$$

III.6. Torsional resistance

As for traction and compression, we define a resistance to torsion practical *Rp* significantly lower than the elastic sliding limit *Re*. For a part subjected to torsion to resist in complete safety, it is necessary that:

$$\tau \leq R_p$$

II.2.5 Application Exercices (Tension System)

Exercise 1:

A solid circular shaft must transmit 75 kW at 200 rpm. The allowable shear stress is 60 MPa and the allowable angle of twist is 1° per metre of shaft length. Using G = 80 GPa:

(a) Determine the required shaft diameter based on stress.

(b) Determine the required shaft diameter based on twist.

(c) Select the appropriate shaft diameter.

Answers:

1. **Given Data**

Power transmitted	P = 75 kW
Rotational speed	n = 200 rpm
Allowable shear stress	τ_allow = 60 MPa
Allowable angle of twist	φ_allow = 1° per metre (= π/180 rad/m)
Shear modulus	G = 80 GPa
Cross-section	Solid circular shaft

2. Torque Transmitted by the Shaft

The angular velocity and torque are found from the power equation: $P = T \cdot \omega$

$$\omega = 2\pi\, n / 60 = 2\pi \times 200 / 60 = 20.944 \text{ rad/s}$$

$$T = P / \omega = 75{,}000 / 20.944 = 3580.99 \text{ N·m}$$

T = 3581.0 N·m

3(a). Required Diameter — Shear Stress Criterion

For a solid circular shaft, the torsion formula gives the maximum shear stress:

$$\tau_{max} = T \cdot c / J = T \cdot (d/2) / (\pi\, d^4/32) = 16T / (\pi\, d^3)$$

Setting $\tau_{max} = \tau_{allow}$ and solving for d:

$$\tau_{allow} \geq 16T / (\pi\, d^3)$$

$$d^3 \geq 16T / (\pi \cdot \tau_{allow})$$

$$d^3 \geq 16 \times 3580.99 / (\pi \times 60 \times 10^6)$$

$$d^3 \geq 303963.5509 \times 10^{-6} \text{ m}^3$$

$$d \geq 67.24 \text{ mm}$$

Result (stress):	$d_{stress} \geq 67.24$ mm

3(b). Required Diameter — Angle of Twist Criterion

The angle of twist per unit length is related to the torque by:

$$\varphi/L = T / (G \cdot J) = T / (G \cdot \pi\, d^4/32) = 32T / (G\, \pi\, d^4)$$

Setting $\varphi/L = \varphi_allow$ and solving for d:

$$\varphi_{allow} \geq 32T / (G\, \pi\, d^4)$$

$$d^4 \geq 32T / (G \cdot \pi \cdot \varphi_allow)$$

$$d^4 \geq 32 \times 3580.99 / (80\times10^9 \times \pi \times \pi/180)$$

$$d^4 \geq 26123742.8909 \times 10^{-9}\ m^4$$

$$d \geq 71.49 \text{ mm}$$

Result (twist):	$d_{twist} \geq 71.49$ mm

3(c). Selection of the Design Diameter

Both criteria must be satisfied simultaneously. The governing (larger) value controls:

Criterion	Min. diameter	Controls?
Shear stress	67.24 mm	No
Angle of twist	71.49 mm	YES — governs

The twist criterion governs (71.49 mm > 67.24 mm).

Rounding up to the nearest whole millimetre for practical manufacture:

$$\mathbf{d_{stress} \geq 67.2\ mm}$$

$$\mathbf{d_{twist} \geq 71.5\ mm \quad \leftarrow governs}$$

Selected shaft diameter: d = 72 mm

---

4. Verification at d = 72 mm

Confirm that d = 72 mm satisfies both criteria:

Polar moment J	$J = \pi\, d^4/32 = \pi \times (72)^4/32 = 2638.33 \times 10^{-6}\ m^4$
Shear stress check	$\tau = T \cdot c/J = 3581.0 \times 36.0 \times 10^{-3} / (2638.33 \times 10^{-6}) = 48.9$ MPa ≤ 60 MPa ✓
Twist check	$\varphi/L = T/(GJ) = 3581.0/(80 \times 10^9 \times 2638.33 \times 10^{-6}) = 0.972°/m \leq 1°/m$ ✓

Both allowable limits are satisfied. The design is valid.

Exercise 12: (Hollow Shaft — Design):

Design a hollow shaft (inner/outer diameter ratio k = d/D = 0.6) to transmit a torque of 5 kN·m with a maximum shear stress of 70 MPa and a maximum twist of 2° over a length of 2 m. Use G = 80 GPa.

(a) Determine D and d from the stress condition.

(b) Determine D and d from the twist condition.

(c) Select the governing design. Compare mass with an equivalent solid shaft. [9]

Answers:

Hollow Shaft Design — Torsion

Stress & Twist Conditions | Mass Comparison

Given Data

Parameter	Symbol	Value	Unit
Applied Torque	T	5 000	N·m
Max. Shear Stress	τ_{max}	70	MPa
Max. Twist Angle	φ_{max}	2	degrees
Shaft Length	L	2	m
Shear Modulus	G	80 000	MPa
Diameter Ratio	k = d/D	0.6	—

Key Formulae for a Hollow Circular Shaft

For a hollow shaft with outer diameter D and inner diameter d = k·D :

Polar second moment of area:

$$J = (\pi/32) \cdot D^4 \cdot (1 - k^4)$$

Shear stress condition (Navier's formula):

$$\tau = T \cdot (D/2) / J \quad \rightarrow \quad \tau = 16 \cdot T / [\pi \cdot D^3 \cdot (1 - k^4)]$$

Angle of twist condition:

$$\varphi = T \cdot L / (G \cdot J) \quad \rightarrow \quad \varphi = 32 \cdot T \cdot L / [\pi \cdot G \cdot D^4 \cdot (1 - k^4)]$$

(a) Design from the Shear Stress Condition

Set $\tau = \tau_{max} = 70$ MPa and solve for D :

$$D^3 = 16 \cdot T / [\pi \cdot \tau_{max} \cdot (1 - k^4)]$$

Substituting $k^4 = (0.6)^4 = 0.1296 \rightarrow (1 - k^4) = 0.8704$:

$$D^3 = (16 \times 5\,000) / [\pi \times 70 \times 10^6 \times 0.8704]$$

$$D^3 = 80\,000 / 191410957.20 = 417948.905178 \times 10^{-6}\ m^3$$

$$D_a = 74.77\ mm$$

> **$D_a = 74.77$ mm | $d_a = k \cdot D_a = 0.6 \times 74.77 = 44.86$ mm**

(b) Design from the Twist Angle Condition

Convert: $\varphi_max = 2° = 2 \times \pi/180 = 0.034907$ rad

Solve for D from the twist equation:

$$D^4 = 32 \cdot T \cdot L / [\pi \cdot G \cdot \varphi_{max} \cdot (1 - k^4)]$$

$$D^4 = (32 \times 5\,000 \times 2) / [\pi \times 80 \times 10^9 \times 0.034907 \times 0.8704]$$

$$D^4 = 320\,000 / 7636003263 = 41906739.557952 \times 10^{-12}\ m^4$$

$$D_b = 80.46\ mm$$

> **$D_b = 80.46$ mm | $d_b = k \cdot D_b = 0.6 \times 80.46 = 48.27$ mm**

(c) Governing Design & Mass Comparison

Selection of Governing Design

Both conditions must be satisfied simultaneously. We adopt the larger outer diameter:

$D_a = 74.77$ mm (stress) $\quad D_b = 80.46$ mm (twist)

$$D_{governing} = \max(D_a, D_b) = 80.46\ mm \quad \text{[Twist condition]}$$

Governing: D = 80.46 mm | d = 48.27 mm (Twist condition)

Verification of the Governing Shaft

$J = (\pi/32) \cdot (80.46^4 - 48.27^4) \times 10^{-12} = 3580.9862 \times 10^{-6}\ m^4$

Actual shear stress: $\tau = T \cdot (D/2)/J = 56.17\ MPa \leq 70\ MPa$ ✓

Actual twist angle : $\varphi = T \cdot L/(G \cdot J) = 2.0000° \leq 2°$ ✓

Mass Comparison with Equivalent Solid Shaft

For an equivalent solid shaft satisfying the same combined requirements:

$D_{solid} = \max(D_{stress}, D_{twist}) = \max(71.39, 77.71) = 77.71\ mm$

Since mass is proportional to cross-sectional area (same material, same length):

$A_{hollow} = (\pi/4) \cdot (D^2 - d^2) = 3253.96 \times 10^{-6}\ m^2$

$A_{solid} = (\pi/4) \cdot D_s^2 = 4743.42 \times 10^{-6}\ m^2$

Mass ratio = $A_{hollow} / A_{solid} = 0.6860$

Mass saving = $(1 - 0.6860) \times 100 = 31.40\ \%$

Summary of Results

Parameter	Stress Cond.	Twist Cond.	Governing
Outer diameter D (mm)	74.77	80.46	**80.46**
Inner diameter d (mm)	44.86	48.27	**48.27**
Actual τ (MPa)	—	—	**56.17**
Actual φ (°)	—	—	**2.0000**
Solid shaft D_s (mm)	—	—	**77.71**
Mass saving vs solid (%)	—	—	**31.40 %**

EXERCISE 2: — Solid Circular Shaft

A solid steel shaft of diameter D = 60 mm and length L = 1.5 m transmits a torque of T = 800 N·m. Given G = 80 GPa, determine:

(a) The maximum shear stress τ_{max}.

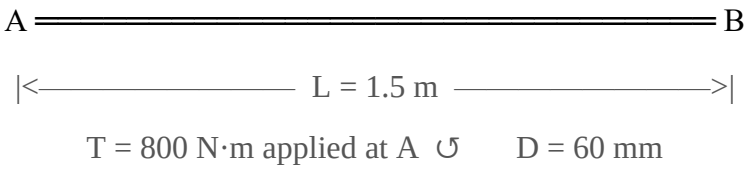
(b) The angle of twist φ (in degrees).

Answers:

Given Data

Parameter	Symbol	Value	Unit
Outer diameter	D	60	mm
Shaft length	L	1.5	m
Applied torque	T	800	N·m
Shear modulus	G	80 000	MPa

Shaft Diagram



(a) Maximum Shear Stress

For a solid circular shaft, the polar second moment of area is:

$$J = \pi \cdot D^4 / 32$$

$$J = \pi \times (0.060)^4 / 32 = 1272.3450 \times 10^{-6}\ m^4$$

The Navier torsion formula gives the maximum shear stress at the outer fibre (r = D/2):

$$\tau_{max} = T \cdot (D/2) / J = T \cdot D / (2 \cdot J)$$

$$\tau_{max} = 800 \times 0.030 / 1272.3450 \times 10^{-6}$$

$\tau_{max} = 18.86$ MPa

(b) Angle of Twist

The angle of twist over the full length L is:

$$\varphi = T \cdot L / (G \cdot J)$$

$$\varphi = (800 \times 1.5) / (80\times10^{9} \times 1272.3450\times10^{-6})$$

$$\varphi = 0.011789 \text{ rad}$$

Converting to degrees:

$$\varphi = 0.011789 \times (180/\pi)$$

$\varphi = 0.6755°$

Summary — Exercise 1

Quantity	Value	Unit
Polar moment J	1272.3450×10^{-6}	m^4
Max. shear stress τ_{max}	**18.86**	MPa
Angle of twist φ	**0.6755**	degrees

EXERCISE 3 — Stepped Shaft (Statically Indeterminate)

A stepped shaft is fixed at both ends A and C. A concentrated torque $T_0 = 1.2$ kN·m is applied at B. The two segments have the following properties:

- Segment AB : diameter $d_1 = 80$ mm, length $L_1 = 1.0$ m
- Segment BC : diameter $d_2 = 50$ mm, length $L_2 = 0.8$ m

G = 80 GPa for both segments. Determine:

(a) The reactive torques T_A and T_C at the fixed ends

(b) The maximum shear stress in each segment

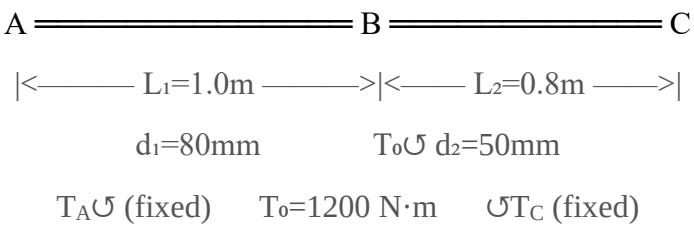
(c) The angle of rotation at B

Answers:

Given Data

Segment	Diameter (mm)	Length (m)	J (×10⁻⁶ m⁴)	G (GPa)
AB	80	1.0	4021.2386	80
BC	50	0.8	613.5923	80

Shaft Diagram



(a) Reactive Torques — Method of Compatibility

The shaft is statically indeterminate to the 1st degree. We have one equilibrium equation and one compatibility equation.

Equilibrium (sum of torques = 0):

$$T_A + T_C = T_0 = 1\,200\ \text{N·m} \quad \text{... (1)}$$

Compatibility (total twist between A and C = 0, since both ends are fixed):

$$\varphi_{AB} + \varphi_{BC} = 0$$

$$T_A \cdot L_1/(G \cdot J_1) - T_C \cdot L_2/(G \cdot J_2) = 0$$

G cancels. Rearranging:

$$T_A \cdot (L_1/J_1) = T_C \cdot (L_2/J_2)$$

$$T_A \cdot (1/4021.2386\times10^{-6}) = T_C \cdot (0.8/613.5923\times10^{-6})$$

$$T_A \cdot 248680 = T_C \cdot 1303797$$

$$T_A = 0.1907 \cdot T_C \quad \text{... (2)}$$

Substituting (2) into (1):

$$(0.1907 + 1) \cdot T_C = 1\,200$$

$$T_C = 1\,200 / 1.1907 = 1007.78\ \text{N·m}$$

$$T_A = 1\,200 - 1007.78 = 192.22\ \text{N·m}$$

$T_A = 192.2$ N·m | $T_C = 1007.8$ N·m

(b) Maximum Shear Stress in Each Segment

Internal torques: segment AB carries T_A ; segment BC carries T_C.

Segment AB ($d_1 = 80$ mm):

$$\tau_{AB} = T_A \cdot (d_1/2) / J_1 = 192.2 \times 0.040 / 4021.2386 \times 10^{-6}$$

$\tau_{AB} = 1.91$ MPa

Segment BC ($d_2 = 50$ mm):

$$\tau_{_BC} = T_{_C} \cdot (d_2/2) / J_2 = 1007.8 \times 0.025 / 613.5923 \times 10^{-6}$$

$\tau_{_BC} = 41.06$ MPa

The critical section is BC with $\tau_{max} = 41.06$ MPa.

(c) Angle of Rotation at B

The angle at B equals the twist of segment AB relative to the fixed end A:

$$\varphi_B = T_A \cdot L_1 / (G \cdot J_1)$$

ADDITIONAL EXERCISES

. **Exercise 1:**

Power Transmission A shaft must transmit P = 200 kW at n = 450 rpm. The allowable shear stress is 60 MPa and the allowable twist is 1° per metre. Using G = 80 GPa,

- determine the minimum solid shaft diameter satisfying both conditions.

Exercise 2:

Statically Indeterminate Shaft A hollow shaft (k = 0.65) is fixed at both ends A and C. A concentrated torque $T_0 = 3$ kN·m is applied at point B located a = 0.8 m from A, with total length L = 2 m. G = 80 GPa, $\tau_max = 60$ MPa. Determine:

(a) The reactive torques at A and C.

(b) The required outer diameter.

D and inner diameter d.

(c) The angle of twist at B

Exercise 3:

Composite Shaft (Steel + Aluminium) A composite shaft consists of a solid aluminium core (diameter 40 mm, G_al = 27 GPa) perfectly bonded inside a steel tube (inner diameter 40 mm, outer diameter 70 mm, G_st = 80 GPa). The assembly is L = 1.2 m long and subjected to a torque T = 2 kN·m. Determine:

(a) The torque carried by each material.

(b) The maximum shear stress in each material

(c) The angle of twist of the assembly

CHAPTER 4: SIMPLE PLANE BENDING

IV.1 Introduction

In practice, structural and machine components are often subjected to static or dynamic loading that induces bending stress in their cross sections, in addition to other stress types such as tension, compression, and shear.

A beam is said to undergo simple, or pure, bending when it deforms under a constant bending moment with no accompanying shear force. The internal stresses produced by this moment are termed flexural or bending stresses. Consider a beam (Fig 4.1) loaded as follows: [2][9]

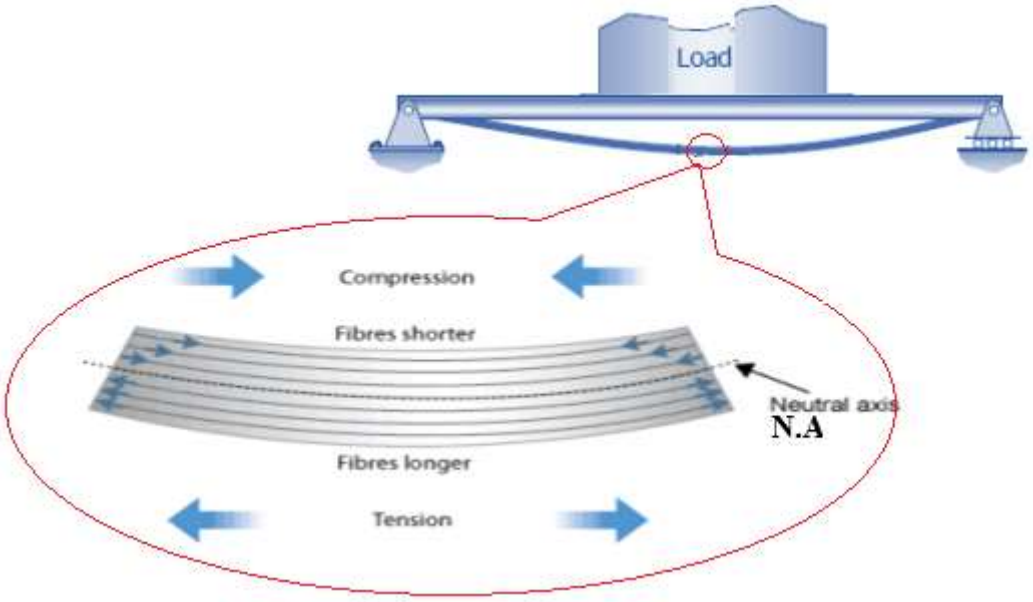


Fig 4.1: simple bending

IV.2 Assumptions in the theory of simple of bending

- The beam material is taken to be perfectly homogeneous (uniform throughout) and isotropic (identical elastic properties in every direction).
- Stresses remain within the material's elastic limit, so Hooke's law applies.
- Young's modulus (E) is assumed to be identical in tension and in compression.
- Cross sections that were plane before bending remain plane after bending (the entire cross section is assumed to rotate about the neutral axis).
- Each fiber of the beam can expand or contract freely, independent of the layers above and below it.
- The beam's radius of curvature is assumed large relative to its cross-sectional dimensions.

- The beam is initially straight, and under bending all longitudinal fibers curve into circular arcs sharing a single center of curvature [2]

IV.3 Theory of Simple Bending:

Take a uniform segment of beam subjected to pure bending; under this action, a small portion of length dx becomes deformed. Define:

- Take two sections AB and CD perpendicular to the axis N-N (Fig 4.2.a).
- Fibers that originally had equal lengths no longer keep the same length after bending
- $\boldsymbol{\rho}$ = Radius of curvature of beam or Radius of neutral layer N'N'
- θ = Angle subtended at o by A'B' and C'D'.

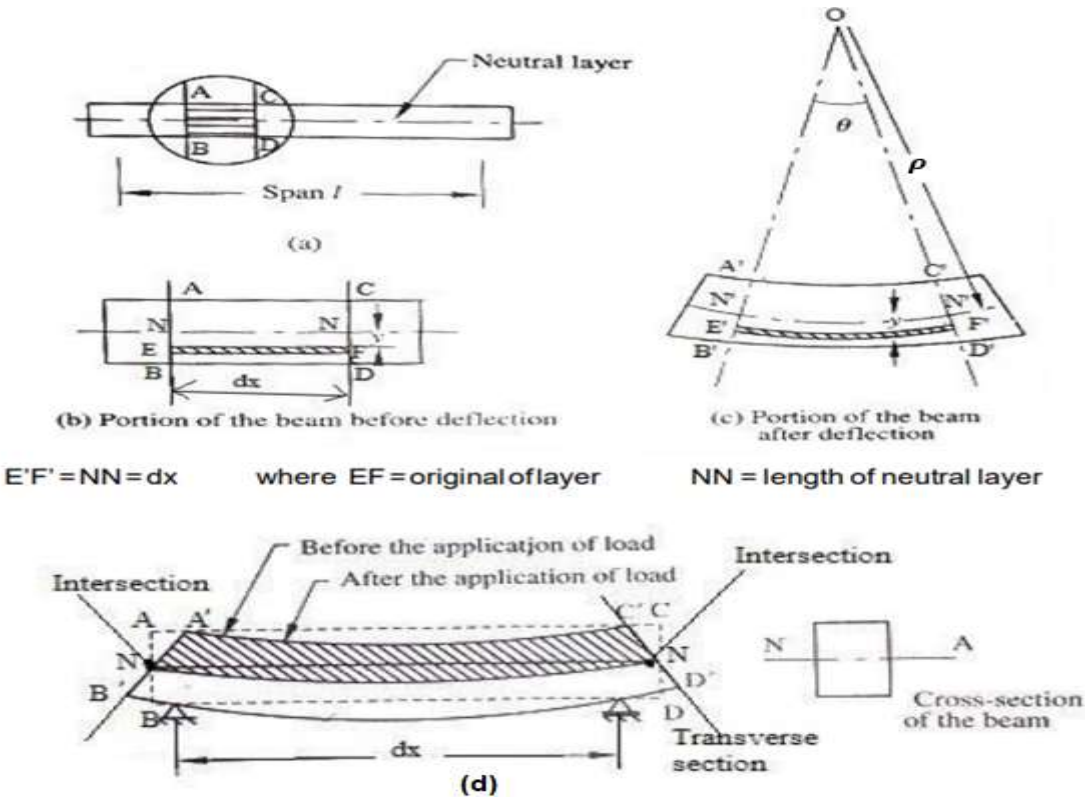


Fig 4.2: Longitudinal and transversal section of the beam.

- Next, take a fiber EF located at a distance y below the neutral layer NN; after bending this fiber stretches to E'F'.
- The upper fiber (above N-N) shortens from AC to A'C' (Fig 4.2.d), having undergone compression.
- Conversely, the lower fiber (below N-N) lengthens from BD to B'D' (Fig.4.2. d), having undergone tension.
- Somewhere between the top and bottom fibers lies one that neither shortens nor lengthens; this is called the neutral layer, or neutral surface.

• Where the neutral layer intersects a given cross section of the beam, this line is called the neutral axis (N.A.).

After bending, the length of neutral layer N'N' will remain unchanged. But length of layer E'F' will increase. After bending, the length of neutral layer N'N' will remain unchanged. But length of layer E'F' will increase.

NN=N'N'=dx because neutral axis

For N'N' **angle= arc / radius** $\boldsymbol{\theta = \frac{N'N'}{\rho}}$

dx = $\boldsymbol{\rho . \theta}$

EF= $\boldsymbol{. \theta}$

For E'F' **angle = arc / radius** $\boldsymbol{\theta = \frac{E'F'}{\rho + Y}}$

E'F = ($\boldsymbol{\rho}$ +Y). $\boldsymbol{. \theta}$

Increase in length of the layer **EF** = E'F' – EF = ($\boldsymbol{\rho}$ +Y) . $\theta - \boldsymbol{\rho}. \theta$= Y. θ

Strain in layer EF = (increase in length)/ (Original length) = Y. θ / $\boldsymbol{\rho}$. θ

$$\boldsymbol{\varepsilon = \frac{Y}{\rho}}$$

By Hooke's law,

As $\boldsymbol{\rho}$ is constant, hence strain in a layer is proportional to its distance from neutral axis.

Young's modulus: $\boldsymbol{E = \frac{stress}{strain} = \frac{\sigma}{\varepsilon}}$

$$\boldsymbol{\sigma = E.\varepsilon = Y . \left(\frac{E}{\rho}\right)}$$

$$\boldsymbol{\sigma \ \alpha \ Y}$$

Since E & $\boldsymbol{\rho}$ **both are constant, therefore stress in any layer is directly proportional to the distance of layer from the neutral layer.**

$$\boldsymbol{\frac{\sigma}{Y} = \frac{E}{\rho}}$$

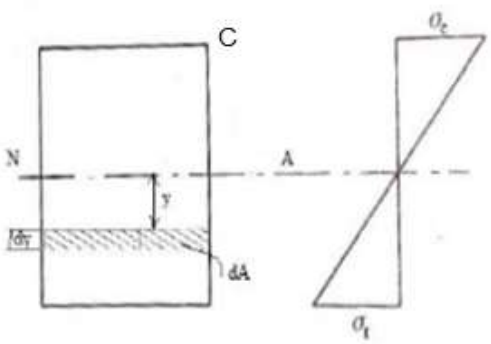
Neutral Axis: at any given cross section of the beam, this is the line along which the neutral layer intersects that section.

We have seen;

- When a section is subjected to a pure sagging moment, every point above the neutral axis experiences compressive stress,
- while every point below it experiences tensile stress,

- and the stress is zero exactly on the neutral axis itself.
- Stress at a distance Y of neutral axis

$$\boldsymbol{\sigma = Y\,.\,\left(\frac{E}{\rho}\right)}$$



Let N.A is neutral axis

Consider a small layer at a distance of Y from N.A.

Let dA= Area of that layer

ce on this layer = stress on layer

Area of layer

$$dF = \left(\boldsymbol{Y\,.\frac{E}{\rho}}\right)\,.\boldsymbol{dA}$$

Total force on beam is equal:

$$F = \int \left(\boldsymbol{Y\,.\frac{E}{\rho}}\right)\,.\boldsymbol{dA} = \boldsymbol{\frac{E}{\rho}\int Y\,.dA}$$

Since the first moment of any area about an axis through its own centroid is zero, it follows that the neutral axis must coincide with the centroidal axis.

- In other words, the centroidal axis of the section locates the N.A.

IV.4 Moment of Resistance

- Under pure bending, fibers above the N.A. carry compressive stress while those below it carry tensile stress.
- Together, these opposing stresses form an internal couple whose moment must balance the external bending moment (M).

This internal resisting moment, which balances the applied bending moment, is called the **moment of resistance.**

Force on layer at a distance Y from **N.A** = $dF = \left(\boldsymbol{Y\,.\frac{E}{R}}\right)\,.\boldsymbol{dA}$

Moment of this force about **N.A = dM = force on layer . Y**

$$= \left(\boldsymbol{Y\,.\frac{E}{R}}\right)\,.\boldsymbol{dA}\;.\;\mathbf{Y}$$

$$= Y^2 . \frac{E}{R} . dA$$

Moment of force on the section of beam (or moment of resistance) is equal:

$$M_f = \int Y^2 . \frac{E}{R} . dA$$

Let M is external moment applied on this section.

At equilibrium, the resisting moment generated by the section must equal the externally applied bending moment**.**

$$M_f = \int Y^2 . \frac{E}{R} . dA$$

$I = \int Y^2 . dA$ **= Second moment of area or moment of inertia of section about N.A.**

$$M_f = \frac{E}{R} . I$$

$$\frac{M}{I} = \frac{E}{R}$$

We know that

$$\frac{\sigma}{Y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{Y}$$

This equation is known as **bending equation or Bernoulli- Euler equation.** [2]

Because E and R do not vary along the section, the bending stress within the elastic range is directly proportional to y, the distance from the neutral axis. From this relation, the bending stress can be written as,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio I/y, denoted Z, defines the section modulus.

- ## Section Modulus

Section modulus is defined as the moment of inertia of a section about the neutral axis divided by the distance to its outermost fiber, denoted Z.

$Z = I / y_{max}$

I = Moment of inertia about N.A.

y_{max} = Distance of outermost layer from the neutral axis

M = Maximum bending moment (or moment of resistance offered by the section)

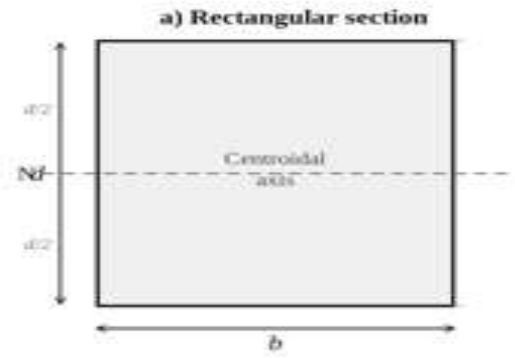
$M / I = \sigma_{max} / y_{max}$

$M = Z \times \sigma_{max}$

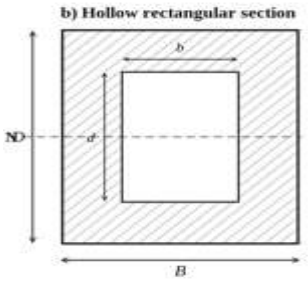
- Moment of resistance offered by the section is maximum when section modulus Z is maximum.
- Section modulus represents strength of the section.

Section modulus for various shapes

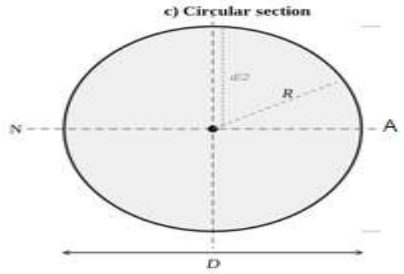
a) Rectangular Section:

Moment of inertia of a rectangular section about an axis through its C.G. (or through N.A.) $I = bd^3 / 12$ Distance of outermost layer from N.A. $y_{max} = d / 2n$ $Z = I / y_{max} = bd^3 / 12 \times 2 / d$ $\boldsymbol{Z = bd^2 / 6}$	

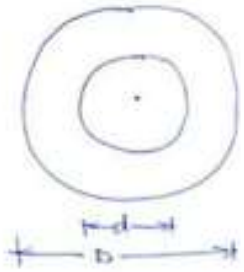
a) Hollow Rectangular Section:

b) $I = (BD^3 - bd^3) / 12$ c) $y_{max} = D / 2$ d) $Z = I / y_{max} = (BD^3 - bd^3) / 12 \times 2 / D$ e) $\boldsymbol{Z = (BD^3 - bd^3) / 6D}$	

c. Circular Section:

Moment of inertia of the section about NA, $\boldsymbol{I} = \pi d^4 / 64$ Distance of extreme fibre from the NA, $y_{max} = d / 2$ Section modulus, $Z = I / y = (\pi d^4 / 64) / (d / 2)$ $Z = \pi d^3 / 32$ or $Z = \pi R^3 / 4$	

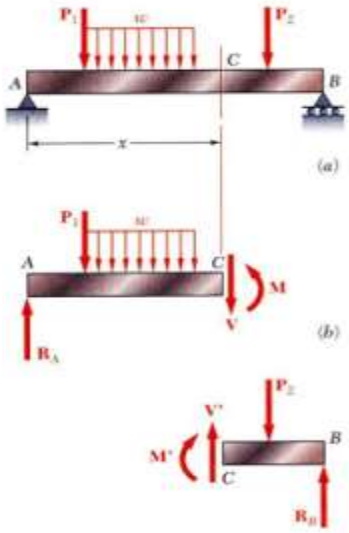
d. hollow circular section :

• Moment of inertia of the section about NA, $I = \pi/64 \times (D^4 - d^4)$ • Distance of extreme fibre from the NA, $y_{max} = D/2$ • Section modulus, $Z = I/y = \pi(D^4 - d^4)/64 \div D/2$ $Z = \pi (D^4 - d^4) / 32D$	

IV.5. Shear stress and bending moment:

Shear Force and Bending Moment Diagrams plot, respectively, how the shear force and bending moment vary along a beam's length, making it easy to identify the maximum values that govern beam design. [9][4]

The most common sign convention for the shear force and bending moment in beams is shown in Fig. 9.12.



- **Sign convention**

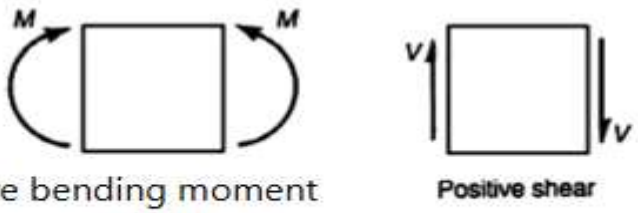


Figure : Sign convention for bending moment and shear

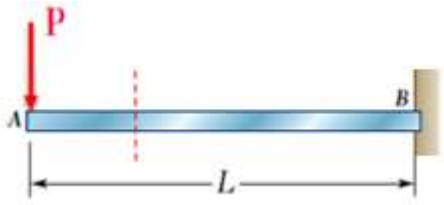
The shear and moment diagrams can be obtained through the following procedure:

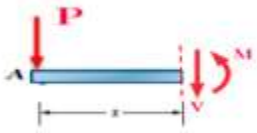
1. Find the support reactions using equilibrium of the whole beam.
2. Make an imaginary cut at any chosen point along the beam.
3. Indicate the unknown shear and moment at the cut, following the positive sign convention of Fig. 9.12.
4. Apply vertical force equilibrium to solve for the unknown shear.
5. Apply moment equilibrium about the cut to solve for the unknown moment.

Example:

For the beam shown, derive equations for shear force and bending moment at any point along the beam.

For the beam shown, derive equations for sear force and bending moment at any point along the beam.



Solution: We cut the beam at a point between A and B at distance x from A and draw the free-body diagram of the left part of the beam, directing V and M as indicated in the figure.	

$\Sigma F_y = 0$:

$P + V = 0$

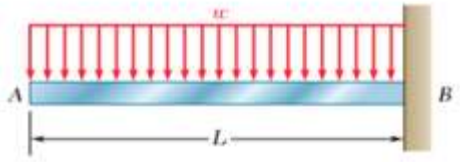
$V = -P \quad (\downarrow)$

$\Sigma M_x = 0$:

$P.x + M = 0$

$M = - Px(\quad) \quad \circlearrowleft$

Note that shear force is constant (equal P) along the beam, and bending moment is a linear function of (x).

Example 2: For a cantilever beam AB of span L supporting a uniformly distributed load w, derive equations for shear force and bending moment at any point along the beam.	

Solution: We cut te beam at a between A and B draw free-body diaram of AC, direct V and M as indicated	**Solution:** We cut the beam at a point C between A and B and draw free-body diagram of AC, directing **V** and **M** as indicated in Fig. Denoting by x the distance from A to C and replacing the distributed load over AC by its resultant (wx) applied at the mid point of AC, we write:

IV.5 Application Exercises (simple Bending)

Exercise 1 — Rectangular Beam under Central Point Load

A simply supported rectangular timber beam (b = 80 mm, h = 160 mm) has span L = 4 m and carries a central point load F = 8 kN. Given allowable bending stress σ_b = 12 MPa, check whether the beam is safe. [4][9]

GIVEN DATA	
Cross-section	b = 80 mm, h = 160 mm
Span	L = 4 m
Central load	F = 8 kN
Allowable stress	σ_b_allow = 12 MPa

Solution:

Step 1 — Maximum bending moment (midspan of simply supported beam with central load):

$$M_{max} = F \cdot L / 4 = 8000 \times 4 / 4 = 8000 \ N \cdot m$$

Step 2 — Second moment of area:

$$I = b \cdot h^3 / 12 = 80 \times 160^3 / 12 = 80 \times 4{,}096{,}000 / 12 = 27.31 \times 10^6 \ mm^4$$

Step 3 — Section modulus (c = h/2 = 80 mm):

$$S = I / c = 27.31 \times 10^{6} / 80 = 341{,}333\ mm^{3} = 3.413 \times 10^{-4}\ m^{3}$$

Step 4 — Maximum bending stress:

$$\sigma_{max} = M_{max} / S = 8000 / 3.413 \times 10^{-4} = 23.44\ MPa$$

Step 5 — Safety check: σ_max = 23.44 MPa > σ_allow = 12 MPa → UNSAFE!

The beam FAILS in bending. The maximum stress exceeds the allowable value by nearly 2.

ANSWERS	
Maximum moment	**M_max = 8000 N·m**
Second moment I	**I = 27.31 × 10^{6} mm^{4}**
Max bending stress	**σ_max = 23.44 MPa**
Safety check	**UNSAFE: 23.44 MPa > 12 MPa**

Exercise 2 — I-Section Beam under UDL

An I-section beam (flange: 100 mm × 10 mm, web: 180 mm × 8 mm) is simply supported over span L = 6 m and carries a UDL of w kN/m. Given σ_allow = 150 MPa, find: (a) the second moment of area I, (b) the maximum allowable w.

GIVEN DATA	
Flange dimensions	b_f = 100 mm, t_f = 10 mm (top & bottom)
Web dimensions	h_w = 180 mm, t_w = 8 mm
Total height	H = 180 + 2×10 = 200 mm
Span	L = 6 m
Allowable stress	σ_allow = 150 MPa

Solution:

(a) Second moment of area — use the subtraction method:

Treat the I-section as a full rectangle (100 × 200) minus two side rectangles cut from the web area:

$$I_{full_rect} = (100 \times 200^{3}) / 12 = 66.67 \times 10^{6}\ mm^{4}$$

Width removed on each side = (100 − 8)/2 = 46 mm, height of removed parts = 180 mm:

$$I_{removed} = 2 \times (46 \times 180^3) / 12 = 2 \times 22.27 \times 10^6 = 44.55 \times 10^6 \ mm^4$$

$$I = I_{full} - I_{removed} = 66.67 \times 10^6 - 44.55 \times 10^6 = 22.12 \times 10^6 \ mm^4$$

(b) Maximum allowable UDL: For a simply supported beam: $M_{max} = wL^2/8$

$$c = H/2 = 100 \ mm \ (symmetric \ section)$$

$$S = I / c = 22.12 \times 10^6 / 100 = 221{,}200 \ mm^3$$

$$\sigma_{max} = M / S \leq \sigma_{allow} \Rightarrow M_{max} = \sigma_{allow} \times S = 150 \times 221{,}200 = 33.18 \times 10^6 \ N \cdot mm$$

$$M_{max} = wL^2/8 \Rightarrow w = 8M_{max} / L^2 = 8 \times 33{,}180 / (6000)^2 \ N/mm = 7.373 \ N/mm$$

$$w_{max} = 7.373 \ N/mm = 7.37 \ kN/m$$

ANSWERS	
Second moment of area	**$I = 22.12 \times 10^6 \ mm^4$**
Section modulus	**$S = 221{,}200 \ mm^3$**
Max allowable UDL	**$w = 7.37 \ kN/m$**

Exercise 3 — Shear Force & Bending Moment Diagrams

A simply supported beam AB (L = 10 m) carries: a UDL of w = 6 kN/m over the left half (0 to 5 m), and a point load P = 20 kN at x = 7 m from A. Draw the SFD and BMD and find the maximum bending moment.

GIVEN DATA	
Span	L = 10 m
UDL	w = 6 kN/m from x = 0 to x = 5 m
Point load	P = 20 kN at x = 7 m

Solution:

Step 1 — Find reactions. UDL resultant = 6 × 5 = 30 kN at x = 2.5 m.

$$\Sigma M_A = 0: \ R_B \times 10 - 30 \times 2.5 - 20 \times 7 = 0$$

$$10 \ R_B = 75 + 140 = 215 \Rightarrow R_B = 21.5 \ kN$$

$$\Sigma F_y = 0: \quad R_A = 30 + 20 - 21.5 = 28.5 \text{ kN}$$

Step 2 — Shear Force V(x):

- $0 \le x < 5$ m: $V(x) = 28.5 - 6x$ [starts at +28.5, reaches 28.5−30=−1.5 at x=5]
- At x = 5 m: $V = 28.5 - 6\times5 = -1.5$ kN
- $5 < x < 7$ m: $V(x) = 28.5 - 30 = -1.5$ kN (constant)
- At x = 7 m (just after P): $V = -1.5 - 20 = -21.5$ kN
- $7 < x \le 10$ m: $V(x) = -21.5$ kN (constant) → at B: $+R_B = +21.5$ checks out

Step 3 — Zero shear in UDL zone: V = 0 at x = 28.5/6 = 4.75 m

Step 4 — Bending Moment M(x):

- M(0) = 0 (simply supported)
- $M(4.75) = 28.5\times4.75 - 6\times4.75^2/2 = 135.375 - 67.69 = 67.69$ kN·m ← MAXIMUM
- $M(5) = 28.5\times5 - 6\times25/2 = 142.5 - 75 = 67.5$ kN·m
- $M(7) = 67.5 - 1.5\times2 = 67.5 - 3 = 64.5$ kN·m
- M(10) = 0 (simply supported)

ANSWERS	
Reaction at A	**$R_A = 28.5$ kN**
Reaction at B	**$R_B = 21.5$ kN**
Zero shear position	**x = 4.75 m from A**
Maximum bending moment	**$M_{max} = 67.69$ kN·m at x = 4.75 m**

QUICK FORMULA REFERENCE [2][4][7]

Topic	Formula	Notes
Equilibrium	$\Sigma Fx=0,\ \Sigma Fy=0,\ \Sigma M=0$	3 equations in 2D
Normal Stress	$\sigma = F / A$	[Pa or MPa]
Elongation	$\Delta L = FL / (EA)$	Elastic range
Safety Condition	$\sigma_{max} \leq \sigma_e / s$	s = 2 to 10
Torsion stress	$\tau_{max} = 16Mt / \pi d^3$	Solid shaft
Polar moment (solid)	$I_n = \pi d^4 / 32$	[m⁴]
Polar moment (hollow)	$I_n = \pi(D^4 - d^4) / 32$	D outer, d inner
Angle of twist	$\alpha = MtL / (GI_n)$	[rad]
Power	$P = 2\pi fMt$	f in Hz
Flexure formula	$\sigma(y) = My / I$	M: moment, I: 2nd moment
Max bending stress	$\sigma_{max} = Mc / I = M/S$	S = I/c: section modulus
Rectangle I	$I = bh^3 / 12$	Centroidal axis
Circle I	$I = \pi d^4 / 64$	Centroidal axis
Beam curvature	$1/\rho = M / (EI)$	EI: flexural rigidity

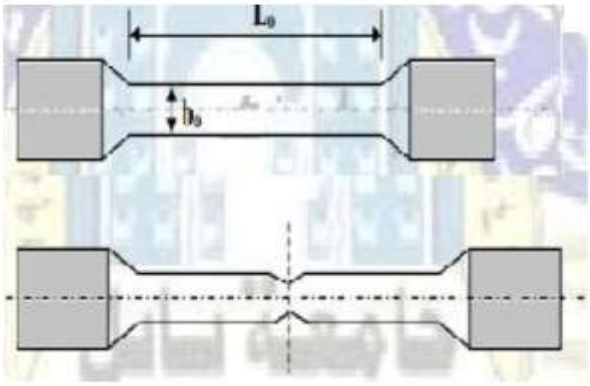
TP N°1 – Tension Test

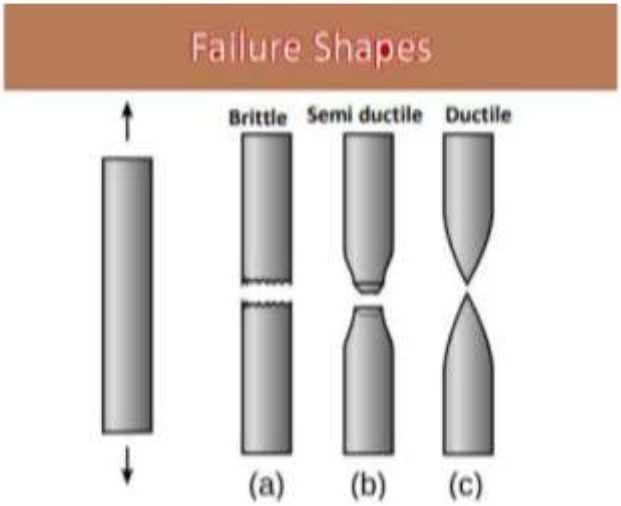
1- Objective of Tension Test

- Calculating Elasticity of Modulus.
- Determine the maximum stress value of the model material (σu, Stress Ultimate).
- Determine the value of the failure stress (Stress Fracture).
- Find the elongation percentage.
- Find the percentage of decrease in cross-sectional area (Area in Reduction %).
- Obtaining the relationship between stress and strain (Curve Strain-Stress). [3]

2- Tensile Sample Description

In Figure (1), the dimensions of the sample used, made of a rectangular cross-section, are noted before and after performing a tensile test on it.





Using the results obtained from the experiment, the following calculations are performed:

The stress and strain are calculated:

$$\sigma_n = F / S_0 \qquad \varepsilon_n = (L - L_0) / L_0$$

Percentage elongation A%:

$$A\% = [(L_final - L_original) / L_original] \times 100\%$$

Percentage reduction in Area Z%:

$$Z\% = [(A_original - A_final) / A_original] \times 100\%$$

3- Data Table

F (N)	ΔL (mm)	S_0 (mm^2)	$\sigma = F/S_0$ (MPa)	$\varepsilon = \Delta L/L$	$E = \sigma/\varepsilon$ (MPa)

4- Questions

A standardized tension test is carried out on a beam of diameter d_0 = ___ mm and of length L_0 ≈ ___ mm.

Using the values from the table above, plot the curve Stress–Strain (σ – ε) and calculate the following values:

- The Young's Modulus: E_th and E_exp → $E = (E_1 + \ldots + E_n) / n$
- The elastic limit stress: σ_e
- Maximum tension strength: σ_max
- The fracture strain: ε_r
- Percentage elongation A%.

Flexural Test N°2

1- Objective

- Determination of the elastic modulus E.
- Basic knowledge of RDM (simple flexure).
- Measurement techniques. [3][9]

2- Beam Used

A beam of rectangular shape with length L and cross-section (b × h) is used as shown in Figure 1.

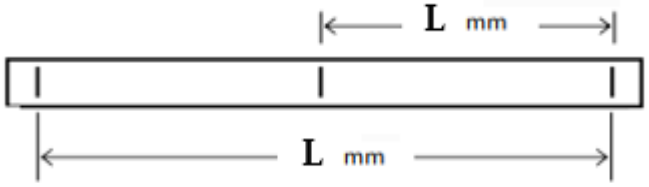


Fig 1

3- Experiment Study

The beam (length L) rests on two simple supports with a variable load applied in the middle.

Masses are varied and the value of the deflection is recorded for each load.

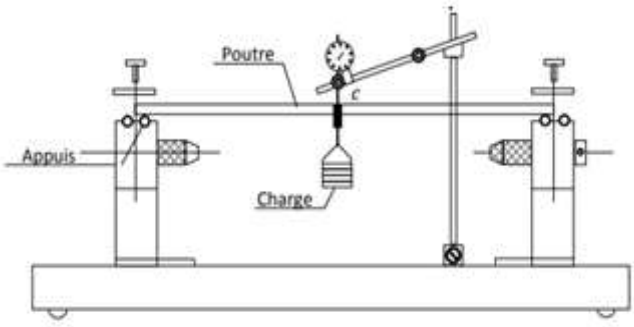


fig 2: flexural test

4- Theory

y represents the value of the deflection at mid-span:

$$y = PL^3 / (48EI) \quad \text{with} \quad I = b \cdot h^3 / 12$$

Therefore the elastic modulus is:

$$E = E_th = PL^3 / (48\, y\, I)$$

Where I is the second moment of area (quadratic moment).

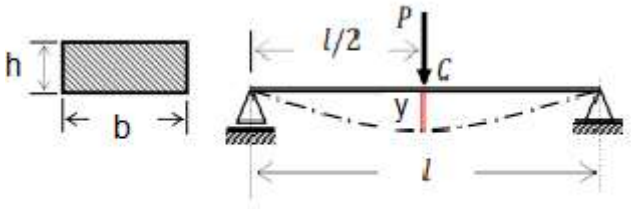


Fig 2

5- Data Table

Weight (g)	P (N)	Y_th (mm)	Y_exp (mm)

6- Tasks

1. Complete the table above.
2. Plot on the same graph y = f(P) experimental and theoretical, and deduce the Young's modulus E for the material and the maximum stress.
3. Interpret the shape of the curves obtained from the results.
4. Conclusion.

TP N°2 – Simple Torsion Test

1- Objective

- Experimental determination of the relationship between the torque M_t and the twist angle α of a bar of circular section.
- Experimental determination of the value of the sliding modulus G (Coulomb modulus). [7][9]

2- Calculation Assumptions

- The bar is rectilinear and of a uniform circular section over its entire length.
- The applied torque is constant over the entire length and acts around the polar axis.
- The induced stresses do not exceed the limit of proportionality.
- The sections normal and perpendicular to the axis of the bar before deformation remain flat and perpendicular to the axis after deformation.
- The straight sections rotate around the axis of the bar (α).
- When the bar deforms, its dimensions do not shift noticeably and the diameters remain straight.

3- Theory

A beam is subjected to simple torsion; it is in cylindrical form (L = ___ mm, d = ___ mm).

$$\theta = \alpha / L \quad (\text{unit twist angle in rad/mm})$$

$$\tau = G \cdot \theta \cdot \rho$$

where: G = transverse elasticity modulus (MPa), ρ = radius, τ = shear stress (N/mm²)

$$M_t = G \cdot \theta \cdot I_0$$

M_t = torsional moment (N·m), I_o = polar moment of inertia = $\pi \cdot d^4/32$ (solid circular shaft)

$$\tau = (M_t / I_0) \cdot \rho$$

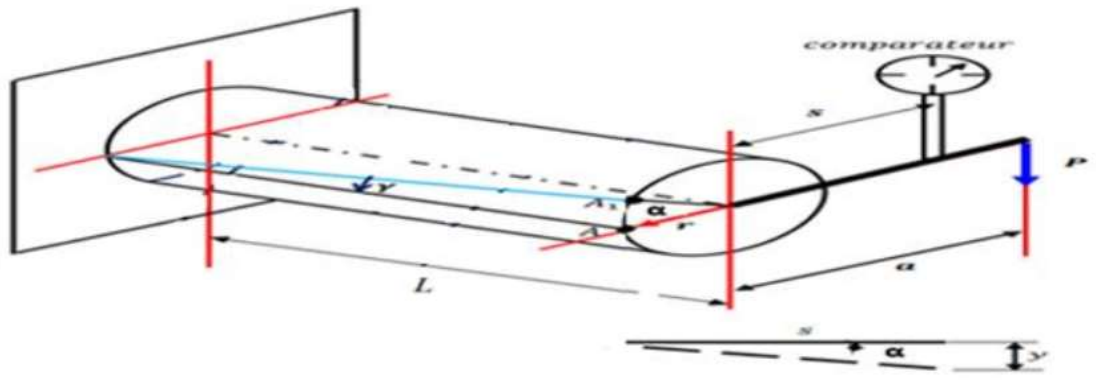
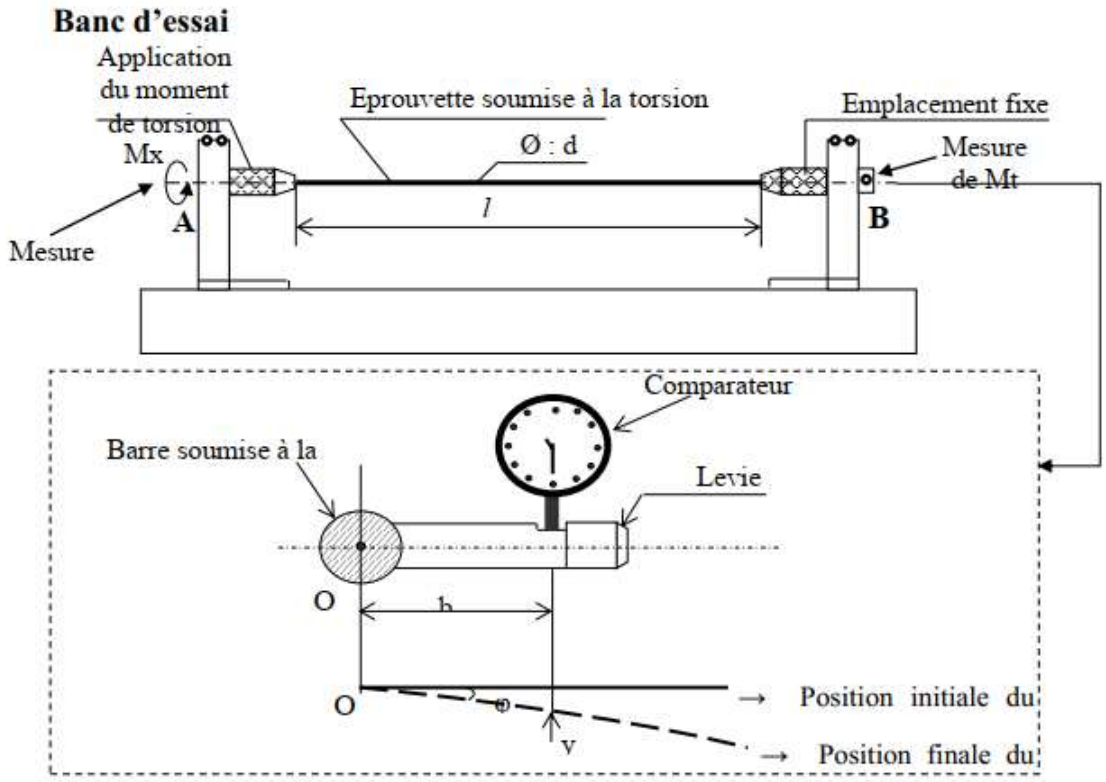


Fig 1



4- Material Reference Values

Material	G (GPa)	E (GPa)
Aluminum	26	70
Steel	80	270
Brass (red)	44	120

5- Required Work

Part A – Torque vs. Angle of Rotation

Complete the table below for both specimens showing the variation of torque M_t as a function of the section rotation angle α:

Force (N)	M_t (N·m)	α (°)

Tasks:

1. Plot on the same graph $M_t = f(\alpha)$ for the two test pieces.
2. Interpret the result: determine the shape of the curves, describe how the torque varies with the angle of rotation, and compare the two curves.

Part B – Determination of Shear Modulus G

1. Measure the diameter of the two rods and calculate the polar moment I_o.
2. Plot the curves $M_t \cdot L = f(I_o \cdot \theta)$ respectively for the two grades of materials.
3. Calculate the slopes of the two straight lines obtained, compare them with the theoretical results and conclude.

Second data table:

M_t (N·m)	θ (°)	$M_t \cdot L$	$I_o \cdot \theta$

TP4 : Hardness test

1- definition

Hardness, with the general symbol H (from the English -hardness), is a mechanical property which expresses the resistance of a material subjected to localized plastic deformation (small indentations or scratches).

Many methods are used to evaluate hardness; the most common are to measure penetration resistance. It is to this category of tests that we will limit ourselves in this TP.

Current practice divides hardness testing into two categories: macro -hardness and micro-hardness.

1) Macro Hardness Testers Loads > 1 kg
   - Rockwell • Brinell • Vickers
2) Micro Hardness Testers < 1 kg
   - Knoop diamond • Vickers diamond pyramid

2-Principal purpose

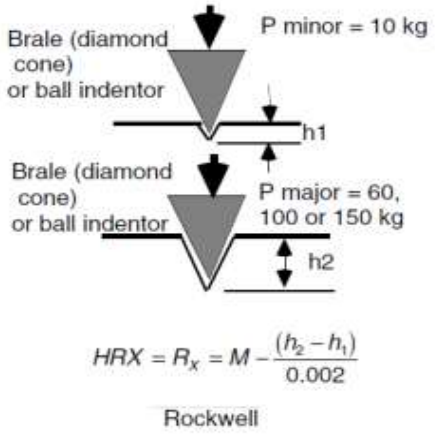
- Understand the concept of hardness
- Discover the different hardness measuring equipment and carry out some tests.
- Measure the hardness of different test pieces and compare the theoretical and numerical results of the hardness of different test pieces.

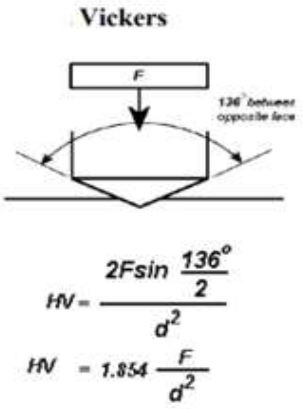
3-Targeted skills

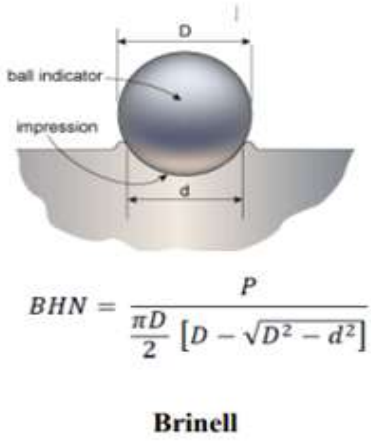
- Know how to measure hardness.
- Know how to analyze and interpret the different results obtained from the hardness test.

4-Theoretical reminders

Hardness can be evaluated by measuring an imprint left on the surface by a punch acting under the action of a known force (Brinell, Vickers and Rockwell tests).







5. Experimental test device:

"UNIVERSAL TESTER 930/250N" a durometer.

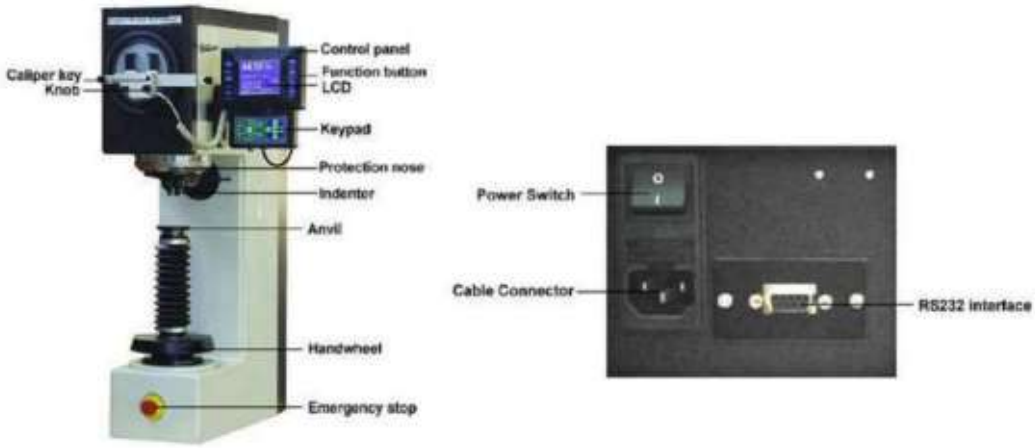


Fig. 1 : Le duromètre « UNIVERSAL TESTER 930/250N »

6. Steps followed:

After observing under the microscope and taking images of the different microstructures materials, we measure the HV hardness of the samples using a durometer.

material	HV(theatrical)	HV (durometer)
Aluminum		
copper		
steel		

2. Measure 3 hardness values on the same surface for the three different specimens under the effect of a constant force and take the middle value and compare the theoretical and numerical results of the hardness of different test pieces.

3. Interpretation the results and Conclusion

TP5: Measurement of elastic moduli by ultrasound

1. Aim

The objective is to determine elastic constants (E, ϑ, G, and K) of each to materials provided below: **steel - aluminum – copper - glass**

2- Definition of ultrasonic waves:

Ultrasonic waves or ultrasound waves are sound waves with frequencies higher than the upper audile limit of human hearing which typically around 20000 Hz (20 kHz). They characterized by their frequency and divided into three classes, as shown in the figure:

3-Types of ultrasonic waves: there are:

3-1-Longitudinal waves (traction or compression)

If the direction of particle vibrations is parallel to the direction of propagation of wave. The velocity of a longitudinal wave is given by the expression: $V_{L=}\sqrt{\frac{E}{\rho}}$

Where: E, ρ are longitudinal modulus of elasticity and density of the material perspectival.

3-2-Transverse (or shear) waves:

The speed of this type of wave is given by the expression**:** $V_T = \sqrt{\frac{G}{\rho}}$

Where: G: the material's slip modulus of elasticity or Coulomb modulus expressed in pascals (Pa) such that : $G = \tau / g$, **τ:** Tangential or shear stress, g: unit slip.

4- Materials use:

- Electrical pulse generator.
- Electrical pulse receiver.
- Longitudinal and transverse wave transducer.
- Oscilloscope

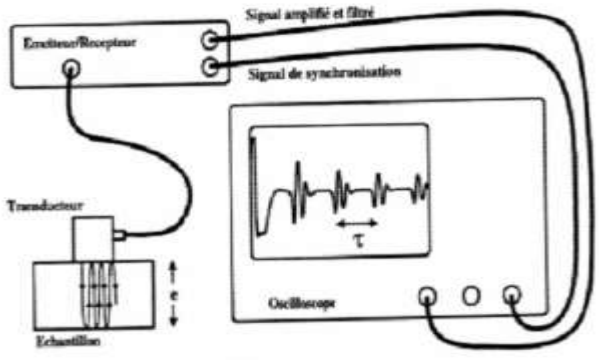


Figure : Technique ultrasonore par réflexion.

5-Principle

The principle of measuring the speed of sound is as follows:

-An electrical pulse is sent to the transducer to generate acoustic waves. With the transducer placed against the medium under study, the acoustic wave penetrates and propagates through it. If the rear face of the medium under study is parallel to the entrance face, a series of echoes resulting from the multiple reflections of the incident acoustic wave are recovered. Measuring the delay between each echo allows the wave speed in the medium under study to be determined if its thickness is known.

-The applied and detected voltages are displayed on an oscilloscope screen. Since the pulse train received as an echo is identical for each pulse sent to the transducer, only a single pulse and the corresponding train will be displayed on the oscilloscope screen. Using the period □ corresponding to the time taken by the wave to make a round trip between two successive echoes, and the thickness of material (e), the propagation speed of the wave is calculated from the following relationship: $V = \frac{2e}{\tau}$

6-Theorie of elastic Moduli:

A solid body is called elastic or brittle, and its mechanical behavior follows Hooke's law $E = \frac{\sigma}{\varepsilon}$. The shear modulus (G) and the compression modulus (K), and all elastic moduli can be related in pairs by the following relationships:

$$E = 2(1+\upsilon)G \qquad E = 3(1-2\upsilon)K$$

6-1-Calculations of elastic moduli:

- Young's modulus E is: $E = \rho V_T^{\,2} \frac{3V_L^2 - 4V_T^2}{V_L^2 - V_T^2}$

- Shear modulus expresses: $G = \rho V_T^2$

- Bulk modulus volumetric printing is: $K = \frac{\rho}{3}(3V_L^2 - 4V_T^2)$

- The longitudinal modulus that expresses the elongation of the sample under the effect of the applied stress is calculated from the relation: $L = \rho V_L^2$

- Debye temperature: the measurement of acoustic speeds makes it possible to determine the Debye temperature θ_D by the following relationship:

$$\theta_D^3 = \frac{9}{4\pi}\left(\frac{h}{K_B}\right)^3 \frac{V_T^3 V_L^3}{V_T^3 + 2V_L^3}\frac{N}{V}$$

With: h = 6.6256 10-34 Js and K_B = 1.3805 10-23 JK-1

N is the number of atoms contained in the volume

7- Questions

1- Measure the density of the samples.

2- Create a table listing the longitudinal and transverse velocities and the thickness of the samples.

3- Calculate the elastic moduli experimentally.

4- Compare these values (Young's modulus) with the results of the bibliography.

5- Calculate the Debye temperature of the materials.

6- Conclusion.

REFERENCES

[1] NPTEL/SWAYAM, “Strength of Materials,” Online course, Prof. S. K. Bhattacharyya, Dept. of Civil Engineering, Indian Institute of Technology Kharagpur, India.

[2] R.S. Khurmi and N. Khurmi, “Strength of Materials (Mechanics of Solids),” S. Chand & Company Ltd., New Delhi, India.

[3] F.P. Beer, E.R. Johnston Jr., J.T. DeWolf and D.F. Mazurek, “Mechanics of Materials,” 7th ed., McGraw-Hill Education, New York, USA.

[4] R.C. Hibbeler, “Mechanics of Materials,” 10th ed., Pearson Prentice Hall, Boston, USA.

[5] A.P. Boresi and R.J. Schmidt, “Advanced Mechanics of Materials,” 6th ed., John Wiley & Sons, New York, USA.

[6] A. Pytel and F.L. Singer, “Strength of Materials,” 4th ed., Harper & Row Publishers, New York, USA.

[7] S.P. Timoshenko, “Strength of Materials, Part I: Elementary Theory and Problems,” 3rd ed., D. Van Nostrand Company, New York, USA.

[8] T.A. Philpot, “MecMovies — Mechanics of Materials Learning Modules,” University of Missouri & University of Florida (educational resource on stress-strain behaviour).

[9] J.M. Gere and B.J. Goodno, “Mechanics of Materials,” 8th ed., Cengage Learning, Stamford, USA.