Fuzzy Adaptive Approach to Nonlinear Systems Control

Noureddine Golea
E.E. Institute,
O.E.B University,
04000 Algeria.
ngolea@lycos.com

Amar Golea & Mohamed Boumehrez
E.E. Institute
Biskra University
07000 Algeria.

Mohamed Kadjoudj
E.E. Institute
Batna University
05000 Algeria
m.kadjoudj@lycos.com

Abstract— **We consider a direct fuzzy adaptive control of continuous-time nonlinear systems. The proposed adaptive scheme uses a Takagi-Seguno (TS) fuzzy controller, which allows the inclusion of a priori information in terms of qualitative knowledge about the plant operating points or analytical conventional regulators for those operating points. The closed loop dynamic is proven to be asymptotically stable, and robust against external disturbance and approximation error. The proposed approach performance is evaluated on an induction motor control problem.**

Keywords— **Non-linear systems, TS fuzzy system, Adaptive control, Stability, Robustness, Induction motor.**

I. Introduction

During the last decade, fuzzy systems have shown to be useful tools for solving the nonlinear systems identification and control problems [1-8]. This is due essentially to three features: First, fuzzy systems are nonlinear, and in this fact are more adapted to nonlinear systems representation, second, fuzzy systems are free model approach, and finally, fuzzy systems provides a simple mean to handle qualitative knowledge in the control design.

Adaptation is used in fuzzy control systems when, qualitative knowledge do not provide enough information to determine all the parameters of the control system. In last years, various kind of fuzzy adaptive systems,which use Mamdani or TS fuzzy systems, were developed and their stability and robustness were checked in the Lyapunov [1-6] and hyperstability [8] frameworks.

This work develops a new stable fuzzy direct adaptive control for disturbed nonlinear continuous systems. The proposed Fuzzy Control system uses an adaptive TS fuzzy system to approximate the nonlinear optimal controller. This adaptive scheme presents the following advantages: i) the qualitative information about the plant operating points can be used to design the fuzzy controller antecedents, ii) if for some operating points, linear regulators are available, they can be directly incorporated into the fuzzy controller rule consequences, and iii) it allows fast control update, which is a limit factor for some applications. The stability and robustness properties of the proposed fuzzy adaptive scheme are proved, in presence of approximation error, external disturbance and input gain variation. The simulation results, for the induction motor control problem, show that the fuzzy adaptive control maintains a consistent performance under approximation error, external disturbance and parameters variation.

II. Problem statement

Consider the continuous-time nonlinear system given by

$$\dot{x}_n = a(x) + b(x)u + \eta \qquad (1)$$

where $a(x)$ and $b(x)$ are smooth unknown functions, η is unknown bounded external disturbance, $u \in R$ is the input of the system, and $x \in R^n$ is the state vector of the system, which is assumed to be available.

The stable, linear time invariant and controllable reference system is defined by the following state equation

$$\dot{x}_m = A_m x_m + b_m r \qquad (2)$$

where $x_m \in R^n$ is the state vector of the reference model, r is a bounded reference input, and A_m, b_m are given by

$$A_m = \begin{bmatrix} 0 & I_{n-1} \\ & -a_m \end{bmatrix}, \quad b_m^T = \begin{bmatrix} 0 & \dots & 0 & b_{nm} \end{bmatrix}$$

where

$$a_m = \begin{bmatrix} a_{1m} & a_{2m} & \dots & a_{nm} \end{bmatrix}$$

The control problem can be stated as: design the control input u such that the states of the plant (1) follow those of the reference system (2), under the condition that all involved signals in the closed loop remain bounded.

The tracking error dynamic is given by

$$\dot{e} = A_m e - b_c\left[a(x) + b(x)u + a_m x - b_{nm} r + \eta\right] \qquad (3)$$

where $e = x_m - x$ and $b_c = \begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix}^T$. If the plant (1) is free of disturbance (i.e. $\eta = 0$), and $a(x)$ and $b(x)$ are known, then, the optimal control input given by

$$u^* = \frac{1}{b(x)}\left[b_{nm} r - a_m x - a(x)\right] \qquad (4)$$

assures the asymptotic convergence of (3) to zero. Since the nonlinear functions are not known, the optimal control will be approximated, in the following, with a TS fuzzy controller.

III. Fuzzy control

The controller to be designed is Multi-input single-output TS fuzzy system [9], constituted by a set of If-Then fuzzy rules of the form

$$\text{R}_i\text{: If } v \text{ is } V_i \text{ Then } u_f = a_i z \quad i = 1 \dots m \qquad (5)$$

where

$$a_i = \left[\begin{array}{cccc} a_{i1} & a_{i2} & ... & a_{in+1} \end{array}\right]$$

$$z = \left[\begin{array}{cc} x^T & r \end{array}\right]^T$$

and $v \in R^p$ is the fuzzy controller input vector, which may contain all or only a part of the plant states following the importance given to each state. The fuzzy sets V_i operate a fuzzy partition of the fuzzy controller input space.

The final output of the fuzzy controller (5) is inferred as follows

$$u_f = \sum_{i=1}^{m} \varphi_i a_i z \tag{6}$$

where

$$\varphi_i = \frac{\mu_i(v)}{\sum_{j=1}^{m} \mu_j(v)} \tag{7}$$

and $\mu_i(v)$ is the grade of membership of v in V_i. In this paper, it assumed that there exist always at least one active rule, i.e. $\sum_{j=1}^{m} \mu_j(v) > 0$.

Using the input control

$$u = u_f + u_s \tag{8}$$

where u_s is a switching control term yet to be defined, the error dynamic becomes

$$\begin{aligned} \dot{e} &= A_m e - b_c[a(x) + b(x) \sum_{i=1}^{m} \varphi_i a_i z + a_m x \\ &\quad -b_{nm} r + b(x) u_s + \eta] \end{aligned} \tag{9}$$

then, adding and substracting $b(x) u^*$ from the right side of (9) yields

$$\dot{e} = A_m e - b_c \left[b(x) \left(\sum_{i=1}^{m} \varphi_i a_i z - u^* \right) + b(x) u_s + \eta \right] \tag{10}$$

Following the universal approximation results, the fuzzy controller (6) can approximate the optimal controller u^* on a compact operating space to any degree of accuracy [1,5,10 and 11]. Next, we define the fuzzy controller optimal parameters a_i^* be such as

$$u^* = \sum_{i=1}^{m} \varphi_i a_i^* z + \omega \tag{11}$$

where ω is the minimum approximation error achieved by the fuzzy controller with the optimal parameters. Introducing (11) in (10) yields

$$\dot{e} = A_m e - b_c \left[b(x) \sum_{i=1}^{m} \varphi_i \widetilde{a}_i z + b(x) u_s + \eta - b(x) \omega \right] \tag{12}$$

where $\widetilde{a} = a - a^*$ is the parameters error.

IV. Stability Analysis

To establish the stability of the feedback system given by (12), the following assumptions are needed:

1. The input gain is bounded by $0 < \underline{b} \leq |b(x)| \leq \overline{b}$, and its variation is bounded by $\left|\dot{b}(x)\right| \leq \beta(x)$ where $\underline{b}$ and $\overline{b}$ are positive constants and $\beta(x)$ is a known function.
2. The approximation error and external disturbance are upper bounded by $\overline{\omega} \geq |\omega|$ and $\overline{\eta} \geq |\eta|$, respectively.

Consider the following Lyapunov function

$$V = \frac{\gamma}{b(x)} e^T P e + \sum_{i=1}^{m} \widetilde{a}_i \widetilde{a}_i^T \tag{13}$$

where P is symmetric positive definite matrix solution of the Lyapunov equation

$$A_m^T P + A_m P = -Q \tag{14}$$

The evaluation of time derivative of (13) along (12) yields

$$\begin{aligned} \dot{V} &= -\frac{\gamma}{b(x)} e^T Q e + 2 \sum_{i=1}^{m} \widetilde{a}_i \left(\dot{\widetilde{a}}_i^T - \gamma \varphi_i z e^T P b_c \right) \\ &\quad -2 \frac{\gamma}{b(x)} \left[b(x) u_s + \eta - b(x) \omega \right] e^T P b_c \\ &\quad - \frac{\gamma \, \dot{b}(x)}{b^2(x)} e^T P e \end{aligned} \tag{15}$$

using the following update law

$$\dot{a}_i = \gamma b_c^T P e \varphi_i z^T \tag{16}$$

in (15) yields

$$\begin{aligned} \dot{V} &= -\frac{\gamma}{b(x)} e^T Q e - \frac{\gamma \, \dot{b}(x)}{b^2(x)} e^T P e \\ &\quad -2 \frac{\gamma}{b(x)} \left[b(x) u_s + \eta - b(x) \omega \right] e^T P b_c \end{aligned} \tag{17}$$

Finally, taking the switching control term as

$$u_s = \frac{\beta(x)}{2\underline{b}^2} e^T P e + \left(\overline{\omega} + \frac{\overline{\eta}}{\underline{b}} \right) \operatorname{sgn}\left(b_c^T P e \right) \tag{18}$$

yields

$$\dot{V} \leq -\frac{\gamma}{\overline{b}} e^T Q e \tag{19}$$

Therefore, $\dot{V}$ is always negative in the e space if $e \neq 0$, then e and $\widetilde{a}_i$ are bounded, thus V is bounded.

Since x_m is bounded by design, x is bounded also, ω is bounded by the universal approximation theorem and under the assumption of bounded external disturbance, then $\dot{e}$ is bounded (φ_i are bounded by the definition of the TS fuzzy controller), and so e is uniformly continuous. Therefore $\dot{V}$ is uniformly continuous.

Hence, using the Barbalat's lemma [12], yields: $\lim_{t \to \infty} \dot{V} = 0$ and $\lim_{t \to \infty} e = 0$.

Remark :

Since the switching control is discontinuous, chattering control may occurred. In practice, chattering is undesirable, because it involves high control organ solicitation, and may excite the plant unmodeled dynamics. To avoid this problem, the switching control is smoothed as

$$u_s = \frac{\beta(x)}{2\underline{b}^2} e^T P e + \left(\overline{\omega} + \frac{\overline{\eta}}{\underline{b}}\right)\left(1 + \frac{\sigma}{\beta}\right) \frac{b_c^T P e}{|b_c^T P e| + \sigma} \tag{20}$$

where σ and β are positive constants. The constant σ is chosen based on engineering considerations to achieve an admissible tracking error. Using (20) in (17) gives

$$\begin{aligned} \dot{V} \;\; \leq \;\; & -\frac{\gamma}{b(x)} e^T Q e \\ & -2\frac{\gamma}{b(x)} \left[\left(\overline{\omega} + \frac{\overline{\eta}}{\underline{b}}\right)\left(1 + \frac{\sigma}{\beta}\right) \frac{\left(b_c^T P e\right)^2}{|b_c^T P e| + \sigma} \right. \\ & \left. + \left(\frac{\eta}{b(x)} - \omega\right) b_c^T P e \right] \end{aligned} \tag{21}$$

Then, if $\left|b_c^T P e\right| \geq \beta$ we get

$$\dot{V} \leq -\frac{\gamma}{\overline{b}} e^T Q e \tag{22}$$

If $\left|b_c^T P e\right| < \beta$ then

$$\dot{V} \leq -\frac{\gamma}{b(x)} e^T Q e + 2\frac{\gamma}{b(x)} \sigma \left| \frac{\eta}{b(x)} - \omega \right| \tag{23}$$

and the tracking error converge to the bounded region given by

$$\Omega(e) = \left\{ e \diagup \|e\| \leq \left(\frac{2\sigma\left(\overline{\omega} + \frac{\overline{\eta}}{\underline{b}}\right)}{\lambda_{\min}(Q)} \right)^{1/2} \right\} \tag{24}$$

In this case, the parameters boundedness is no longer ensured, and it is necessary to modify the update law (16) to remedy to this problem. Various modifications are possible, such the dead zone or the projection [12].

V. Simulation

The fuzzy adaptive control is tested on an induction motor model. The dynamic of the current-controlled induction motor in the $d-q$ axis, and under the field orientation assumption (the rotor flux ϕ_r is oriented according to the d-axis, i.e. $\phi_d = \phi_r$ and $\phi_q = 0$) is given by

$$\dot{\omega}_r = -\tfrac{f}{J}\omega_r + \tfrac{p^2 M}{J L_r}\phi_r u_1 - \tfrac{p}{J} T_l \tag{25}$$

$$\dot{\phi}_r = -\tfrac{1}{T_r}\phi_r + \tfrac{M}{T_r} u_2 \tag{26}$$

where ω_r is the rotor speed; u_1, u_2 are the stator current $d-q$ components, and T_l is the load torque. The motor physical parameters are described in the appendix.

The reference models for the speed and the flux are given, respectively, by

$$\dot{\omega}_m = -20\omega_m + 20\omega_{ref} \tag{27}$$

$$\dot{\phi}_m = -50\phi_m + 50\phi_{ref} \tag{28}$$

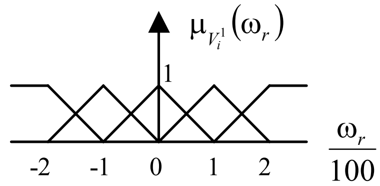

Fig. 1: Speed controller membership functions.

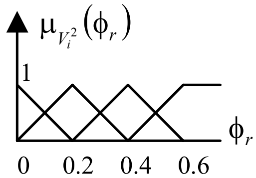

Fig. 2: Flux controller membership functions.

The controllers design steps are:

1. To design the speed and flux fuzzy controllers, the relevant region of the operating space is partitioned using fuzzy sets as in Fig. 1 and Fig. 2.
The speed controller is constituted by five rules of the form

$$\mathrm{R}_i^1: \text{ If } \omega_r \text{ is } V_i^1 \text{ Then } u_{f_1} = a_{i1}^1\omega_r + a_{i2}^1\phi_r + a_{i3}^1\omega_{ref}$$

The flux controller is constituted by four rules of the form

$$\mathrm{R}_i^2: \text{ If } \phi_r \text{ is } V_i^2 \text{ Then } u_{f_2} = a_{i1}^2\phi_r + a_{i2}^2\phi_{ref}$$

No a priori knowledge is assumed in this simulation, and the controllers parameters are initialized to zero.
2. The selection of $q_1 = q_2 = 10$, and the solution of (14) yields $p_1 = 1/4$ and $p_2 = 1/10$.
3. The adaptive controller (8) is used, with the switching control term as defined in (18) with $k_{s1} = 10$, $\sigma_1 = 0.05$ for the speed controller and $k_{s2} = 2$, $\sigma_2 = 0.005$ for the flux controller.
4. The fuzzy controllers parameters are updated using (16), with $\gamma_1 = 0.05$ for the speed and $\gamma_2 = 0.3$ for the flux.

The first test concerns the speed inversion under a load torque change of 5 Nm, with the rotor flux maintained to the rated value. As can be seen from fig. 3(a) the speed tracks closely the reference, with full rejection of the load torque perturbation. Fig. 3(b) show the flux fast convergence to the reference flux. As depicted in fig. 3(c-d) the control inputs respond rapidly without any oscillations, also it is remarkable from fig. 3(c) that u_1 is above the saturation value, i.e. the rated value.

The second test consider the motor working in the flux weakening region. As pointed out by the fig. 4(a) the speed tracks accurately the reference speed, which passes from 100 to 200 rad/s. The rotor flux tracks closely the reference flux variation (fig. 4(b)). The control inputs dynamics are depicted in fig. 4(c) and (d).

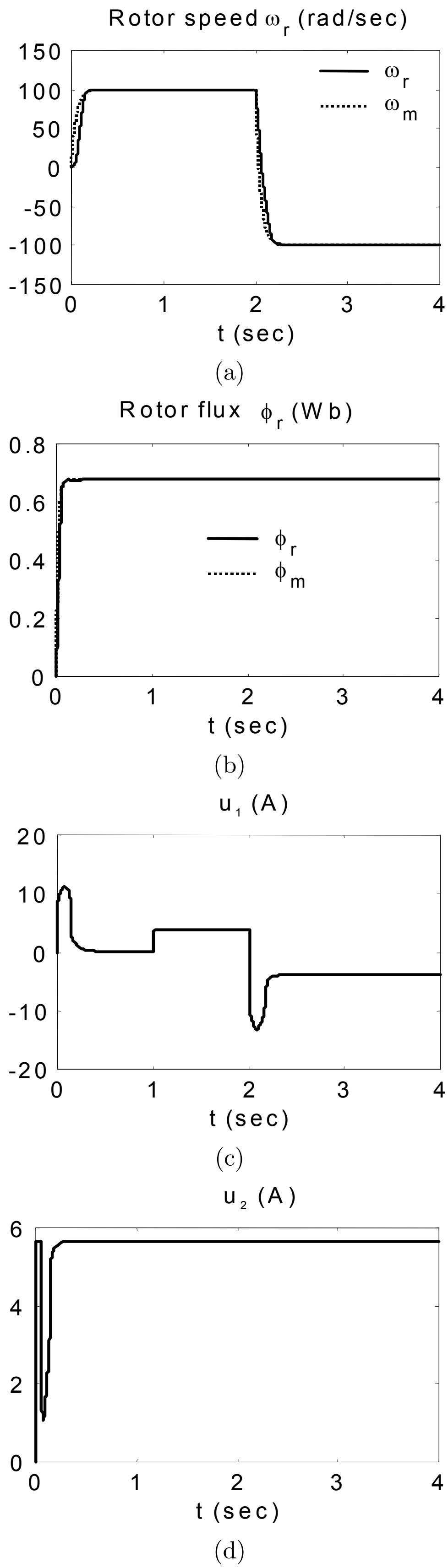

Fig. 3: Speed inversion under load torque change: (a) Speed, (b) Rotor flux, (c) Speed controller input and (d) Flux controller input.

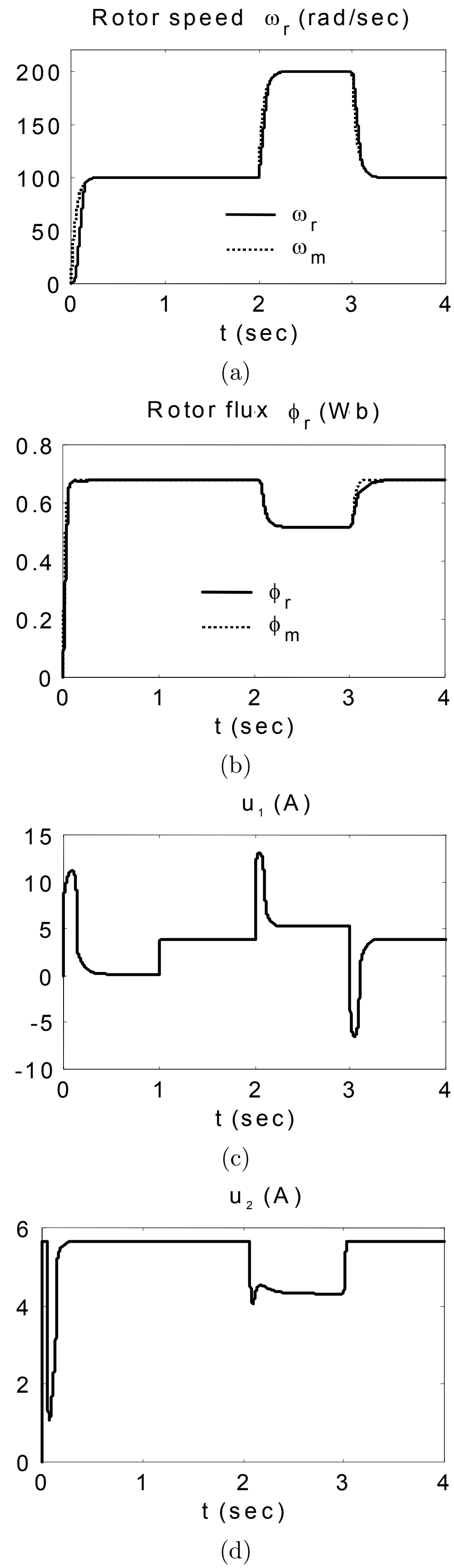

Fig. 4: Flux weakening under load torque change: (a) Speed, (b) Rotor flux, (c) Speed controller input and (d) Flux controller input.

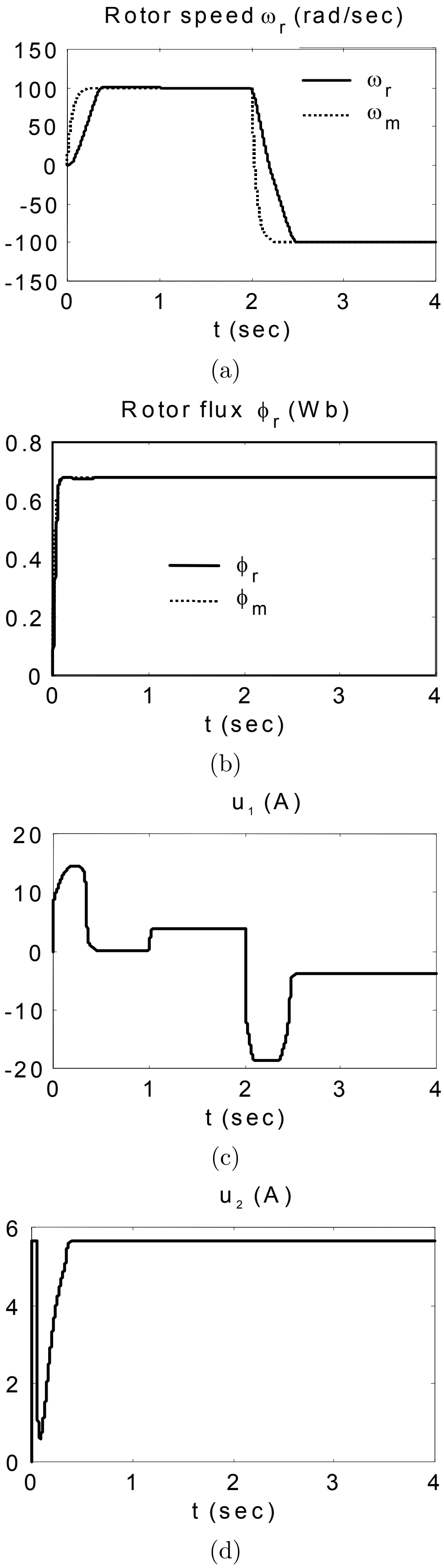

Fig. 5: Speed inversion under load torque change and inertia variation: (a) Speed, (b) Rotor flux, (c) Speed controller input and (d) Flux controller input.

The last test concerns the speed inversion under load torque change and inertia moment variation (the inertia is made four times its nominal value). Fig. 5(a) show that the settling time increases but the speed tracks the reference speed without steady state error. From fig. 5(b) it is clear that the flux response is the same as previously. The control inputs are shown in fig. 5(c-d).

VI. Conclusion

Fuzzy direct adaptive control of nonlinear continuous-time systems, is developed. The stability of this adaptive scheme, under weak requirements on the nonlinear system and the uncertainties, is established. Robustness against approximation error, external disturbance and input gain variation is shown using Lyapunov theory. Simulation results show that good performance is achieved even when the plant parameters change. Major features of this approach are: low computation cost, effective perturbations rejection, and fast tracking performance. Further research are directed to introduce an state observer and the extension of the approach to multivariable nonlinear systems.

References

[1] L. X. Wang, *Adaptive fuzzy systems and control: Design and stability analysis*, Printice Hall, 1994.

[2] C. Y. Su and Y. Stepanenko, Adaptive control of a class of nonlinear systems with fuzzy logic, *IEEE Trans. Fuzzy Syst., vol.* 2, pp. 285-294,1994.

[3] J. F. Spooner and K. M. Passino, Stable adaptive control using fuzzy systems and neural networks, *IEEE Trans. Fuzzy Syst.,* vol. 4, pp. 339-359, 1996.

[4] B.-S. Chen, C.-H. Lee and Y.-C. Chang, H^∞ Tracking design of uncertain nonlinear SISO systems, Adaptive fuzzy approach, *IEEE Trans. Fuzzy Syst.,* vol. 4, pp. 32-43, 1996.

[5] K. M. Passino and S. Yurkovich, *Fuzzy control*, Addison-Wesley, 1998.

[6] K. Fischle and D. Schroder, An improved stable adaptive fuzzy control method, *IEEE Trans. Fuzzy Syst.,* vol. 7, pp. 27-40, 1999.

[7] N. Golea and K. Benmahammed, Iterative construction and optimization of fuzzy models, *Int. J. Appl. Math. & Comp. Sci.*, vol. 9, no.4, pp. 899-919, 1999.

[8] N. Golea, A. Golea and K. Benmahammed, Fuzzy adaptive control: An hyperstability approach, 7th *IEEE Int. Conf. on Electronics, Circuits and Systems*, Kaslik, Lebanon, Dec. 17-20, 2000.

[9] T. Takagi and M. Seguno, Fuzzy identification of systems and its application to controllering and control, *IEEE Trans. Syst., Man, Cybernet., vol.* 15, pp. 116-132, 1985.

[10] H. Ying,, Sufficient conditions on general fuzzy systems as function approximators, *Automatica,* vol. 30, pp. 521-525, 1994.

[11] J. J. Buckly, Seguno type controllers are universal controllers, *Fuzzy Sets Syst.,* vol. 53, pp. 299-303, 1995.

[12] S. Sastry and M. Bosdon, *Adaptive control: Stability, Convergence and Robustness*, Printice-Hall, 1989.

Appendix

Motor parameters	Values
Inertia moment	$J = 0.013$ kgm^2
friction coefficient	$f = 0.002$ Nm/rad/s
Poles pairs	$p = 2$
Rotor inductance	$L_r = 0.121$ H
Mutual inductance	$M = 0.1198$ H
Rotor time constant	$T_r = 0.083$ s
Rated speed	$\omega_n = 152$ rd/s
Rated flux	$\phi_n = 0.67$ Wb