

Constrained Non-linear Model Based Predictive Control using Genetic Algorithms

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Abstract : Model based predictive control (MBPC) is one of the most powerful techniques in process control, however, two main problems need to be considered : obtaining a suitable nonlinear model and an efficient optimization procedure. In this paper, non-linear models are used to predict the plant future response. A genetic algorithm based approach is then used in an MBPC structure to deal with the problem of optimisation which is non convex and thus difficult to solve. The efficiency of this approach had been demonstrated with simulation examples.

1. Introduction

Model based predictive control MBPC was developed in the process industries in the 1960's and 70's, based primarily on heuristic ideas and input-output step and impulse response models[1,2]. The basic principle is to solve an open-loop optimal control problem at each time step. The decision variables are a set of future manipulated variables and the objective function is to minimize deviations from a desired trajectory; constraints on manipulated, state and output variables are naturally handled in this formulation. Feedback is handled by providing a model update at each time, and performing the optimization again[3,4].

The classical MBPC algorithm use linear models of the process to predict the output of the process over the prediction horizon. When no model of the system is available, the classical system identification theory provides possible solutions to the problem, but when the process is non-linear and it is driven over a wide dynamic operating range, the use of linear models becomes impractical, and the use of non-linear models for control becomes a necessity[5]. The success of MBPC is highly dependent on a reliable process model. Since most of the industrial processes exhibit a complex and often non linear behaviour, modelling can take a major part of the design time and costs of a predictive controller. In most of the cases deriving white-box process model based on first principles is a difficult, expensive and time consuming task and so

MBPC must rely on suitable grey-box models or black-box model descriptions of the process.

Various model forms (Wiener models, neural networks, fuzzy models, Voltera models, piece-wise linear models, etc.) have been proposed.

The use of non-linear prediction models in the predictive control scheme results in a non-linear and non convex optimisation problem which must be solved at each control sample. The optimisation problems to be solved on line are generally nonlinear programmes without any redeeming features, which implies that convergence to global optimum cannot be assured[2]. Often the nonlinear optimisation problem is solved by iterative methods such as sequential quadratic programming (SQP), which is computationally very expensive with no guarantee of convergence to a global optimum. Genetic Algorithms (GAs) [5] are potential methods as optimisation techniques for complex problems. The aim of this paper is to use non linear models of the plant in an MBPC strategy and to solve the non-linear constrained optimisation problem by genetic algorithms. The paper is organized as follows. Section 2 provide elementary ground on MBPC. Section 3 describes the concept of non-linear system modelling . Section 4 deals with the use of genetic algorithms to solve constrained optimisation problems in MBPC. Section 5, presents simulation examples to demonstrate the effectiveness of the

proposed approach. Section 6 draws some conclusions from the presented work.

2. Basic elements of model based predictive control

MBPC also known as receding horizon control (RHC) is a general methodology for solving control problems in the time domain. It is based on three main concepts [3,9]:

1. Explicit use of a model to predict the process output.
2. Computation of a sequence of future control actions by minimizing a given objective function.
3. The use of the receding horizon strategy: only the first control action in the sequence is applied, the horizons are moved one sample period towards the future, and optimization is repeated.

Because of the optimization approach and the explicit use of the process model, MBPC can realize multivariable optimal control, deal with nonlinear processes and handle constraints efficiently. The three basic elements of MBPC: (i) a model which describes the process, (ii) a goal, defined by an objective function and constraints (optional), and (iii) an optimization procedure.

The future process outputs are predicted over the prediction horizon H_p using the model of the process : $\hat{y}(k+i)$ for $i=1, \dots, H_p$. These values depend on the current process state, and the future control signal $u(k+i)$ for $i=0, \dots, H_c-1$, where $H_c \leq H_p$ is the control horizon. The control variable is manipulated only within the control horizon and remains constant afterwards.

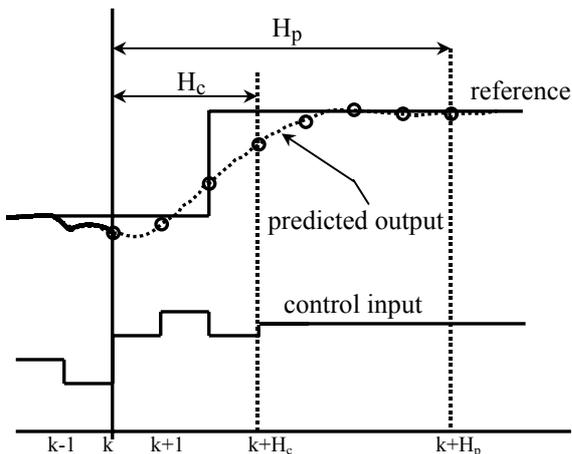


Figure 1. The basic principle of model based predictive control

2.1. Process model

The model must describe the system well and it does not matter what type of model is used to this end: a black-box, a gray-box, or a white-box[9,10]. The future process outputs $\hat{y}(k+i)$ for $i=1, \dots, H_p$, are predicted over the prediction horizon H_p using a model of the process.

2.2. Objective function

The objective function mathematically describes the control goal. In general, good tracking of the reference trajectory is required, with low control energy consumption. These requirements can be expressed by the general form [4]:

$$J = \sum_{i=0}^{H_p} [r(k+i) - y(k+i)] Q [r(k+i) - y(k+i)] + \sum_{i=1}^{H_c} u(k+i) P u^T(k+i) + \sum_{i=0}^{H_p} [\Delta r(k+i) - \Delta y(k+i)] \Delta Q [\Delta r(k+i) - \Delta y(k+i)] + \sum_{i=1}^{H_c} \Delta u(k+i) \Delta P \Delta u^T(k+i) \quad (1)$$

Where $r(k)$ is the reference, P , ΔP , Q and ΔQ are positive definite weight matrices. Level and rate constraints of the control input and/or other process variables can be specified as a part of the optimization problem.

In MBPC equation (1) is usually used in combination with input and output constraints:

$$\begin{aligned} u_{\min} &\leq u \leq u_{\max} \\ \Delta u_{\min} &\leq \Delta u \leq \Delta u_{\max} \\ y_{\min} &\leq y \leq y_{\max} \\ \Delta y_{\min} &\leq \Delta y \leq \Delta y_{\max} \end{aligned} \quad (2)$$

Other constraints can be implemented in a straightforward way, e.g. state constraints for state space models [2].

2.3 Optimisation

Model predictive control requires an optimization procedure by which a sequence of optimal control signals can be found at each step.

Linear MBPC problem with constraints form a convex optimization problem, that

can be efficiently solved by numerical methods[2].

In the presence of nonlinearities and constraints, a non-convex optimization problem must be solved at each sampling period. This hampers the application of nonlinear MBPC to fast systems where iterative optimization techniques cannot be properly used, due to short sampling periods and extensive computation times[9].

Moreover, iterative optimization algorithms, such as the Nelder-Mead method, the multi-step Newton-type algorithm[11], or sequential quadratic programming(SQP)[12], usually converge to local minima, which results in poor solutions of the optimization problem. For efficiency many vendors use heuristic methods, for example, by using dynamic matrices[2].

In this paper, a genetic algorithm based approach is used to solve the MBPC constrained optimisation problem.

3. Non linear modelling

Unlike the linear case, there is no established method to construct a non linear model through a plant tests. Recognition of the need has made empirical modelling of non-linear systems a focal research topic within the process control community.

Most non-linear empirical models studied in the literature fit into the following form:

$$y(k) = F(\phi(k), \theta) + \varepsilon(k)$$

ϕ and y are all real vectors of dimensions n_ϕ and n_y respectively. F is a function that maps ϕ to the output vector y . The transformation is usually estimated in a parameterised form (in terms of the vector denoted by θ). $\varepsilon(k)$ is the residual error sequence.

Before the application of a parameter estimation method the user must first make the following choices:

Input Vector : One has to decide which variables to include in ϕ as well as how many. This is critical as different choices

lead to models with fundamentally different characteristics.

$$\phi(k) = [y^T(k-1), \dots, y^T(k-m_y),$$

$$u^T(k-1), \dots, u^T(k-m_u)]^T$$

gives a Non-linear Auto Regressive model with eXogenous input (NARX).

Once the model is decided the number of lagged inputs/outputs (m_y, m_u) should be determined. Available methods for determining these parameters include: the False Nearest Neighbourhood (FNN) method, Akaike Information Criteria (AIC) and its related concepts and cross-validation. The FNN method has the advantage over the others that the order selection can be carried out independently of finding the non linear operator.

F may be parameterised as a series sum a of a priori chosen basis functions :

$$y(k) = \sum_{i=1}^M w_i f_i(\phi(k))$$

Basis functions can be classified into global functions (e.g. polynomials) and local functions (e.g. radial basis and fuzzy systems).

4. Optimisation

Genetic Algorithms (GAs) as an optimization method have been lately applied as an alternative to classical optimization methods. Their ability to find the optimum of functions where classical methods have difficulties (e.g. non derivative functions), is one of the most properties of this technique. In this paper, a genetic algorithm is used to solve the MBPC optimization problem. The algorithm is derived from the steady-state GA and utilizes floating point encoding. A fitness function of the optimizer is defined by the objective function of the model predictive control formulation.

4.1. Encoding

Every individual $\{p_i ; i = 1, \dots, N_{pop}\}$ in the population of a genetic algorithm determines a control sequence:

$$p_i = \{u_i(k), u_i(k+1), \dots, u_i(k+H_c-1)\} \quad (6)$$

the elements of which are represented as floating point numbers. An individual p_i is described by a set of H_c numbers which are selected within the admissible interval $[u_{\min}, u_{\max}]$ with absolute differences $\{\Delta u_i(k+j); j=1, \dots, H_c-1\}$ not exceeding the prescribed value Δu_{\max} .

4.2 Initialization

In order to provide for faster convergence of the genetic algorithm, suitable initialization procedure should be specified. In this paper we combine random initialisation with the interevolution steady-state principle :

Randomly Initialization : Random control sequences are generated in accordance with the constraints presented in Eq.(2).

Inter-evolution exchange : The best solutions of the last optimization cycle are used in the next period.

4.2 Termination conditions

The termination function is used to determine when the optimization loop should be finished. Selection of a fixed number of generations is not very suitable because evolution may converge earlier. Therefore we introduce a new convergence measure to determine the termination condition. Deviations of all signals of the best individual in the population are scanned for the last N_{conv} generations. The termination condition is fulfilled when either the relative maximum deviation becomes smaller than a prescribed value or the maximum number of generations N_{gen} is exceeded.

4.3 Constraints handling

Manipulated variables (MVs) Constraints are directly handled in the AG reproduction procedure. Each individual p_i is described by a set of H_c numbers which are selected within the admissible interval $[u_{\min}, u_{\max}]$ with absolute differences $\{\Delta u_i(k+j); j=1, \dots, H_c-1\}$ not exceeding the prescribed value $\Delta u_{i \min}$ and $\Delta u_{i \max}$.

Controlled variables (CVs) constraints are handled by penalizing infeasible individuals[14]. The fitness function is modified and the violation of constraints is

specified by penalties. The modified fitness function for an individual p is evaluated by :

$$eval(p) = f(p) + Q(p) \quad (7)$$

where $f(p)$ is the fitness function without constraints and $Q(p)$ is a penalty function corresponding to constraints violation. The value of $Q(p)$ is proportional to the amplitude and the time of the constraint violation.

5. Simulation

Example 1 : Consider the non-linear discrete system described by the equation :

$$y(k+1) = \frac{y(k)}{1+y^2(k)} + u(k) \quad (8)$$

A neural model is obtained using input/output data sets generated by random values of $u(k) \in [-1.0, 1.0]$. The model is a feedforward neural network with three layers : one input layer, one hidden layer and one output layer. The activation function of the three hidden units is the sigmoid. The activation function of the output node is linear. The model has two inputs $y(k)$ and $u(k)$ and one output $y(k+1)$.

Levenberg-Marquardt algorithm is used to train the neural model using the input output data generated randomly. The structure of the neural model is represented in Figure 2.

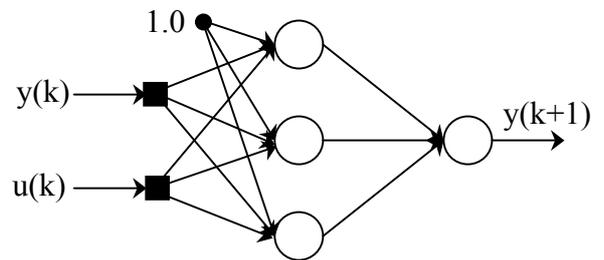


Figure 2. The Neural Model

The goal of the predictive control is to generate suitable sequence of actions $u(k) \in [-1.0, 1.0]$ so to minimize the objective function given by equation (1) where the reference signal is : $r(k) = 0.5$ for $k=1, \dots, 50$;

$r(k)=-0.2$ for $k=51,\dots,100$ and $r(k) = 0.2$ for $k=101,\dots,200$.

The constraints are :

$$\begin{aligned} -1.0 \leq u(k) \leq 1.0 \\ -1.0 \leq y(k) \leq 1.0 \end{aligned} \quad (8)$$

The prediction horizon $H_p=4$ and the control horizon is $H_c = 2$. The weight matrices in equation (1) are $P = 1.0$, $Q = 1.0$ $\Delta P=0$ and $\Delta Q = 0$.

Figure 3 represents the system output and the reference, the corresponding control input is represented in figure 4.

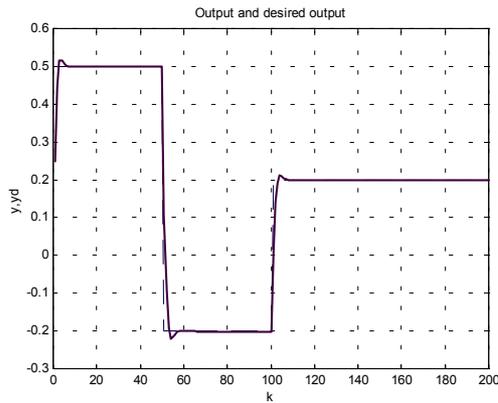


Figure 3. System output (dashed line) and the desired response (solid line)

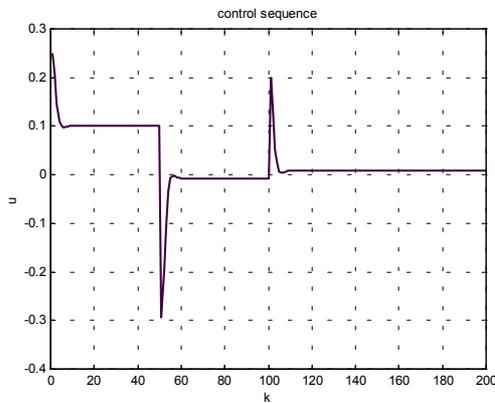


Figure 4. Control sequence

Example 2 :

let us consider an exothermic continuous stirred tank reactor (CSTR) described by the following differential equations [15]:

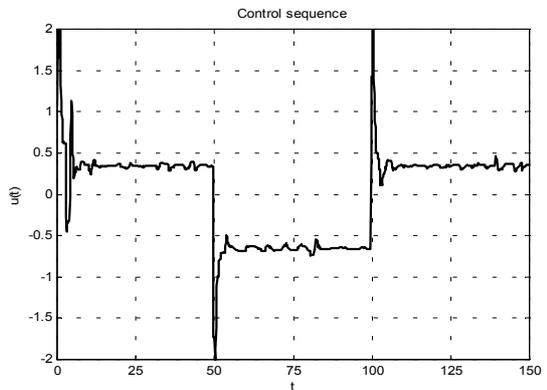
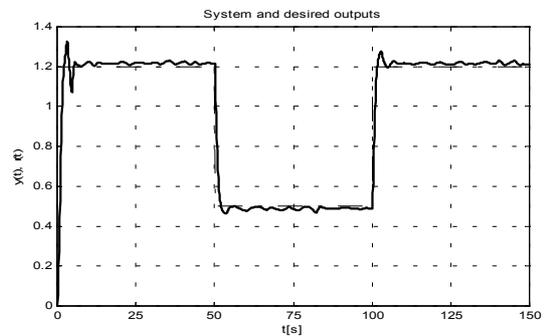
$$\begin{cases} \frac{dx_1(t)}{dt} = -x_1(t) + D_a(1-x_1(t)) \exp\left(\frac{x_2(t)}{1+\frac{x_2(t)}{\phi}}\right) \\ \frac{dx_2(t)}{dt} = -x_2(t) + B D_a(1-x_1(t)) \exp\left(\frac{x_2(t)}{1+\frac{x_2(t)}{\phi}}\right) + \beta(u(t-\tau) - x_2(t)) \end{cases}$$

Where x_1 and x_2 represent dimensionless reactant conversion and temperature, u is the coolant temperature which is used as a manipulated variable. The parameters in the process are : $B=8$, $Da=0.072$, $\phi=20$, $\beta=0.3$ and $\tau=0.4$. The sampling time is 0.1. The process output is the reactor temperature x_2 . The purpose of the control is to keep the temperature to track the reference setpoint.

A sequence of random steps with amplitude between $[-1,1]$ is used to excite the process. Then the produced data are employed for identification. A fuzzy model with four inputs ($y(k-1), y(k-2), u(k-5), u(k-6)$) and one output $y(k)$ is constructed to model the process. Fuzzy model membership functions are obtained by the Gustafson-Kessel[16] clustering algorithm and the consequent parameters are derived with a least squares algorithm. The obtained model is a collection of seven rules of the form:

If $y(t-1)$ est W_{1i} et $y(t-2)$ W_{1i} est et $u(t-5)$ W_{1i} est et $u(t-6)$ est W_{1i} alors $y(t) = C_{i0} + C_{i1}y(t-1) + C_{i2}y(t-2) + C_{i3}u(t-5) + C_{i4}u(t-6)$

The plant response is represented in figure and the control sequence in figure



6. Conclusions

A non-linear model based predictive control strategy based on algorithms had been presented. This strategy is a very efficient non-linear model based predictive control approach.

Future work should be done to improve the computation time of the optimiser by choosing special operators to enhance the convergence of the genetic algorithm. A combination with iterative methods may decrease the computational time and avoid the convergence to local minima.

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