T–S Fuzzy Discrete Systems Stabilization using Switching PDC Controller

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Abstract. In this paper a new stability condition is derived for discrete T-S fuzzy systems. The discrete T–S fuzzy system is represented by a hybrid system, each hybrid system state corresponds to a group of rules that can be fired in the same time. The discrete T-S fuzzy system stability test consists to find a set of symmetric positive definite matrices that ensure stability in local regions and verify the global stability during transition between subregions. This idea is used to stabilize discrete T-S fuzzy systems by the use of a local linear state feedback or a local PDC controller in each subregion.

Key words: Discrete Takagi-Sugeno fuzzy model, stability, PDC, LMI.

Résumé. Dans ce papier, une nouvelle condition de stabilité des systèmes flous discrets de type Takagi–Sugeno est proposée. Le système flou discret de Takagi–Sugeno est représenté par un système hybride, chaque état du système hybride correspond à un groupe de règles qui peuvent etre actives en meme temps. La verification de la stabilite consiste a trouver un ensemble de matrices symetriques definies positives qui assurent la stabilite locale et verifient la stabilite globale pendant les transitions entre les differentes regions. Cette idee est ensuite utilisee pour la stabilisation des systmes flous discrets par l'utilisation de lois de commande lineaires locales ou des lois de commandes locales de type PDC.

Mots-clés: Modèle flou T–S discret,, stabilité, PDC,LMI

1 Introduction

During the last few years, the analysis and design of fuzzy logic controllers based on the Takagi-Sugeno fuzzy model have been a popular research topic in control community . Tanaka et al. discussed the stability and the design of fuzzy control systems in [1, 2]. They gave some checking conditions for stability, which can be used to design fuzzy control laws. Unfortunately, the stability conditions require the existence of a common positive definite matrix for all the local linear models. Linear matrix inequalities (LMI) tools have been used to find the common matrix. However, this is a difficult problem to be solved in many cases, especially when the number of rules is large. Several methods have been proposed to relax the stability conditions. The existence of a common matrix means that the stability is independent of the rules that can be active simultaneously. However, at each time, only few rules are fired and the unfired rules are not necessary to be considered in local regions. Instead of analyzing the stability of the whole polyhedron composed by system matrices we can analyze the stability by considering the global polyhedron as a union of small polyhedrons composed by rules that may be inferred in the same time. The fuzzy system rule set can be decomposed into several subsets, each subset contains a group of rules that may be inferred in the same time. This is equivalent to a state space partition into several subregions, each subregion corresponds to a rule subset and the discrete fuzzy system can be modeled by a hybrid system. Each hybrid system state corresponds to a group of rules. Local common matrices are used to guarantee the local stability of each rule subset. Additional relations between different local matrices are used to ensure the global stability of the fuzzy system. Based on this approach, two fuzzy regulator design methods are proposed. The rest of the paper is organized as follows. Section 2 introduces the T-S discrete time fuzzy model and reviews the existing stability conditions. Section 3 presents the proposed relaxed method. In section 4, based on this relaxation method two types of fuzzy regulators are proposed. Finally, conclusions are given in section 4.

2 Takagi-Sugeno fuzzy model

The Takagi-Sugeno fuzzy dynamic model is a piecewise interpolation of several linear models through membership functions. The discrete T-S fuzzy model is a set of fuzzy if-then rules. The ith rule of a discrete time fuzzy model take the form:

Rule i:

$$\text{If } z_1(k) \text{ is } F_1^i, \cdots, \text{ and } z_g(k) \text{ is } F_g^i \text{ Then } \begin{cases} \mathbf{x}(k+1) = \mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}_i\mathbf{x}(k) \end{cases} \tag{1}$$

where $\mathbf{x}(k) \in \mathbb{R}^n$ denotes the state vector, $\mathbf{u}(k) \in \mathbb{R}^m$ the control vector, $\mathbf{y}(k) \in \mathbb{R}^p$ the output vector, $F_j^i,\ j = 1, 2, \dots, g$ are fuzzy sets, $\mathbf{A}_i \in \mathbb{R}^{n\times n}, \mathbf{B}_i \in \mathbb{R}^{n\times m}$ and $\mathbf{C}_i \in \mathbb{R}^{p\times n}$ are the state matrix, the input matrix and the output matrix for the ith local model, r is the number of if-then rules, and $z_1(t), z_2(t) \cdots, z_g(t)$ are some measurable system variables; the premise variable. It is assumed that premise variables do not depend on control variables. By using a center-average defuzzifer, product inference and singleton fuzzifier, the discrete time fuzzy system can be expressed by:

$$\begin{cases} \mathbf{x}(k+1) = \sum\limits_{i=1}^{r} \alpha_i(\mathbf{z}(k))\{\mathbf{A}_i\mathbf{x}(k) + \mathbf{B}_i\mathbf{u}(k)\} \\ \mathbf{y}(k) = \sum\limits_{i=1}^{r} \alpha_i(\mathbf{z}(k))\mathbf{C}_i\mathbf{x}(k) \end{cases} \tag{2}$$

Where

$$\alpha_i(\mathbf{z}(k)) = \frac{\omega_i(\mathbf{z}(k))}{\sum\limits_{i=1}^{r} \omega_i(\mathbf{z}(k))} \tag{3}$$

The scalars $\alpha_i(\mathbf{z}(t))$ are characterized by:

$$0 \leq \alpha_i(\mathbf{z}(k)) \leq 1 \text{ and } \quad \sum_{i=1}^{r} \alpha_i(\mathbf{z}(k)) = 1 \tag{4}$$

3 Stability analysis

The discrete fuzzy system (2) without input can be written as :

$$\mathbf{x}(k+1) = \sum_{i=1}^{r} \alpha_i(\mathbf{z}(k))\mathbf{A}_i\mathbf{x}(k) \tag{5}$$

The stability condition of the unforced discrete fuzzy system (2) can be formulated by the following theorems

Theorem 1. *[3] The equilibrium of the discrete fuzzy system (5) is globally asymptotically stable if there exists a common symmetric positive definite matrix* $\mathbf{P}$ *such that:*

$$\mathbf{A}_i^T\mathbf{P}\mathbf{A}_i - \mathbf{P} < 0, \qquad i = 1, 2, \ldots, r \tag{6}$$

Theorem 2. *If there exist symmetric positive definite matrices* $\mathbf{P}_i,\ i = 1, ..., r$ *such that:*

$$\mathbf{A}_k^T\mathbf{P}_i\mathbf{A}_k - \mathbf{P}_j < 0, \qquad i, j, k = 1, ..., r \tag{7}$$

then the discrete time fuzzy system (5) is globally asymptotically stable.

To guarantee the stability of the unforced discrete fuzzy system(5) the most used approach is to find a common matrix $\mathbf{P}$ that satisfy r inequalities (6). LMI methods are always used to find the common matrix. The existence of a common matrix means that the stability of the discrete fuzzy system is independent of the fired rules at each time, so we can add any combination of the initial rules to the fuzzy system without affecting its stability, and this is the origin of the conservativeness of this method. The first theorem is obtained by considering the stability of the convex hull of all subsystem matrices $\mathbf{A}_i,\ i = 1, \ldots, r$, this is true if all rules may be inferred in the same time. However, in general, only few rules are fired in the same time, particularly when the number of rules is large. The stability condition can be relaxed by considering the stability of the union of smaller convex hulls. The original convex hull can be partitioned into several subregions, each subregion corresponds to a combination of rules that may be fired in the same time. The discrete fuzzy system rules $R_i, i = 1, \cdots, r$ can be divided into several subsets containing rules that may be inferred in the same time. Let $\mathcal{R} = \{R_1, R_2, \cdots, R_r\}$ the T-S fuzzy system rule set. It can be divided into N subsets:

$$\mathcal{R} = \bigcup_{i=1}^{N} \mathcal{R}_i \tag{8}$$

where $\mathcal{R}_i$ is a rule subset containing rules that may be active simultaneously. The fuzzy system can be modeled by a hybrid system in which each state corresponds to a rule subset.

Example 1. Consider the fuzzy system composed by four rules $\mathcal{R} = \{R_1, R_2, R_3, R_4\}$, the membership functions are shown in figure 1. Rules R_1 and R_3 can not be fired in the same time. The rule set can be divided into 3 subsets; $\mathcal{R}_1 = \{R_1, R_2\}$, $\mathcal{R}_2 = \{R_1, R_3\}$ and $\mathcal{R}_3 = \{R_3, R_4\}$

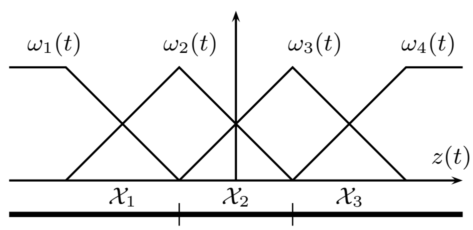

Fig. 1. Membership functions

The discrete fuzzy system can be modeled by a three states hybrid system as shown in figure 2.

The partition of the rule set can be seen as a partition of the state space $\mathcal{X} \subseteq \mathbb{R}^n$ into N subspaces and the fuzzy system can be modeled by a hybrid system with N states.
The T–S fuzzy fuzzy model (2) can be written as:

$$\begin{cases} \mathbf{x}(k+1) = \mathcal{A}_{\mathcal{R}_l}\mathbf{x}(k) + \mathcal{B}_{\mathcal{R}_l}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathcal{C}_{\mathcal{R}_l}\mathbf{x}(k) \end{cases} \quad l = 1, ..., N \quad \mathbf{x}(k) \in \mathcal{X}_l \tag{9}$$

with :

$$\mathcal{A}_{\mathcal{R}_l} = \sum_{R_i \in \mathcal{R}_l} \alpha_i(\mathbf{z}(k))\mathbf{A}_i, \ \mathcal{B}_{\mathcal{R}_l} = \sum_{R_i \in \mathcal{R}_l} \alpha_i(\mathbf{z}(k))\mathbf{B}_i, \mathcal{C}_{\mathcal{R}_l} = \sum_{R_i \in \mathcal{R}_l} \alpha_i(\mathbf{z}(k))\mathbf{C}_i \tag{10}$$

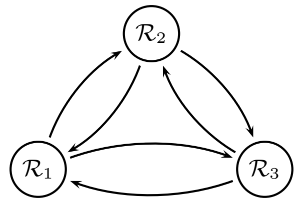

Fig. 2. Hybrid system corresponding to the T-S fuzzy model

and

$$\sum_{R_i \in \mathcal{R}_l} \alpha_i(\mathbf{z}(k)) = 1 \tag{11}$$

The autonomous discrete fuzzy system (5) can be written as:

$$\mathbf{x}(k+1) = \sum_{R_i \in \mathcal{R}_l} \alpha_i(\mathbf{z}(k))\mathbf{A}_i\mathbf{x}(k), \quad \mathbf{x}(k) \in \mathcal{X}_l \tag{12}$$

The following lemma will be used in the proof of relaxed stability theorem.

Lemma 1. *If symmetric positive definite matrices* $\mathbf{P}_i$, $\mathbf{P}_j$ *and matrices* $\mathbf{A}$ *and* $\mathbf{B}$ *of appropriate dimension are such that:*

$$\mathbf{A}^T\mathbf{P}_i\mathbf{A} - \mathbf{P}_j < 0 \text{ and } \mathbf{B}^T\mathbf{P}_i\mathbf{B} - \mathbf{P}_j < 0 \tag{13}$$

then

$$\mathbf{A}^T\mathbf{P}_i\mathbf{B} + \mathbf{B}^T\mathbf{P}_i\mathbf{A} - 2\mathbf{P}_j < 0 \tag{14}$$

Theorem 3. *If there exists symmetric positive definite matrices* $\mathbf{P}_i, i = 1, 2, \ldots, N$ *such that the following LMIs are satisfied:*

$$\begin{aligned} &\mathbf{P}_i = \mathbf{P}_i^T > 0 \\ &\mathbf{A}_k^T\mathbf{P}_j\mathbf{A}_k - \mathbf{P}_i < 0 \\ &i, j = 1, \ldots, N, \; R_k \in \mathcal{R}_i \end{aligned} \tag{15}$$

then the discrete time fuzzy system (5) is globally asymptotically stable.

Proof. Let the discrete time Lyapunov function candidate be defined as:

$$V(k) = \mathbf{x}^T(k)\mathbf{P}_i\mathbf{x}(k), \quad \mathbf{x}(k) \in \mathcal{X}_i, \; i = 1, 2, \ldots, N \tag{16}$$

At time k, $\mathbf{x}(k) \in \mathcal{X}_i$, only the rules of the subset $\mathcal{R}_i$ are fired. At time $k+1$ the state moves from subregion $\mathcal{X}_i$ to subregion $\mathcal{X}_j$, It is possible that the state stays in the same subregion. To guarantee the stability we must have:

$$\Delta V(k) = V(k+1) - V(k) < 0 \tag{17}$$

$$\begin{aligned}
\Delta V(k) =& V(k+1) - V(k) \\
=& \mathbf{x}^T(k+1)\mathbf{P}_j\mathbf{x}(k+1) - \mathbf{x}^T(k)\mathbf{P}_i\mathbf{x}(k) \\
=& \sum_{R_l \in \mathcal{R}_i} \alpha_l^2(\cdot)\mathbf{x}^T(k)\mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_l\mathbf{x}(k) \\
&+ \sum_{R_l \in \mathcal{R}_i} \sum_{\substack{R_m \in \mathcal{R}_i \\ l \neq m}} \alpha_l(\cdot)\alpha_m(\cdot)\mathbf{x}^T(k)\mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_m\mathbf{x}(k) - \mathbf{x}^T(k)\mathbf{P}_i\mathbf{x}(k) \\
=& \sum_{R_l \in \mathcal{R}_i} \alpha_l^2(\cdot)\mathbf{x}^T(k)(\mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_l - \mathbf{P}_i)\mathbf{x}(k) \\
&+ \sum_{R_l \in \mathcal{R}_i} \sum_{\substack{R_m \in \mathcal{R}_i \\ l \neq m}} \alpha_l(\cdot)\alpha_m(\cdot)\mathbf{x}^T(k)\left(\mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_m - \mathbf{P}_i\right)\mathbf{x}(k) \\
=& \sum_{l \in \mathcal{R}_i} \alpha_l^2(\cdot)\mathbf{x}^T(k)(\mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_l - \mathbf{P}_i)\mathbf{x}(k) \\
&+ \frac{1}{2} \sum_{R_l \in \mathcal{R}_i} \sum_{\substack{R_m \in \mathcal{R}_i \\ l < m}} \alpha_l(\cdot)\alpha_m(\cdot)\mathbf{x}^T(k)\left(\mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_m + \mathbf{A}_m^T\mathbf{P}_j\mathbf{A}_l - 2\mathbf{P}_i\right)\mathbf{x}(k) \\
\Delta V(k) \leq& \sum_{R_l \in \mathcal{R}_i} \alpha_l^2(\cdot)\mathbf{x}^T(k)(\mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_l - \mathbf{P}_i)\mathbf{x}(k) \\
& \mathbf{A}_l^T\mathbf{P}_j\mathbf{A}_l - \mathbf{P}_i < 0 \Rightarrow \Delta V(k) < 0
\end{aligned}$$

Remark 1. The number of LMIs N_{LMI} depends on the number of subsets N and the number of elements in each rule subset $card(\mathcal{R}_i)$:

$$N_{LMI} = N\sum_{i=1}^{N} card(\mathcal{R}_i) \tag{18}$$

- If $N = 1$, the number of LMIs is equal to r, and this theorem is the same as theorem 1.
- If $N = r$, the number of subsets is the same as the number of rules, each rule subset contains only a single rule, the number of LMIs is equal to r^2 and this theorem is the same as theorem 2.

Remark 2. The number of inequalities can be reduced by the elimination LMIs corresponding to impossible transitions.

3.1 LMIs reduction

We define x_i^{min} and x_i^{max} as the minimal norm value and the maximal norm value of states belonging to the subspace $\mathcal{X}_i$:

$$x_i^{min} = \min_{\mathbf{x}(k)\in\mathcal{X}_i} \|\mathbf{x}(k)\|,\ x_i^{max} = \max_{\mathbf{x}(k)\in\mathcal{X}_i} \|\mathbf{x}(k)\|, \tag{19}$$

and d_{ij} as the distance between the subregions $\mathcal{X}_i$ and $\mathcal{X}_j$ given by:

$$d_{ij} = \min_{\mathbf{x}^i(k)\in\mathcal{X}_i, \mathbf{x}^j(k)\in\mathcal{X}_j} \|\mathbf{x}^j(k) - \mathbf{x}^i(k)\| \tag{20}$$

Lemma 2. *If*

$$x_j^{min} > \underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i\| x_i^{max} \quad (21)$$

then the state can't move from the subregion $\mathcal{X}_i$ to the subregion $\mathcal{X}_j$.

Proof. We have:

$$\mathbf{x}(k+1) = \sum_{R_i \in \mathcal{R}_l} \alpha_i(\mathbf{z}(k))\mathbf{A}_i\mathbf{x}(k) \quad (22)$$

$$\mathbf{x}(k) \in \Omega_i \Rightarrow \|\mathbf{x}(k+1)\| \leq \underset{R_l \in \mathcal{R}_i}{max} \|\mathbf{A}_l\| \|\mathbf{x}(k)\| \quad (23)$$

The state can reach the subregion $\mathcal{X}_j$ from the subregion $\mathcal{X}_i$ if :

$$\mathbf{x}(k) \in \mathcal{X}_i \text{ and } x_j^{min} \leq \|\mathbf{x}(k+1)\| \leq x_j^{max} \quad (24)$$

$$\mathbf{x}(k) \in \mathcal{X}_i \Rightarrow x_i^{min} \leq \|\mathbf{x}(k)\| \leq x_i^{max} \Rightarrow \\ \|\mathbf{x}(k+1)\| \leq \underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i\| x_i^{max} \quad (25)$$

then the transition from subregion $\mathcal{X}_i$ to subregion $\mathcal{X}_j$ is possible if:

$$x_j^{min} \leq \underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i\| x_i^{max} \quad (26)$$

Lemma 3. *If*

$$\underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i - \mathbf{I}\| x_i^{max} < d_{ij} \quad (27)$$

then the state can't move from the subregion $\mathcal{X}_i$ to the subregion $\mathcal{X}_j$.

Proof. The state can move from subregion $\mathcal{X}_i$ to subregion $\mathcal{X}_j$ if:

$$\|\Delta\mathbf{x}(k)\| = \|\mathbf{x}(k+1) - \mathbf{x}(k)\| \geq d_{ij}$$

$$\|\Delta\mathbf{x}(k)\| \leq \left\| \sum_{R_i \in \mathcal{R}_l} \alpha_i(\mathbf{z}(k))\mathbf{A}_i - \mathbf{I} \right\| \|\mathbf{x}(k)\| \Rightarrow \|\Delta\mathbf{x}(k)\| \leq \underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i - \mathbf{I}\| \|\mathbf{x}(k)\|$$

$$\|\Delta\mathbf{x}(k)\| \geq d_{ij} \Rightarrow \underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i - \mathbf{I}\| \|\mathbf{x}(k)\| \geq d_{ij}$$

$$\underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i - \mathbf{I}\| \|\mathbf{x}(k)\| \geq d_{ij} \Rightarrow \underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i - \mathbf{I}\| x_i^{max} \geq d_{ij}$$

A transition from $\mathcal{X}_i$ to $\mathcal{X}_j$ is possible if:

$$\underset{R_i \in \mathcal{R}_l}{max} \|\mathbf{A}_i - \mathbf{I}\| x_i^{max} \geq d_{ij}$$

Example 2. consider the following free fuzzy system:

$$R^i: \text{ If } x_1(k) \text{ is } F_1^i \text{ and } x_2(k) \text{ is } F_2^i \text{ Then } \mathbf{x}(k+1) = \mathbf{A}_1\mathbf{x}(k), \quad i = 1, ..., 16$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0.80 & 0.25 \\ 0.10 & 0.76 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0.79 & 0.25 \\ 0.10 & 0.78 \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} 0.81 & 0.25 \\ 0.10 & 0.77 \end{bmatrix},$$

$$\mathbf{A}_4 = \begin{bmatrix} 0.79 & 0.24 \\ 0.1 & 0.76 \end{bmatrix}, \quad \mathbf{A}_5 = \begin{bmatrix} 0.78 & 0.10 \\ 0.20 & 0.77 \end{bmatrix}, \quad \mathbf{A}_6 = \begin{bmatrix} 0.82 & 0.30 \\ 0.08 & 0.77 \end{bmatrix},$$

$$\mathbf{A}_7 = \begin{bmatrix} 0.80 & 0.25 \\ 0.10 & 0.81 \end{bmatrix}, \quad \mathbf{A}_8 = \begin{bmatrix} 0.9 & -0.15 \\ 0.10 & 0.65 \end{bmatrix}, \quad \mathbf{A}_9 = \begin{bmatrix} 0.9 & 0.52 \\ -0.10 & 0.66 \end{bmatrix},$$

$$\mathbf{A}_{10} = \begin{bmatrix} 0.81 & 0.10 \\ 0.10 & 0.80 \end{bmatrix}, \quad \mathbf{A}_{11} = \begin{bmatrix} 0.82 & 0.11 \\ 0.09 & 0.83 \end{bmatrix}, \quad \mathbf{A}_{12} = \begin{bmatrix} 0.77 & 0.05 \\ 0.23 & 0.60 \end{bmatrix}$$

$$\mathbf{A}_{13} = \begin{bmatrix} 0.78 & 0.10 \\ 0.13 & 0.60 \end{bmatrix}, \quad \mathbf{A}_{14} = \begin{bmatrix} 0.75 & 0.16 \\ 0.25 & 0.78 \end{bmatrix}, \quad \mathbf{A}_{15} = \begin{bmatrix} 0.78 & 0.12 \\ 0.28 & 0.76 \end{bmatrix},$$

$$\mathbf{A}_{16} = \begin{bmatrix} 0.76 & 0.14 \\ 0.30 & 0.76 \end{bmatrix}$$

The membership functions are shown in figure 3. The system is stable as shown by its phase portrait in figure 5, but a common symmetric positive definite matrix can't be found.

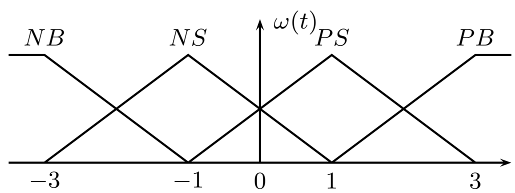

Fig. 3. Membership functions

The fuzzy system rule set $\mathcal{R} = \{R_i,\ i = 1, ..., 16\}$ can be divided into nine subsets : $\mathcal{R}_1 = \{R_1, R_2, R_5, R_6\}$, $\mathcal{R}_2 = \{R_2, R_3, R_6, R_7\}$, $\mathcal{R}_3 = \{R_3, R_4, R_7, R_8\}$, $\mathcal{R}_4 = \{R_5, R_6, R_9, R_{10}\}$, $\mathcal{R}_5 = \{R_6, R_7, R_{10}, R_{11}\}$, $\mathcal{R}_6 = \{R_7, R_8, R_{11}, R_{12}\}$, $\mathcal{R}_7 = \{R_9, R_{10}, R_{13}, R_{14}\}$, $\mathcal{R}_8 = \{R_{10}, R_{11}, R_{14}, R_{15}\}$, and $\mathcal{R}_4 = \{R_{11}, R_{12}, R_{15}, R_{16}\}$.
By using MATLAB LMI toolbox we found the nine local matrices:

$$\mathbf{P}_1 = \begin{bmatrix} 6.641 & 1.4032 \\ 1.4032 & 9.2465 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 6.5884 & 1.4839 \\ 1.4839 & 9.2418 \end{bmatrix}, \quad \mathbf{P}_3 = \begin{bmatrix} 6.8266 & 1.1464 \\ 1.1464 & 9.4444 \end{bmatrix},$$

$$\mathbf{P}_4 = \begin{bmatrix} 6.5652 & 1.7825 \\ 1.7825 & 8.9617 \end{bmatrix}, \quad \mathbf{P}_5 = \begin{bmatrix} 6.6568 & 1.3671 \\ 1.3671 & 9.3240 \end{bmatrix}, \quad \mathbf{P}_6 = \begin{bmatrix} 6.8599 & 1.0682 \\ 1.0682 & 9.4586 \end{bmatrix},$$

$$\mathbf{P}_7 = \begin{bmatrix} 6.5635 & 1.7886 \\ 1.7886 & 8.9501 \end{bmatrix}, \quad \mathbf{P}_8 = \begin{bmatrix} 6.6243 & 1.7826 \\ 1.7826 & 8.8084 \end{bmatrix}, \quad \mathbf{P}_9 = \begin{bmatrix} 6.1343 & 2.7877 \\ 2.7877 & 7.2537 \end{bmatrix}$$

By applying lemma 2 and lemma 3 the number of LMIs can be reduced from $81 * 4 = 324$ to $64 * 4 = 256$. Possible transitions are shown in table 1.

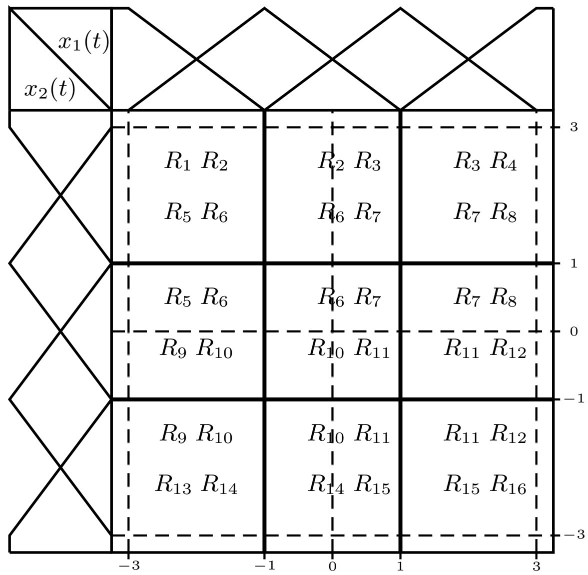

Fig. 4. State space partition

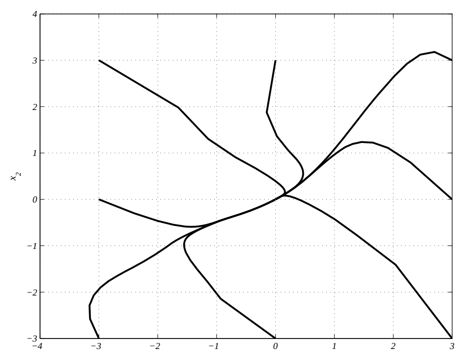

Fig. 5. Phase portrait

	$\mathcal{X}_1$	$\mathcal{X}_2$	$\mathcal{X}_3$	$\mathcal{X}_4$	$\mathcal{X}_5$	$\mathcal{X}_6$	$\mathcal{X}_7$	$\mathcal{X}_8$	$\mathcal{X}_9$
$\mathcal{X}_1$	1	1	1	1	1	1	1	1	0
$\mathcal{X}_2$	1	1	1	1	1	1	0	0	0
$\mathcal{X}_3$	1	1	1	1	1	1	0	1	1
$\mathcal{X}_4$	1	1	1	1	1	1	1	1	1
$\mathcal{X}_5$	0	0	0	0	1	0	0	0	0
$\mathcal{X}_6$	1	1	1	1	1	1	1	1	1
$\mathcal{X}_7$	1	1	1	1	1	1	1	1	1
$\mathcal{X}_8$	0	0	0	1	1	1	1	1	1
$\mathcal{X}_9$	0	1	1	1	1	1	1	1	1

Table 1. Possible transitions (0 : impossible, 1 : possible)

4 Stabilization by state feedback

4.1 Stabilisation using local linear state feedback

We assume that the discrete fuzzy system (2) is locally controllable, that is, the pairs $(\mathbf{A}_l, \mathbf{B}_l)$, $l = 1, \ldots, r$, are controllable. The basic idea is to design a local feedback controller for each subregion in the state space and the control law is given by:

$$\mathbf{u}(k) = \mathbf{F}_i \mathbf{x}(k), \quad \mathbf{x}(k) \in \mathcal{X}_i,\ i = 1, 2, \ldots, N \tag{28}$$

Theorem 4. *If there exists symmetric positive definite matrices $\mathbf{P}_i$ and gain matrices $\mathbf{F}_i, i = 1, 2, \ldots, N$ such that the following inequalities hold:*

$$\begin{aligned} &\mathbf{P}_i = \mathbf{P}_i^T > 0 \\ &(\mathbf{A}_k + \mathbf{B}_k \mathbf{F}_i)^T \mathbf{P}_j (\mathbf{A}_k + \mathbf{B}_k \mathbf{F}_i) - \mathbf{P}_i < 0 \\ &\textit{for} \quad i, j = 1, \ldots, N,\ k \in \mathcal{R}_i \end{aligned} \tag{29}$$

Then the closed loop fuzzy system is globally asymptotically stable.

Proof. It follows directly from Theorem 1

The design problem to determine the feedback gains can be reformulated as an LMI problem:
Find $\mathbf{X}_i > 0$ and $\mathbf{M}_i \quad (i = 1, ..., N)$ satisfying:

$$\begin{aligned} &\begin{bmatrix} \mathbf{X}_i & \mathbf{X}_i \mathbf{A}_k^T + \mathbf{M}_i^T \mathbf{B}_k^T \\ \mathbf{A}_k \mathbf{X}_i + \mathbf{B}_k \mathbf{M}_i & \mathbf{X}_j \end{bmatrix} > 0 \\ &\text{for } j = 1, ..., N; k \in \mathcal{R}_i \\ &\text{where } \mathbf{X}_i = \mathbf{P}_i^{-1}, \quad \mathbf{M}_i = \mathbf{K}_i \mathbf{X}_i \end{aligned} \tag{30}$$

Example 3. consider the following fuzzy system:

$$R^1: \text{ If } x_1(k) \text{ is } F_i^1 \text{ and } x_2(k) \text{ is } F_i^2 \text{ Then } \mathbf{x}(k+1) = \mathbf{A}_1 \mathbf{x}(k) + \mathbf{B}_1 \mathbf{u}(k) \qquad i = 1, ..., 9$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 1.0 & 0.2 \\ -0.7 & 1.5 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 1.2 & 0.3 \\ -0.1 & 1.0 \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} 1.0 & 0.5 \\ 1.8 & 0.8 \end{bmatrix},$$
$$\mathbf{A}_4 = \begin{bmatrix} 1.25 & 0.35 \\ -0.3 & 1.0 \end{bmatrix}, \quad \mathbf{A}_5 = \begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.6 \end{bmatrix}, \quad \mathbf{A}_6 = \begin{bmatrix} 1.2 & 0.6 \\ 2.0 & 1.0 \end{bmatrix},$$
$$\mathbf{A}_7 = \begin{bmatrix} 1.2 & 0.1 \\ -2.2 & 0.9 \end{bmatrix}, \quad \mathbf{A}_8 = \begin{bmatrix} 0.9 & 0.3 \\ -0.2 & 1.1 \end{bmatrix}, \quad \mathbf{A}_9 = \begin{bmatrix} 0.8 & 0.4 \\ 1.5 & 0.6 \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0.0 \\ 0.6 \end{bmatrix}, \quad \mathbf{B}_3 = \begin{bmatrix} 0 \\ 1.3 \end{bmatrix}, \quad \mathbf{B}_4 = \begin{bmatrix} 0.3 \\ 0.9 \end{bmatrix}, \quad \mathbf{B}_5 = \begin{bmatrix} -0.2 \\ 0.8 \end{bmatrix},$$
$$\mathbf{B}_6 = \begin{bmatrix} 0.2 \\ 1.5 \end{bmatrix}, \quad \mathbf{B}_7 = \begin{bmatrix} -0.2 \\ 1.19 \end{bmatrix}, \quad \mathbf{B}_8 = \begin{bmatrix} 0.2 \\ 1.0 \end{bmatrix}, \quad \mathbf{B}_9 = \begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix}$$

The membership functions are shown in figure 6.

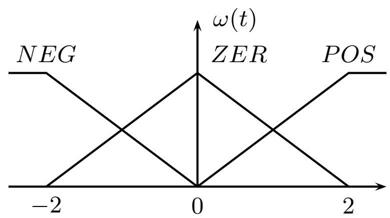

Fig. 6. Membership functions

The fuzzy system rule set $\mathcal{R} = \{R_i,\ i = 1, ..., 9\}$ can be divided into four subsets : $\mathcal{R}_1 = \{R_1, R_2, R_4, R_5\}, \mathcal{R}_2 = \{R_2, R_3, R_5, R_6\}, \mathcal{R}_3 = \{R_4, R_5, R_7, R_8\}$ and $\mathcal{R}_4 = \{R_5, R_6, R_8, R_9\}$. By using MATLAB LMI toolbox we found the four local matrices:

$$\mathbf{P}_1 = \begin{bmatrix} 0.5842 & 0.1662 \\ 0.1662 & 0.0822 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 0.5715 & 0.1842 \\ 0.1842 & 0.0883 \end{bmatrix}, \quad \mathbf{P}_3 = \begin{bmatrix} 0.5744 & 0.1593 \\ 0.1593 & 0.0788 \end{bmatrix},$$
$$\mathbf{P}_4 = \begin{bmatrix} 0.5538 & 0.1749 \\ 0.1749 & 0.0853 \end{bmatrix}$$

and the local state feedbacks

$$\mathbf{K}_1 = \begin{bmatrix} 1.5487 & 1.3649 \end{bmatrix}, \quad \mathbf{K}_2 = \begin{bmatrix} 2.2211 & 1.3323 \end{bmatrix}, \quad \mathbf{K}_3 = \begin{bmatrix} 0.7264 & 0.9715 \end{bmatrix},$$
$$\mathbf{K}_4 = \begin{bmatrix} 1.7784 & 1.1528 \end{bmatrix}$$

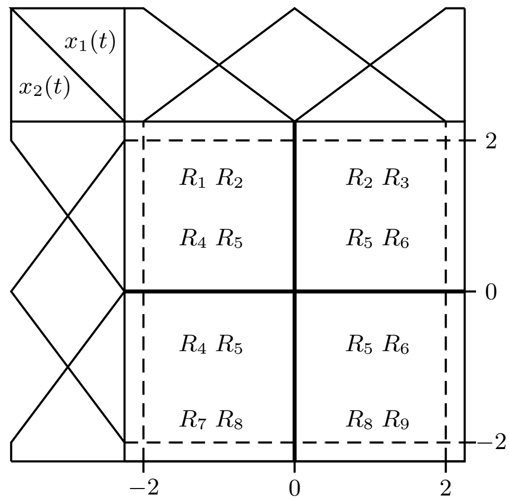

Fig. 7. State space partition

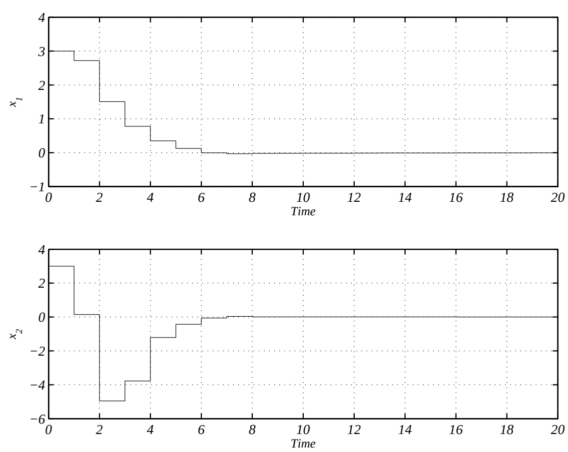

Fig. 8. States evolution for initial condition $\mathbf{x}(t) = [3, 3]^T$

4.2 Stabilisation using local PDC controller

In the precedent section a local linear state feedback is used in each subregion. However, the design LMIs may be infeasible particularly if the number of rules in a subspace is large. The PDC(Parallel Distributed Compensation) controller can be used to overcome this problem. The control law is then given by:

$$\mathbf{u}(k) = \sum_{R_l \in \mathcal{R}_i} \alpha_l(\mathbf{z}(k))\mathbf{F}_{il}\mathbf{x}(k), \quad \mathbf{x}(k) \in_l,\ R_l \in \mathcal{R}_i, i = 1, 2, \ldots, N \tag{31}$$

Each rule subset is considered as a fuzzy system with its own PDC control law and each rule has as many feedbacks as the number of rule subsets containing it.

Theorem 5. *If there exists symmetric positive definite matrices $\mathbf{P}_i$ and gain matrices $\mathbf{F}_{il}, i = 1, 2, \ldots, N$ $R_l \in \mathcal{R}_i$ so that the following inequalities are satisfied:*

$$\mathbf{P}_i = \mathbf{P}_i^T > 0, \mathbf{Q}_i = \mathbf{Q}_i^T > 0 \tag{32}$$

$$\mathbf{G}_{ill}^T \mathbf{P}_j \mathbf{G}_{ill} - \mathbf{P}_i + (n_i - 1)\mathbf{Q}_i < 0 \tag{33}$$

$$\left(\frac{\mathbf{G}_{ikl} + \mathbf{G}_{ilk}}{2}\right)^T \mathbf{P}_j \left(\frac{\mathbf{G}_{ikl} + \mathbf{G}_{ilk}}{2}\right) - \mathbf{P}_i - \mathbf{Q}_i \leq 0 \tag{34}$$

$$\textit{for } R_l \in \mathcal{R}_i, R_k \in \mathcal{R}_i, k < l$$

$$\textit{and } n_i = \textit{card}(\mathcal{R}_i), \quad \mathbf{G}_{ilk} = \mathbf{A}_l - \mathbf{B}_l \mathbf{F}_{ik}$$

then the closed loop fuzzy system is globally asymptotically stable.

Proof. Let the discrete time Lyapunouv function candidate be defined as:

$$V(k) = \mathbf{x}^T(k)\mathbf{P}_i\mathbf{x}(k), \quad \mathbf{x}(k) \in \mathcal{X}_i,\ i = 1, 2, \ldots, N \tag{35}$$

$$\mathbf{G}_{ilk} = \mathbf{A}_l - \mathbf{B}_l\mathbf{F}_{ik} \tag{36}$$

$$\Delta V(k) = \left\{ \sum_{R_l \in \mathcal{R}_i} \sum_{R_k \in \mathcal{R}_i} \alpha_l(\cdot)\alpha_k(\cdot)\mathbf{G}_{ilk}\mathbf{x}(k) \right\}^T \cdot \mathbf{P}_j \cdot \left\{ \sum_{R_l \in \mathcal{R}_i} \sum_{R_k \in \mathcal{R}_i} \alpha_l(\cdot)\alpha_k(\cdot)\mathbf{G}_{ilk}\mathbf{x}(k) \right\} - \mathbf{x}^T(k)\mathbf{P}_i\mathbf{x}(k)$$

$$\begin{aligned}
\Delta V(k) = & \sum_{R_l\in\mathcal{R}_i}\sum_{R_k\in\mathcal{R}_i}\sum_{R_m\in\mathcal{R}_i}\sum_{R_n\in\mathcal{R}_i}\alpha_l(\cdot)\alpha_k(\cdot)\alpha_m(\cdot)\alpha_n(\cdot)\\
& \mathbf{x}^T(k)\mathbf{G}_{ilk}^T\mathbf{P}_j\mathbf{G}_{imn}\mathbf{x}(k)-\mathbf{x}^T(k)\mathbf{P}_i\mathbf{x}(k)\\
= & \sum_{R_l\in\mathcal{R}_i}\sum_{R_k\in\mathcal{R}_i}\sum_{R_m\in\mathcal{R}_i}\sum_{R_n\in\mathcal{R}_i}\alpha_l(\cdot)\alpha_k(\cdot)\alpha_m(\cdot)\alpha_n\cdot)\\
& \mathbf{x}^T(k)\left\{\mathbf{G}_{ilk}^T\mathbf{P}_j\mathbf{G}_{imn}-\mathbf{P}_i\right\}\mathbf{x}(k)\\
= & \frac{1}{4}\sum_{R_l\in\mathcal{R}_i}\sum_{R_k\in\mathcal{R}_i}\sum_{R_m\in\mathcal{R}_i}\sum_{R_n\in\mathcal{R}_i}\alpha_l(\cdot)\alpha_k(\cdot)\alpha_m(\cdot)\alpha_n(\cdot)\\
& \mathbf{x}^T(k)\left\{(\mathbf{G}_{ikl}+\mathbf{G}_{ilk})^T\mathbf{P}_j\left(\mathbf{G}_{imn}+\mathbf{G}_{inm}\right)-4\mathbf{P}_i\right\}\mathbf{x}(k)\\
\leq & \frac{1}{4}\sum_{R_l\in\mathcal{R}_i}\sum_{R_k\in\mathcal{R}_i}\alpha_l(\cdot)\alpha_k(\cdot)\mathbf{x}^T(k)\left\{(\mathbf{G}_{ikl}+\mathbf{G}_{ilk})^T\right.\\
& \left.\mathbf{P}_j\left(\mathbf{G}_{ikl}+\mathbf{G}_{ilk}\right)-4\mathbf{P}_i\right\}\mathbf{x}(k)\\
\leq & \sum_{R_l\in\mathcal{R}_i}\sum_{R_k\in\mathcal{R}_i}\alpha_l(\cdot)\alpha_k(\cdot)\mathbf{x}^T(k)\left\{\left(\frac{\mathbf{G}_{ikl}+\mathbf{G}_{ilk}}{2}\right)^T\right.\\
& \left.\mathbf{P}_j\left(\frac{\mathbf{G}_{ikl}+\mathbf{G}_{ilk}}{2}\right)-\mathbf{P}_i\right\}\mathbf{x}(k)\\
\leq & \sum_{R_l\in\mathcal{R}_i}\alpha_l^2(\cdot)\mathbf{x}^T(k)\left\{\mathbf{G}_{ill}^T\mathbf{P}_j\mathbf{G}_{ill}-\mathbf{P}_i\right\}\mathbf{x}(k)\\
& +2\sum_{\substack{R_l,R_k\in\mathcal{R}_i\\ l<k}}\alpha_l(\cdot)\alpha_k(\cdot)\mathbf{x}^T(k)\left\{\left(\frac{\mathbf{G}_{ikl}+\mathbf{G}_{ilk}}{2}\right)^T\mathbf{P}_j\left(\frac{\mathbf{G}_{ikl}+\mathbf{G}_{ilk}}{2}\right)-\mathbf{P}_i\right\}\mathbf{x}(k)
\end{aligned}$$

By using the property [2];

$$\sum_{R_l\in\mathcal{R}_i}\alpha_l^2-\frac{1}{n_i-1}\sum_{\substack{R_l,R_k\in\mathcal{R}_i\\ l<k}}2\alpha_l\alpha_k\geq 0\Rightarrow 2\sum_{\substack{R_l,R_k\in\mathcal{R}_i\\ l<k}}\alpha_l\alpha_k\leq(n_i-1)\sum_{R_l\in\mathcal{R}_i}\alpha_l^2$$

$$\begin{aligned}
\Delta V(k)\leq & \sum_{R_l\in\mathcal{R}_i}\alpha_l^2(\cdot)\mathbf{x}^T(k)\left\{\mathbf{G}_{ill}^T\mathbf{P}_j\mathbf{G}_{ill}-\mathbf{P}_i\right\}\mathbf{x}(k)\\
& +(n_i-1)\sum_{R_l\in\mathcal{R}_i}\alpha_l^2(\cdot)\mathbf{x}^T(k)\left\{\left(\frac{\mathbf{G}_{ikl}+\mathbf{G}_{ilk}}{2}\right)^T\right.\\
& \left.\mathbf{P}_j\left(\frac{\mathbf{G}_{ikl}+\mathbf{G}_{ilk}}{2}\right)-\mathbf{P}_i\right\}\mathbf{x}(k)
\end{aligned}$$

If condition (34) holds then

$$\begin{aligned}\Delta V(k) \le & \sum_{R_l\in\mathcal{R}_i} \alpha_l^2(\cdot)\mathbf{x}^T(k)\left\{\mathbf{G}_{ill}^T\mathbf{P}_j\mathbf{G}_{ill}-\mathbf{P}_i\right\}\mathbf{x}(k)\\ & + (n_i-1)\sum_{R_l\in\mathcal{R}_i}\alpha_l^2\mathbf{x}^T(k)\mathbf{Q}_i\mathbf{x}(k)\\ \le & \sum_{R_l\in\mathcal{R}_i}\alpha_l^2(\cdot)\mathbf{x}^T(k)\left\{\mathbf{G}_{ill}^T\mathbf{P}_j\mathbf{G}_{ill}-\mathbf{P}_i\right.\\ & \left.+(n_i-1)\mathbf{Q}_i\right\}\mathbf{x}(k)\end{aligned}$$

If the condition (33) holds then $\Delta V(k) < 0$

The design problem to determine the feedback gains can be reformulated as a LMI problem.
By right and left multiplying by $\mathbf{X}_i = \mathbf{P}_i^{-1}$

$$\begin{aligned}&\mathbf{X}_i\mathbf{G}_{ill}^T\mathbf{P}_j\mathbf{G}_{ill}\mathbf{X}_i - \mathbf{X}_i + (n_i-1)\mathbf{Y}_i < 0\\ &\mathbf{X}_i\left(\frac{\mathbf{G}_{ilk}^T+\mathbf{G}_{ikl}}{2}\right)^T\mathbf{P}_j\left(\frac{\mathbf{G}_{ilk}+\mathbf{G}_{ikl}}{2}\right)\mathbf{X}_i - \mathbf{X}_i \le 0\\ &\text{for } R_l\in\mathcal{R}_i, R_k\in\mathcal{R}_i, k<l\\ &\text{and } n_i = card(\mathcal{R}_i), \quad \mathbf{G}_{ilk} = \mathbf{A}_l - \mathbf{B}_l\mathbf{F}_{ik}\end{aligned} \tag{37}$$

And by using Schur Complement we get:
Find $\mathbf{X}_i > 0$ and $\mathbf{M}_{il}$ $\quad (i = 1, ..., N,\ R_l \in \mathcal{R}_i)$ satisfying:

$$\begin{aligned}&\begin{bmatrix}\mathbf{X}_j-(n_i-1)\mathbf{Y}_i & \mathbf{X}_i\mathbf{A}_k^T-\mathbf{M}_{il}^T\mathbf{B}_l^T\\ \mathbf{A}_l\mathbf{X}_i-\mathbf{B}_l\mathbf{M}_{il} & \mathbf{X}_i\end{bmatrix} < 0\\ &\begin{bmatrix}\mathbf{X}_j+\mathbf{Y}_i & *\\ \frac{1}{2}\left\{\mathbf{A}_l\mathbf{X}_i+\mathbf{A}_k\mathbf{X}_i-\mathbf{B}_k\mathbf{M}_{il}-\mathbf{B}_l\mathbf{M}_{ik}\right\} & \mathbf{X}_i\end{bmatrix} \le 0\end{aligned} \tag{38}$$

for $j = 1, ..., N$ and $R_k \in \mathcal{R}_i,\ R_l \in \mathcal{R}_i \quad k < l$
where
$\mathbf{X}_i = \mathbf{P}_i^{-1}, \quad \mathbf{M}_{il} = \mathbf{K}_{il}\mathbf{X}_i$

Example 4. To illustrate the controller synthesis approach, we consider the following discrete T-S fuzzy system:
R^1 : if $x_1(k)$ is NEG Then $\mathbf{x}(k+1) = \mathbf{A}_1\mathbf{x}(k) + \mathbf{B}_1\mathbf{u}(k)$
R^2 : if $x_1(k)$ is ZER Then $\mathbf{x}(k+1) = \mathbf{A}_2\mathbf{x}(k) + \mathbf{B}_2\mathbf{u}(k)$
R^3 : if $x_1(k)$ is POS Then $\mathbf{x}(k+1) = \mathbf{A}_3\mathbf{x}(k) + \mathbf{B}_3\mathbf{u}(k)$
Where

$$\mathbf{A}_1 = \begin{bmatrix}1.2 & 0.2\\ -2.0 & 0.8\end{bmatrix}, \qquad \mathbf{A}_2 = \begin{bmatrix}1.0 & 0.2\\ -0.1 & 0.7\end{bmatrix}, \qquad \mathbf{A}_3 = \begin{bmatrix}1.2 & 0.2\\ -2 & 0.8\end{bmatrix},$$

$$\mathbf{B}_1 = \begin{bmatrix}0\\ 0.14\end{bmatrix}, \qquad \mathbf{B}_2 = \begin{bmatrix}0\\ 1.0\end{bmatrix}, \qquad \mathbf{B}_3 = \begin{bmatrix}0\\ 1.86\end{bmatrix}$$

The membership functions NEG, ZER and POS are shown in Fig. 2. The fuzzy system is composed of three rules, $\mathcal{R} = \{R_1, R_2, R_3\}$, but R_1 and R_3 can not be inferred in the same time. The rule set can be decomposed into:
$\mathcal{R}_1 = \{R_1, R_2\}$ and $\mathcal{R}_2 = \{R_2, R_3\}$
By using the method proposed in[2] we can not find stabilizing feedback gains, by using the local PDC approach the gains obtained are:

$$\mathbf{P}_1 = \begin{bmatrix} 0.9541 & 0.2119 \\ 0.2119 & 0.1113 \end{bmatrix}, \qquad \mathbf{P}_2 = \begin{bmatrix} 0.9605 & 0.2856 \\ 0.2856 & 0.1403227 \end{bmatrix}$$
$$\mathbf{K}_1 = \begin{bmatrix} 1.5877 & 3.5270 \end{bmatrix}, \qquad \mathbf{K}_{12} = \begin{bmatrix} 2.8630 & 0.7789 \end{bmatrix},$$
$$\mathbf{K}_{22} = \begin{bmatrix} 2.1467 & 1.1877 \end{bmatrix}, \qquad \mathbf{K}_3 = \begin{bmatrix} 2.0180 & 0.9426 \end{bmatrix}$$

and as is shown in Fig.9, the fuzzy TS system is stabilized by the local PDC controllers.

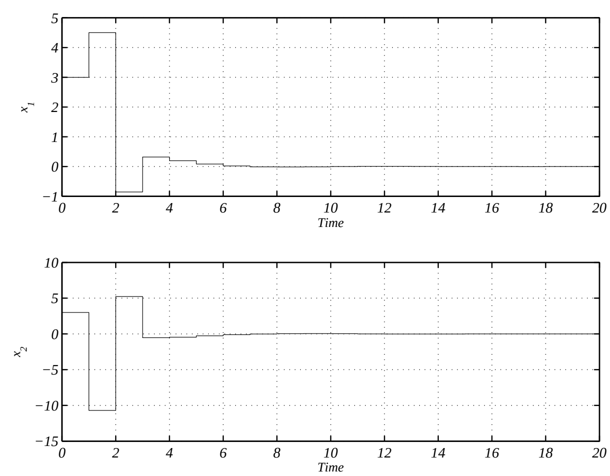

Fig. 9. Evolution of the states for initial condition $\mathbf{x}(t) = [3, 4]^T$

5 Conclusion

In this paper we studied the global stability of discrete T-S fuzzy system by the decomposition of the fuzzy rule subset. Each subset contains a group of rules that can be inferred in the same time. To check the stability we have to find a set of symmetric positif definite matrices, each local matrix guarantee the local stability and the global stability can be ensured by additional relations between different local matrices. An LMI approach has been used to find the set of local matrices. The number of LMIs can be reduced by the elimination of matrix inequalities corresponding to impossible transition between different subregions. This approach has been used to stabilize discrete T-S fuzzy systems by state feedback. Two types of regulators have been studied. In the first type, a linear state feedback is used in each subregion corresponding to a rule subset. While in the second, a local PDC controller is used. The use of local PDC controller outperform the famous PDC control law.

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