Non-linear systems control via fuzzy models: a multicontroller approach

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Abstract: This paper presents a Lyapunov-based switching controller design method for non-linear systems using Takagi-Sugeno fuzzy models. The basic idea of the proposed approach is to represent the fuzzy model as a set of uncertain linear systems. The controller is obtained by solving the corresponding set of Algebraic Ricatti Equations (AREs). A simulation example is given to illustrate the effectiveness of this approach.

Keywords: switching control; fuzzy systems; uncertain system.

Reference to this paper should be made as follows: Boumehraz, M. and Benmahammed, K. (2007) 'Non-linear systems control via fuzzy models: a multicontroller approach', *Int. J. Modelling, Identification and Control*, Vol. 2, No. 1, pp.16–23.

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1 Introduction

Over the past several years, fuzzy systems have attracted considerable attention from scientists and engineers. Fuzzy modelling is an efficient method to represent complex non-linear systems by fuzzy sets and fuzzy reasoning. By using a Takagi-Sugeno (T-S) fuzzy model, a non-linear system can be expressed as a weighted sum of simple subsystems. This model gives a fixed structure to some non-linear systems and thus facilitates their analysis. There are two ways to obtain the fuzzy model:

1 by applying identification methods with input–output data from the plant

2 or directly from the mathematical model of the non-linear plant (Cao et al., 1997; Sugeno and Kang, 1988; Takagi and Sugeno, 1985).

More recently, a number of systematic stability analysis and controller design results have appeared in the fuzzy control literature. Tanaka et al. (1998) discussed the stability and the design of fuzzy control systems. They gave some checking conditions for stability, which can be used to design fuzzy control laws. Unfortunately, the stability conditions require the existence of a common positive definite matrix for all the local linear models. However, this is a difficult problem to be solved in many cases, especially when the

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number of rules is large. Representation of fuzzy models by a set of linear uncertain systems has been suggested by Cao et al. (1996). Based on linear uncertain system theory several control design approaches have been proposed. The drawback of the preceding approaches is that the LMIs or the algebraic Riccati equations used to check the stability can be infeasible. Based on the representation of Cao et al. (1996, 2001) and Feng (2001) we propose, in this work, a switching control design approach. The proposed approach is based on the resolution of a set of independent algebraic Ricatti equation. To overcome the problem of infeasibility the fulfilment degree of each rule is incorporated in the algebraic Riccati equation and a minimization program is used to determine the minimal degree for which the algebraic Riccati equation has a solution. The rest of this paper is organised as follows: Section 2 introduces the fuzzy dynamic model. Section 3 presents the switching controller design approach for fuzzy dynamic models based on the resolution of a set of algebraic Ricatti equations. To demonstrate the efficiency of the proposed approach, a simulation example is given in Section 4. Finally, conclusions are given in Section 5.

2 T-S fuzzy model

Many physical systems are so complex in practice that rigorous mathematical models can be very difficult to obtain, if not impossible. However, many of these systems can be expressed in some form of mathematical models. T-S fuzzy models have been largely used to model complex non-linear systems (Takagi and Sugeno, 1985). The continuous-time T-S fuzzy dynamic model is a piecewise interpolation of several linear models through membership functions. The fuzzy model is described by a set of fuzzy if-then rules. The ith rule of the fuzzy model take the form:
Rule i:

If $z_1(t)$ is $F_1^i, \ldots,$ and $z_g(t)$ is F_g^i

$$\text{Then } \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t) \\ \mathbf{y}(t) = \mathbf{C}_i\mathbf{x}(t) \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbf{R}^n$ denotes the state vector, $\mathbf{u}(t) \in \mathbf{R}^m$ the control vector, $\mathbf{y}(t) \in \mathbf{R}^p$ the output vector, F_j^i is the jth fuzzy set of the ith rule, $\mathbf{A}_i \in \mathbf{R}^{n\times n}, \mathbf{B}_i \in \mathbf{R}^{n\times m}$ and $\mathbf{C}_i \in \mathbf{R}^{p\times n}$ are the state matrix, the input matrix and the output matrix for the ith local model, r is the number of if-then rules and $z_1(t), z_2(t), \ldots, z_g(t)$ are some measurable system variables. The final output of the fuzzy model can be expressed as:

$$\begin{cases} \dot{\mathbf{x}}(t) = \sum_{i=1}^r \alpha_i(\mathbf{z}(t))\{\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\} \\ \mathbf{y}(t) = \sum_{i=1}^r \alpha_i(\mathbf{z}(t))\mathbf{C}_i\mathbf{x}(t) \end{cases} \tag{2}$$

where

$$\alpha_i(\mathbf{z}(t)) = \frac{\omega_i(\mathbf{z}(t))}{\sum_{i=1}^r \omega_i(\mathbf{z}(t))} \tag{3}$$

and

$$\omega_i(\mathbf{z}(t)) = \prod_{j=1}^g F_j^i(\mathbf{z}(t)) \tag{4}$$

F_j^i is the grade of membership of $z_j(t)$ in F_j^i.

The scalars $\alpha_i(\mathbf{z}(t))$ are characterised by:

$$0 \le \alpha_i(\mathbf{z}(t)) \le 1 \quad \text{and} \quad \sum_{i=1}^r \alpha_i(\mathbf{z}(t)) = 1 \tag{5}$$

The T-S fuzzy model (2) has strong non-linear interactions among its fuzzy rules which complicates its analysis and control. In order to overcome these difficulties the T-S fuzzy model is represented as a set of uncertain linear systems (Cao et al., 1996). The global state space $\Omega \subseteq \mathbf{R}^n$ is partitioned into r subspaces, each subspace is defined by:

$$\Omega_l = \{\Omega |\, \alpha_l(\mathbf{z}(t)) > 0\} \tag{6}$$

Each subspace Ω_l is the union of two subsets:

$$\Omega_l = \bar{\Omega}_l \cup \Delta\Omega_l \tag{7}$$

where

$$\bar{\Omega}_l = \{\Omega |\, \alpha_l(\mathbf{z}(t)) = 1\} \tag{8}$$

and

$$\Delta\Omega_l = \{\Omega |\, 0 < \alpha_l(\mathbf{z}(t)) < 1\} \tag{9}$$

These subspaces are characterised by:

$$\bigcup_{i=1}^r \Omega_i = \Omega \tag{10}$$

If the rules i and j can be inferred in the same time then:

$$\Omega_i \cap \Omega_j \ne \phi \tag{11}$$

If the rules i and j cannot be inferred in the same time then:

$$\Omega_i \cap \Omega_j = \phi \tag{12}$$

In each subspace the T-S fuzzy model (2) can be represented as:

$$\begin{cases} \dot{\mathbf{x}}(t) = \{\mathbf{A}_l + \sum_{i=1, i\ne l}^r \alpha_i(\mathbf{z}(t))\mathbf{A}_{li}\}\mathbf{x}(t) \\ \qquad +\{\mathbf{B}_l + \sum_{i=1, i\ne l}^r \alpha_i(\mathbf{z}(t))\mathbf{B}_{li}\}\mathbf{u}(t) \\ \mathbf{y}(t) = \{\mathbf{C}_l + \sum_{i=1, i\ne l}^r \alpha_i(\mathbf{z}(t))\mathbf{C}_{li}\}\mathbf{x}(t) \end{cases} \tag{13}$$

where

$$\mathbf{A}_{li} = \mathbf{A}_i - \mathbf{A}_l, \quad \mathbf{B}_{li} = \mathbf{B}_i - \mathbf{B}_l, \quad \mathbf{C}_{li} = \mathbf{C}_i - \mathbf{C}_l \tag{14}$$

Since

$$\sum_{\substack{i=1\\ i\ne l}}^r \alpha_i(\mathbf{z}(t)) = 1 - \alpha_l(\mathbf{z}(t)) \tag{15}$$

The T-S fuzzy model can be written as:

$$\begin{cases} \dot{\mathbf{x}}(t) = \{\mathbf{A}_l + (1 - \alpha_l(\mathbf{z}(t)))\Delta\mathbf{A}_l(\mathbf{z}(t))\}\mathbf{x}(t) \\ \qquad +\{\mathbf{B}_l + (1 - \alpha_l(\mathbf{z}(t)))\Delta\mathbf{B}_l(\mathbf{z}(t))\}\mathbf{u}(t) \\ \mathbf{y}(t) = \{\mathbf{C}_l + (1 - \alpha_l(\mathbf{z}(t)))\Delta\mathbf{C}_l(\mathbf{z}(t))\}\mathbf{x}(t) \end{cases} \tag{16}$$

where

$$\Delta\mathbf{A}_l(\mathbf{z}(t)) = \sum_{\substack{i=1\\ i\neq l}}^{r} \alpha'_i(\mathbf{z}(t))(\mathbf{A}_i - \mathbf{A}_l) \tag{17}$$

$$\Delta\mathbf{B}_l(\mathbf{z}(t)) = \sum_{\substack{i=1\\ i\neq l}}^{r} \alpha'_i(\mathbf{z}(t))(\mathbf{B}_i - \mathbf{B}_l) \tag{18}$$

$$\Delta\mathbf{C}_l(\mathbf{z}(t)) = \sum_{\substack{i=1\\ i\neq l}}^{r} \alpha'_i(\mathbf{z}(t))(\mathbf{C}_i - \mathbf{C}_l) \tag{19}$$

and

$$\alpha'_i(\mathbf{z}(t)) = \frac{\alpha_i(\mathbf{z}(t))}{1-\alpha_l(\mathbf{z}(t))} \tag{20}$$

If $\alpha_l(\mathbf{z}(t)) = 1$ then the fuzzy system can be represented by the corresponding linear local model. In each subspace, the fuzzy model consists of a dominant nominal system $(\mathbf{A}_l, \mathbf{B}_l, \mathbf{C}_l)$ and a set of interacting systems representing the effect of other active rules. In this paper, we suppose that the state vector is measurable and $\mathbf{y}(t) = \mathbf{x}(t)$. The fuzzy system can be simplified to:

$$\dot{\mathbf{x}}(t) = \tilde{\mathbf{A}}_l(\alpha'(\mathbf{z}(t)))\mathbf{x}(t) + \tilde{\mathbf{B}}_l(\alpha'(\mathbf{z}(t)))\mathbf{u}(t) \tag{21}$$

with

$$\tilde{\mathbf{A}}_l(\alpha'(\mathbf{z}(t))) = \mathbf{A}_l + (1-\alpha_l(\mathbf{z}(t)))\Delta\mathbf{A}_l(\alpha'(\mathbf{z}(t))) \tag{22}$$

$$\tilde{\mathbf{B}}_l(\alpha'(\mathbf{z}(t))) = \mathbf{B}_l + (1-\alpha_l(\mathbf{z}(t)))\Delta\mathbf{B}_l(\alpha'(\mathbf{z}(t))) \tag{23}$$

Suppose that the matrices $\mathbf{A}_i - \mathbf{A}_l$ and $\mathbf{B}_i - \mathbf{B}_l$ can be written as:

$$\mathbf{A}_i - \mathbf{A}_l = \mathbf{M}^A_{li}\,\mathbf{N}^A_{li}, \quad \mathbf{B}_i - \mathbf{B}_l = \mathbf{M}^B_{li}\,\mathbf{N}^B_{li} \tag{24}$$

Then $\Delta\mathbf{A}_l(\alpha'(\mathbf{z}(t)))$ and $\Delta\mathbf{B}_l(\alpha'(\mathbf{z}(t)))$ can be expressed as:

$$\Delta\mathbf{A}_l(\alpha'(\mathbf{z}(t))) = \mathbf{M}_{A_l}\,\mathbf{F}_{A_l}(\alpha'(\mathbf{z}(t)))\,\mathbf{N}_{A_l} \tag{25}$$

$$\Delta\mathbf{B}_l(\alpha'(\mathbf{z}(t))) = \mathbf{M}_{B_l}\,\mathbf{F}_{B_l}(\alpha'(\mathbf{z}(t)))\,\mathbf{N}_{B_l} \tag{26}$$

where

$$\begin{aligned}\mathbf{M}_{A_l} &= \begin{bmatrix}\mathbf{M}^A_{l1} & \mathbf{M}^A_{l2} & \cdots & \mathbf{M}^A_{lr}\end{bmatrix}\\ \mathbf{M}_{B_l} &= \begin{bmatrix}\mathbf{M}^B_{l1} & \mathbf{M}^B_{l2} & \cdots & \mathbf{M}^B_{lr}\end{bmatrix}\end{aligned} \tag{27}$$

$$\mathbf{N}_{A_l} = \begin{bmatrix}\mathbf{N}^A_{l1}\\ \mathbf{N}^A_{l2}\\ \vdots\\ \mathbf{N}^A_{lr}\end{bmatrix}, \quad \mathbf{N}_{B_l} = \begin{bmatrix}\mathbf{N}^B_{l1}\\ \mathbf{N}^B_{l2}\\ \vdots\\ \mathbf{N}^B_{lr}\end{bmatrix} \tag{28}$$

$$\mathbf{F}_{A_l}(\alpha'(\mathbf{z}(t))) = \begin{bmatrix}\alpha'_1(\mathbf{z}(t))\mathbf{I}_{ql} & 0 & \cdots & 0\\ 0 & \alpha'_2(\mathbf{z}(t))\mathbf{I}_{q2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \alpha'_r(\mathbf{z}(t))\mathbf{I}_{q_r}\end{bmatrix} \tag{29}$$

$$\mathbf{F}_{B_l}(\alpha'(\mathbf{z}(t))) = \begin{bmatrix}\alpha'_1(\mathbf{z}(t))\mathbf{I}_{p_l} & 0 & \cdots & 0\\ 0 & \alpha'_2(\mathbf{z}(t))\mathbf{I}_{p2} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \alpha'_r(\mathbf{z}(t))\mathbf{I}_{p_r}\end{bmatrix} \tag{30}$$

$$0 \le \alpha'_i(\mathbf{z}(t)) \le 1 \implies \begin{cases}\mathbf{F}_{A_l}(\alpha'(\mathbf{z}(t)))\mathbf{F}^T_{A_l}(\alpha'(\mathbf{z}(t))) \le \mathbf{I}\\ \mathbf{F}_{B_l}(\alpha'(\mathbf{z}(t)))\mathbf{F}^T_{B_l}(\alpha'(\mathbf{z}(t))) \le \mathbf{I}\end{cases} \tag{31}$$

3 Controller design

We assume that the fuzzy system (2) is locally controllable, that is, the pairs $(\mathbf{A}_l, \mathbf{B}_l)$, $l = 1, \ldots, r$, are controllable. The basic idea is to design local feedback controllers that maximise the stability region of each closed-loop local model. The switching controller, represented in Figure 1 consists of r linear state feedback controllers that will be switched from one to another to control the system. The switching controller can be described by

$$\mathbf{u}(t) = \sum_{l=1}^{r} \zeta_l(\mathbf{z}(t))\mathbf{u}_l(t) \tag{32}$$

with

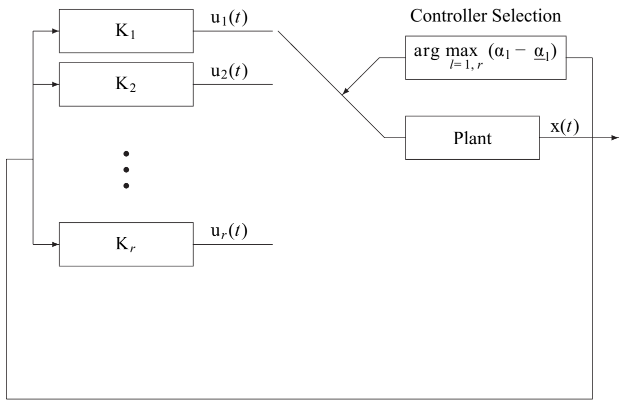
$$\mathbf{u}_l(t) = \mathbf{K}_l\mathbf{x}(t), \quad \mathbf{K}_l = -\widetilde{\mathbf{R}}_l^{-1}\mathbf{B}_l^T\mathbf{P}_l \tag{33}$$

and

$$\sum_{l=1}^{r} \zeta_l(\mathbf{z}(t)) = 1, \quad \zeta_l(\mathbf{z}(t)) \in \{0, 1\} \tag{34}$$

$\mathbf{K}_l$ is the local state feedback gain in subspace Ω_l to be designed. It can be seen that (32) is a linear combination of r linear state feedback controllers. At each moment, only one of the linear state feedback controllers is chosen to generate the control signal.

Figure 1 Structure of the switching controller



Theorem 1: *If there exist positive definite matrices* $\boldsymbol{R}_l \in \boldsymbol{R}^{m\times m}, \boldsymbol{Q}_l \in \boldsymbol{R}^{n\times n}$ *positive scalars* $\mu^1_l > 0, \mu^2_l > 0$

and $0 \leq \underline{\alpha}_l \leq 1$ *such that the following algebraic Ricatti equation*

$$\boldsymbol{A}_l^T \boldsymbol{P}_l + \boldsymbol{P}_l \boldsymbol{A}_l - \boldsymbol{P}_l \boldsymbol{B}_l \widetilde{\boldsymbol{R}}_l^{-1} \boldsymbol{B}_l^T \boldsymbol{P}_l + \widetilde{\boldsymbol{Q}}_l + \boldsymbol{P}_l \boldsymbol{H}_l \boldsymbol{P}_l = 0 \quad (35)$$

has a solution $\boldsymbol{P}_l = \boldsymbol{P}_l^T > 0$ *where*

$$\widetilde{\boldsymbol{Q}}_l = \boldsymbol{Q}_l + \frac{1}{\mu_l^1}(1 - \underline{\alpha}_l)\boldsymbol{N}_{A_l}^T \boldsymbol{N}_{A_l} \quad (36)$$

$$\boldsymbol{H}_l = (1 - \underline{\alpha}_l)\big(\mu_l^1 \boldsymbol{M}_{A_l} \boldsymbol{M}_{A_l}^T + \mu_l^2 \boldsymbol{M}_{B_l} \boldsymbol{M}_{B_l}^T\big) \quad (37)$$

$$\widetilde{\boldsymbol{R}}_l = \boldsymbol{R}_l + \frac{1}{\mu_l^2}(1 - \underline{\alpha}_l)\boldsymbol{N}_{B_l}^T \boldsymbol{N}_{B_l} \quad (38)$$

then the state feedback control law (33) *quadratically stabilise the fuzzy system* (21) *in the subregion:*

$$\Omega_l^s = \{\Omega | \alpha_l(\mathbf{z}(t)) \geq \underline{\alpha}_l\} \quad (39)$$

Proof: Consider the following Lyapunouv function candidate:

$$V_l(t) = \mathbf{x}^T(t)\mathbf{P}_l \mathbf{x}(t) \quad (40)$$

where $\mathbf{P}_l$ is a symmetric positive definite matrix. The time derivative of $V_l(t)$ along the trajectory of the fuzzy system is given by

$$\dot{V}_l(t) = \dot{\mathbf{x}}^T(t)\mathbf{P}_l\mathbf{x}(t) + \mathbf{x}^T(t)\mathbf{P}_l\dot{\mathbf{x}}(t)$$
$$\dot{V}_l(t) = \mathbf{x}^T(t)\left[\widetilde{\mathbf{A}}_l^T(\alpha(\mathbf{z}(t)))\mathbf{P}_l + \mathbf{P}_l\widetilde{\mathbf{A}}_l(\alpha(\mathbf{z}(t)))\right]\mathbf{x}(t) + \mathbf{u}^T(t)\widetilde{\mathbf{B}}_l^T(\alpha(\mathbf{z}(t)))\mathbf{P}_l\mathbf{x}(t) + \mathbf{x}^T(t)\mathbf{P}_l\widetilde{\mathbf{B}}_l(\alpha(\mathbf{z}(t)))\mathbf{u}(t)$$

For simplicity of notation $\alpha(\mathbf{z}(t))$ and t will be omitted from matrix and function expressions.

$$\begin{aligned}
\dot{V}_l(t) &= \mathbf{x}^T[\mathbf{A}_l + (1-\alpha_l)\Delta\mathbf{A}_l]^T\mathbf{P}_l\mathbf{x} \\
&\quad + \mathbf{x}^T\mathbf{P}_l[\mathbf{A}_l + (1-\alpha_l)\Delta\mathbf{A}_l]\mathbf{x} \\
&\quad + \mathbf{u}^T[\mathbf{B}_l + (1-\alpha_l)\Delta\mathbf{B}_l]^T\mathbf{P}_l\mathbf{x} \\
&\quad + \mathbf{x}^T\mathbf{P}_l[\mathbf{B}_l + (1-\alpha_l)\Delta\mathbf{B}_l]\mathbf{u} \\
&= \mathbf{x}^T\left[\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l + (1-\alpha_l)(\Delta\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\Delta\mathbf{A}_l)\right]\mathbf{x} \\
&\quad + \mathbf{x}^T\left[\mathbf{K}_l^T\mathbf{B}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{B}_l\mathbf{K}_l\right. \\
&\quad \left.+ (1-\alpha_l)(\mathbf{K}_l^T\Delta\mathbf{B}_l^T\mathbf{P}_l + \mathbf{P}_l\Delta\mathbf{B}_l\mathbf{K}_l)\right]\mathbf{x} \\
&= \mathbf{x}^T\left[\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l + \mathbf{K}_l^T\mathbf{B}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{B}_l\mathbf{K}_l\right]\mathbf{x} \\
&\quad + (1-\alpha_l)\mathbf{x}^T\left[\Delta\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\Delta\mathbf{A}_l + \mathbf{K}_l^T\Delta\mathbf{B}_l^T\mathbf{P}_l\right. \\
&\quad \left.+ \mathbf{P}_l\Delta\mathbf{B}_l\mathbf{K}_l\right]\mathbf{x} \\
&= \mathbf{x}^T\left[\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l + \mathbf{K}_l^T\mathbf{B}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{B}_l\mathbf{K}_l\right]\mathbf{x} \\
&\quad + (1-\alpha_l)\mathbf{x}^T\left[\mathbf{N}_{A_l}^T\mathbf{F}_{A_l}^T\mathbf{M}_{A_l}^T\mathbf{P}_l + \mathbf{P}_l\mathbf{M}_{A_l}\mathbf{F}_{A_l}\mathbf{N}_{A_l}\right]\mathbf{x} \\
&\quad + (1-\alpha_l)\mathbf{x}^T\left[\mathbf{K}_l^T\mathbf{N}_{B_l}^T\mathbf{F}_{B_l}^T\mathbf{M}_{B_l}^T\mathbf{P}_l + \mathbf{P}_l\mathbf{M}_{B_l}\mathbf{F}_{B_l}\mathbf{N}_{B_l}\mathbf{K}_l\right]\mathbf{x}
\end{aligned}$$

Since for any positive scalar $\mu > 0$ and real matrices $\mathbf{Y}$ and $\mathbf{Z}$ we have (Petersen, 1987):

$$\mathbf{Z}\mathbf{Y}^T + \mathbf{Y}\mathbf{Z}^T \leq \mu\mathbf{Y}\mathbf{Y}^T + \frac{1}{\mu}\mathbf{Z}\mathbf{Z}^T \quad (41)$$

It follows that:

$$\begin{aligned}
&\mathbf{N}_{A_l}^T\mathbf{F}_{A_l}^T\mathbf{M}_{A_l}^T\mathbf{P}_l + \mathbf{P}_l\mathbf{M}_{A_l}\mathbf{F}_{A_l}\mathbf{N}_{A_l} \\
&\quad \leq \mu_l^1\mathbf{P}_l\mathbf{M}_{A_l}\mathbf{M}_{A_l}^T\mathbf{P}_l + \frac{1}{\mu_l^1}\mathbf{N}_{A_l}^T\mathbf{N}_{A_l} \\
&\mathbf{K}_l^T\mathbf{N}_{B_l}^T\mathbf{F}_{B_l}^T\mathbf{M}_{B_l}^T\mathbf{P}_l + \mathbf{P}_l\mathbf{M}_{B_l}\mathbf{F}_{B_l}\mathbf{N}_{B_l}\mathbf{K}_l \\
&\quad \leq \mu_l^2\mathbf{P}_l\mathbf{M}_{B_l}\mathbf{M}_{B_l}^T\mathbf{P}_l + \frac{1}{\mu_l^2}\mathbf{K}_l^T\mathbf{N}_{B_l}^T\mathbf{N}_{B_l}\mathbf{K}_l
\end{aligned}$$

$$\begin{aligned}
\dot{V}_l(t) &\leq \mathbf{x}^T\left[\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l + \mathbf{K}_l^T\mathbf{B}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{B}_l\mathbf{K}_l\right]\mathbf{x} \\
&\quad + (1-\alpha_l)\mathbf{x}^T\left[\mu_l^1\mathbf{P}_l\mathbf{M}_{A_l}\mathbf{M}_{A_l}^T\mathbf{P}_l + \frac{1}{\mu_l^1}\mathbf{N}_{A_l}^T\mathbf{N}_{A_l}\right]\mathbf{x} \\
&\quad + (1-\alpha_l)\mathbf{x}^T\left[\mu_l^2\mathbf{P}_l\mathbf{M}_{B_l}\mathbf{M}_{B_l}^T\mathbf{P}_l + \frac{1}{\mu_l^2}\mathbf{K}_l^T\mathbf{N}_{B_l}^T\mathbf{N}_{B_l}\mathbf{K}_l\right]\mathbf{x}
\end{aligned}$$

Since

$$\mathbf{x}(t) \in \boldsymbol{\Omega}_l^s \Rightarrow 1 - \alpha_l \leq 1 - \underline{\alpha}_l$$

then

$$\begin{aligned}
\dot{V}_l(t) &\leq \mathbf{x}^T\left[\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l + \mathbf{K}_l^T\mathbf{B}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{B}_l\mathbf{K}_l\right]\mathbf{x} \\
&\quad + (1-\underline{\alpha}_l)\mathbf{x}^T\mathbf{P}_l\left[\mu_l^1\mathbf{M}_{A_l}\mathbf{M}_{A_l}^T\mathbf{P}_l + \mu_l^2\mathbf{P}_l\mathbf{M}_{B_l}\mathbf{M}_{B_l}^T\right]\mathbf{P}_l\mathbf{x} \\
&\quad + (1-\underline{\alpha}_l)\mathbf{x}^T\left[\frac{1}{\mu_l^1}\mathbf{N}_{A_l}^T\mathbf{N}_{A_l} + \frac{1}{\mu_l^2}\mathbf{K}_l^T\mathbf{N}_{B_l}^T\mathbf{N}_{B_l}\mathbf{K}_l\right]\mathbf{x}
\end{aligned}$$

$$\begin{aligned}
\dot{V}_l(t) &\leq \mathbf{x}^T\big(\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l - \mathbf{P}_l\mathbf{B}_l\tilde{\mathbf{R}}_l^{-1}\mathbf{B}_l^T\mathbf{P}_l + \tilde{\mathbf{Q}}_l + \mathbf{P}_l\mathbf{H}_l\mathbf{P}_l\big)\mathbf{x} \\
&\quad - \mathbf{x}^T\big(\mathbf{Q}_l + \mathbf{K}_l^T\mathbf{R}_l\mathbf{K}_l\big)\mathbf{x} + \mathbf{x}^T\mathbf{K}_l^T\big(\mathbf{B}_l^T\mathbf{P}_l + \tilde{\mathbf{R}}_l\mathbf{K}_l\big)\mathbf{x}
\end{aligned}$$

Since

$$\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l - \mathbf{P}_l\mathbf{B}_l\tilde{\mathbf{R}}_l^{-1}\mathbf{B}_l^T\mathbf{P}_l + \tilde{\mathbf{Q}}_l + \mathbf{P}_l^T\mathbf{H}_l\mathbf{P}_l = 0$$

and

$$\mathbf{K}_l = -\tilde{\mathbf{R}}_l^{-1}\mathbf{B}_l^T\mathbf{P}_l \Rightarrow \tilde{\mathbf{R}}_l\mathbf{K}_l = -\mathbf{B}_l^T\mathbf{P}_l \Rightarrow \mathbf{B}_l^T\mathbf{P}_l + \tilde{\mathbf{R}}_l\mathbf{K}_l = 0$$

It yields

$$\dot{V}_l(t) \leq -\mathbf{x}^T\left(\mathbf{Q}_l + \mathbf{K}_l^T\mathbf{R}_l\mathbf{K}_l\right)\mathbf{x} < 0 \quad (42)$$

$$\dot{V}_l(t) \leq -\lambda_{\min}\left(\mathbf{Q}_l + \mathbf{K}_l^T\mathbf{R}_l\mathbf{K}_l\right)\|\mathbf{x}(t)\|^2 \quad (43)$$

In each subspace, the command is given by

$$\mathbf{u}_l(t) = -\tilde{\mathbf{R}}_l^{-1}\mathbf{B}_l^T\mathbf{P}_l\mathbf{x}(t) \quad (44)$$

In order to maximise the region of stability of each subregion Ω_l^s, the minimal value that guarantee the stability is obtained by solving the following minimization program:

$$\begin{aligned}
&\underset{\mathbf{P}_l,\mathbf{Q}_l,\mathbf{R}_l,\mu_l^1,\mu_l^2}{\text{minimize}} \ \alpha_l \\
&\text{s.t. } \mathbf{P}_l = \mathbf{P}_l^T > 0, \mathbf{Q}_l > 0, \mathbf{R}_l > 0, \mu_l^1 > 0, \mu_l^2 > 0 \\
&\mathbf{A}_l^T\mathbf{P}_l + \mathbf{P}_l\mathbf{A}_l - \mathbf{P}_l\mathbf{B}_l\widetilde{\mathbf{R}}_l^{-1}\mathbf{B}_l^T\mathbf{P}_l + \widetilde{\mathbf{Q}}_l + \mathbf{P}_l\mathbf{H}_l\mathbf{P}_l = 0
\end{aligned} \quad (45)$$

Note that this minimization program has always a solution $\underline{\alpha}_l < 1$ since we assume that the local systems are controllable.

Let $\Omega_l^c \subseteq \Omega_l^s$ be the state subspace associated with the state feedback $\mathbf{K}_l$ and $\tau_i, i = 1, 2, \ldots, N$, the ith time instant at which the state meets the boundary of a subregion $\Omega_j^c, j = 1, 2, \ldots, r$. We assume that the state $\mathbf{x}(t)$ does not jump at the transition time τ_i, that is (Feng, 2001)

$$\mathbf{x}(\tau_i^-) = \mathbf{x}(\tau_i) = \mathbf{x}(\tau_i^+), \quad i = 1, 2, \ldots, N \tag{46}$$

Lemma 1: *The fuzzy system (21) is globally stable if N is finite ($N < \infty$) and there exists, at each moment t, at least one integer $1 \leq k \leq r$ so that:*

$$\alpha_k(z(t)) \geq \underline{\alpha}_k \tag{47}$$

or

$$\bigcup_{i=1}^{r} \Omega_i^s = \Omega \tag{48}$$

Proof: Consider the following piecewise quadratic Lyapunouv function candidate:

$$V(t) = \sum_{l=1}^{r} \zeta_l(\mathbf{x}(t))\mathbf{x}^T(t)\mathbf{P}_l\mathbf{x}(t) \tag{49}$$

where

$$\zeta_l(\mathbf{x}(t)) = \begin{cases} 1 & \mathbf{x}(t) \in \Omega_l^c \\ 0 & \text{otherwise} \end{cases} \tag{50}$$

if τ_i is the time instant at which the state leaves the subregion Ω_j^c and enters into the subregion Ω_k^c then

$$V(\tau_i^-) = \mathbf{x}^T(\tau_i^-)\mathbf{P}_j\mathbf{x}(\tau_i^-) = \mathbf{x}^T(\tau)\mathbf{P}_j\mathbf{x}(\tau) \tag{51}$$

$$V(\tau_i^+) = \mathbf{x}^T(\tau_i^+)\mathbf{P}_k\mathbf{x}(\tau_i^+) = \mathbf{x}^T(\tau)\mathbf{P}_k\mathbf{x}(\tau) \tag{52}$$

The local symmetric positive matrices $\mathbf{P}_l, l = 1, 2, \ldots, r$, are determined so as to guarantee the local stability:

$$(42) \Rightarrow \frac{\dot{V}(t)}{V(t)} \leq -\frac{\mathbf{x}^T(t)\left(\mathbf{Q}_l + \mathbf{K}_l^T\mathbf{R}_l\mathbf{K}_l\right)\mathbf{x}(t)}{\mathbf{x}^T(t)\mathbf{P}_l\mathbf{x}(t)}$$

$$\leq -\sigma_l, \sigma_l = \frac{\lambda_{\min}\left(\mathbf{Q}_l + \mathbf{K}_l^T\mathbf{R}_l\mathbf{K}_l\right)}{\lambda_{\max}(\mathbf{P}_l)} \quad \mathbf{x}(t) \in \Omega_l^c,$$

$$\tau_i^+ < t < \tau_{i+1}^-, \quad i = 1, 2, \ldots, N$$

$$V(t) > 0, \quad \mathbf{x}(t) \neq 0 \Rightarrow \frac{d(\ln(V(t)))}{\mathrm{d}t} \leq -\sigma_l \Rightarrow V(t)$$

$$\leq V(\tau_i^+)e^{-\sigma_l(t-\tau_i^+)}\tau_i^+ < t < \tau_{i+1}^-, \quad i = 1, 2, \ldots, N$$

since

$$\lambda_{\min}(\mathbf{P}_l)\,\|\mathbf{x}(t)\|^2 \leq V(t) \leq \lambda_{\max}(\mathbf{P}_l)\,\|\mathbf{x}(t)\|^2,$$

$$\tau_i^+ < t < \tau_{i+1}^-, \quad i = 1, 2, \ldots, N$$

It follows that

$$\|\mathbf{x}(t)\| \leq C_l\|\mathbf{x}(\tau_i)\|e^{-\frac{\sigma_l}{2}(t-\tau_i^+)}, \quad C_l = \sqrt{\frac{\lambda_{\max}(\mathbf{P}_l)}{\lambda_{\min}(\mathbf{P}_l)}}$$

$$\tau_i^+ < t < \tau_{i+1}^-, \quad i = 1, 2, \ldots, N$$

Since the number of transition is finite, $N < \infty$ then

$$\|\mathbf{x}(t)\| \leq C_{l_0}\|\mathbf{x}(\tau_N)\|e^{-\frac{\sigma_{l_0}}{2}(t-\tau_N^+)}, \quad t > \tau_N^+ \tag{53}$$

At the Nth transition ($t = \tau_N^+$) the state enters into the subregion $\Omega_{l_0}^c$ containing the origin and converges to the origin at $t \to \infty$.

$$\mathbf{x}(t) \in \Omega_{l_0}^c, \quad t > \tau_N^+ \quad \text{and} \quad \|\mathbf{x}(t)\| \underset{t\to\infty}{\to} 0 \tag{54}$$

The fuzzy system is globally stable.

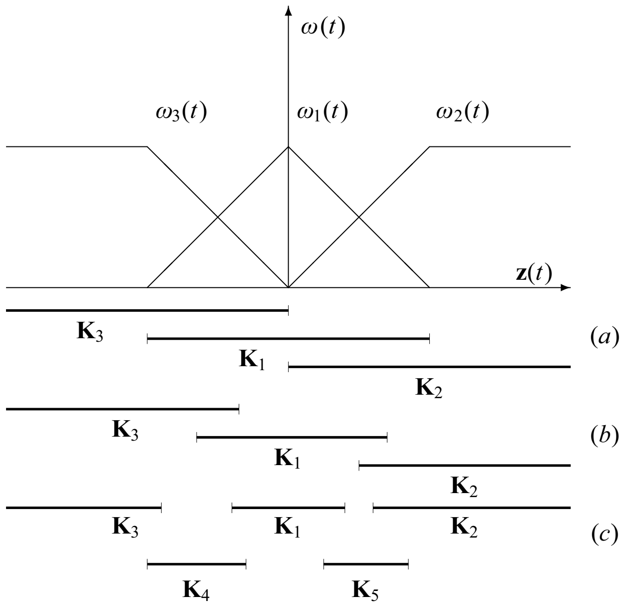
Remark: *If the transition time instants are not finite, some other conditions have to be imposed (Feng, 2001).*

Since several rules may satisfy the condition (47), in this case the control is inferred by selecting the control of the dominant system whose membership degree is of maximum distance from its guaranteed stability boundary:

$$\mathbf{u}(t) = \mathbf{K}_l\mathbf{x}(t), \quad l = \arg\max_{i=1,r}\,(\alpha_i(\mathbf{z}(t)) - \underline{\alpha}_i) \tag{55}$$

The resolution of the r independent minimization programs (45) leads to three possible cases as shown in Figure 2.

Figure 2 Possible cases



Case 1: *Several or all $\underline{\alpha}_l = 0$, $l = 1, 2, \ldots, r$, Figure 2(a), a local controller can be used to stabilise the fuzzy system in its own local subregion and in adjacent subregions and the number of controllers can be reduced. The number of controllers is inferior to the number of rules. In Figure 2(a), the state feedback gain $\boldsymbol{K}_1$ is sufficient to control the fuzzy system.*

Case 2: *If the number of controllers cannot be reduced and the condition (48) is fulfilled then the number of controllers is equal to the number of rules, Figure 2(b).*

Case 3: *If the condition (48) is not fulfilled, the global system may be unstable. To solve this problem, we can add new rules to the model since we know exactly in which region, in the state space, we need new ones, or we can add new controllers, $\boldsymbol{K}_4$ and $\boldsymbol{K}_5$ in Figure 2(c), without changing the model by using new nominal local systems which is equivalent to the addition of new rules to the model.*

In Tanaka et al. (1998), the controller has the same number of rules as the model, the design procedure is based on checking the existence of a common symmetric positive definite matrix using LMIs. The solvability of the LMIs may be impossible in many cases especially when the number of rules is large. Using a simplified model with few rules is the alternative proposed in such situations, but this reduction of rules decreases the accuracy of the model. In our approach the number of controllers can be less than the number of rules. Adding new rules rather than simplifying the original model, using more accurate model, is used when the stability conditions are not fulfilled. Since we know exactly in which regions in the state space we need new rules the addition of new rules is straightforward. However, the global stability is assured with the assumption that the transition time instants are finite.

3.1 Design procedure

The design procedure of the switching controller can be summarised in the following steps:

Step 1 Obtain the fuzzy plant model of the non-linear plant by means of the methods in Takagi and Sugeno (1985), Sugeno and Kang (1988) and Cao et al. (1997), or other suitable ways.

Step 2 Determine the subsystems matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ $i = 1, \cdots, r$.

Step 3 Choose the suitable matrices $\mathbf{M}_{A_i}$, $\mathbf{N}_{A_i}$, $\mathbf{M}_{B_i}$ and $\mathbf{N}_{B_i}$ for each local model.

Step 4 For each subsystem, solve the corresponding minimization program (45).

Step 5 Check if the stability condition (48) is satisfied, otherwise, go to *Step 2* and choose other values for the free design parameters or add new controllers until the stability condition (48) is fulfilled.

4 Simulation example

To show the effectiveness of the proposed method, we simulate the control of the chaotic Lorenz system. The control objective is to drive its chaotic trajectory to the origin. The Lorenz equations are as follows (Lee et al., 2001):

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -\sigma x_1(t) + \sigma x_2(t) \\ r x_1(t) - x_2(t) - x_1(t)x_3(t) \\ x_1(t)x_2(t) - b x_3(t) \end{bmatrix} \tag{56}$$

The nominal values of (σ, r, b) are (10,28,8/3) for chaos to emerge. An exact fuzzy modelling is employed to construct fuzzy model for the chaotic systems. It utilises the concept of sector non-linearity (Takagi and Sugeno, 1985). Assume that $x_1(t) \in [M_1, M_2]$, then we can have the following two rules fuzzy model which exactly represents the non-linear equation under $x_1(t) \in [M_1, M_2]$.

R^1: if $x_1(t)$ is about M_1 Then $\dot{x}(t) = \mathbf{A}_1\mathbf{x}(t)$

R^2: if $x_1(t)$ is about M_2 Then $\dot{x}(t) = \mathbf{A}_2\mathbf{x}(t)$.

where

$$\mathbf{A}_1 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_1 \\ 0 & M_1 & -b \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} -\sigma & \sigma & 0 \\ r & -1 & -M_2 \\ 0 & M_2 & -b \end{bmatrix} \tag{57}$$

and

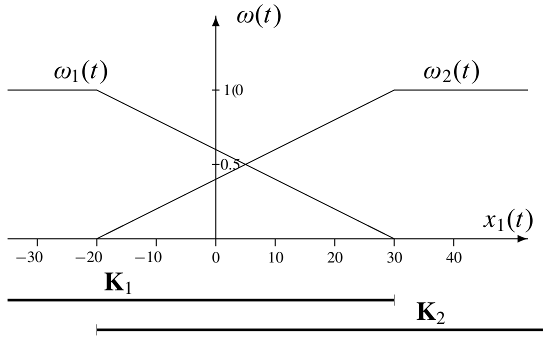
$$M_1 = -20, \quad M_2 = 30 \tag{58}$$

The membership functions, shown in Figure 3, are chosen as:

$$\omega_1(\mathbf{x}(t)) = \begin{cases} \dfrac{-x_1(t) + M_2}{M_2 - M_1} & \text{if } -20 \le x_1(t) \le 30 \\ 1.0 & \text{if } x_1(t) < -20 \\ 0 & \text{if } x_1(t) > 30 \end{cases} \tag{59}$$

$$\omega_2(\mathbf{x}(t)) = \begin{cases} \dfrac{x_1(t) - M_1}{M_2 - M_1} & \text{if } -20 \le x_1(t) \le 30 \\ 1.0 & \text{if } x_1(t) > 30 \\ 0 & \text{if } x_1(t) < -20 \end{cases} \tag{60}$$

Figure 3 Possible cases



The input matrices $\mathbf{B}_1$ and $\mathbf{B}_2$ are chosen as

$$\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{61}$$

The fuzzy model can be decomposed into two subsystems:

- *Subsystem 1:*

$$\dot{\mathbf{x}}(\mathbf{t}) = [\mathbf{A}_1 + (1 - \alpha_1)\Delta\mathbf{A}_1]\,\mathbf{x}(t) + [\mathbf{B}_1 + (1 - \alpha_1)\Delta\mathbf{B}_1]\,\mathbf{u}(t)$$

$$\mathbf{A}_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 20 \\ 0 & -20 & -2.6667 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Delta\mathbf{A}_1 = \alpha_2'(t)\,(\mathbf{A}_2 - \mathbf{A}_1) = \alpha_2'(t) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 50 \\ 0 & -50 & 0 \end{bmatrix},$$

$$\Delta\mathbf{B}_1 = 0$$

- *Subsystem 2:*

$$\dot{\mathbf{x}}(\mathbf{t}) = [\mathbf{A}_2 + (1 - \alpha_2)\Delta\mathbf{A}_2]\,\mathbf{x}(t) + [\mathbf{B}_2 + (1 - \alpha_2)\Delta\mathbf{B}_2]\,\mathbf{u}(t)$$

$$\mathbf{A}_2 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -30 \\ 0 & 30 & -2.6667 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Delta\mathbf{A}_2 = \alpha_1'(t)\,(\mathbf{A}_1 - \mathbf{A}_2) = \alpha_1'(t)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -50 \\ 0 & 50 & 0 \end{bmatrix},$$

$$\Delta\mathbf{B}_2 = 0$$

$\Delta\mathbf{A}_1$ and $\Delta\mathbf{A}_2$ can be written as:

$$\Delta\mathbf{A}_1 = \mathbf{M}_{A_1}\mathbf{F}_{A_1}(\alpha_2'(t))\mathbf{N}_{A_1}$$

$$\mathbf{M}_{A_1} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \\ 5 & 0 \end{bmatrix}, \quad \mathbf{N}_{A_1} = \begin{bmatrix} 0 & 10 & 0 \\ 0 & 0 & -10 \end{bmatrix}$$

$$\mathbf{F}_{A_1}(\alpha_2'(t)) = \begin{bmatrix} \alpha_2'(t) & 0 \\ 0 & \alpha_2'(t) \end{bmatrix}$$

$$\Delta\mathbf{A}_2 = \mathbf{M}_{A_2}\mathbf{F}_{A_2}(\alpha_1'(t))\mathbf{N}_{A_2}$$

$$\mathbf{M}_{A_2} = \begin{bmatrix} 0 & 0 \\ 0 & -5 \\ -5 & 0 \end{bmatrix}, \quad \mathbf{N}_{A_2} = \begin{bmatrix} 0 & 10 & 0 \\ 0 & 0 & -10 \end{bmatrix}$$

$$\mathbf{F}_{A_2}(\alpha_1'(t)) = \begin{bmatrix} \alpha_1'(t) & 0 \\ 0 & \alpha_1'(t) \end{bmatrix}$$

The values obtained after the resolution of the minimization program (45) with:

$$\mathbf{Q}_1 = \mathbf{Q}_2 = \mathbf{I}_3, \quad \mu_1^1 = \mu_2^1 = 0.1, \quad \mathbf{R}_1 = \mathbf{R}_2 = 0.1\mathbf{I}_2$$

- *Subsystem 1:*

$$\underline{\alpha}_1 = 0$$

$$\mathbf{P}_1 = \begin{bmatrix} 5.0595 & 3.6705 & 0.7583 \\ 3.6705 & 11.8115 & 0.1092 \\ 0.7583 & 0.1092 & 11.2293 \end{bmatrix},$$

$$\mathbf{K}_1 = \begin{bmatrix} -36.7048 & -118.1146 & -1.0918 \\ -7.5833 & 1.0918 & -112.2734 \end{bmatrix}$$

- *Subsystem 2:*

$$\underline{\alpha}_2 = 0$$

$$\mathbf{P}_2 = \begin{bmatrix} 4.8170 & 3.4679 & -1.0947 \\ 3.4679 & 11.7632 & -0.1478 \\ -1.0947 & -0.1478 & 11.2528 \end{bmatrix},$$

$$\mathbf{K}_2 = \begin{bmatrix} -34.6792 & -117.6321 & 1.4783 \\ 10.7489 & 1.4783 & -112.5280 \end{bmatrix}$$

The boundary of the two subspaces are determined by $\underline{\alpha}_1 = \underline{\alpha}_2 = 0$, Figure 3, which means that the chaotic system can be controlled using only one state feedback $\mathbf{u}(t) = \mathbf{K}_1\mathbf{x}(t)$ or $\mathbf{u}(t) = \mathbf{K}_2\mathbf{x}(t)$. The initial states are $\mathbf{x}(0) = [10, 10, 10]^T$ and the simulation time is 40 sec. The control input is activated at $t = 10$ sec using the linear state feedback $\mathbf{u}(t) = \mathbf{K}_1\mathbf{x}(t)$. Before the activation of the control the phase trajectory of the Lorenz system was chaotic. However, after the activation of the control the phase trajectory is quickly directed to the origin as shown in Figures 4 and 5.

Figure 4 The phase trajectory of the controlled Lorenz chaotic system

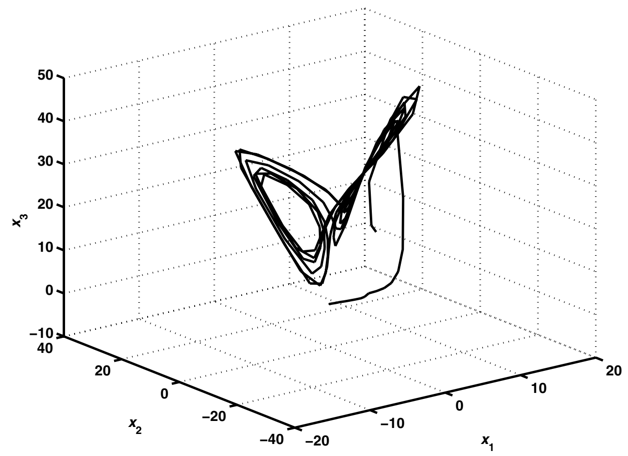
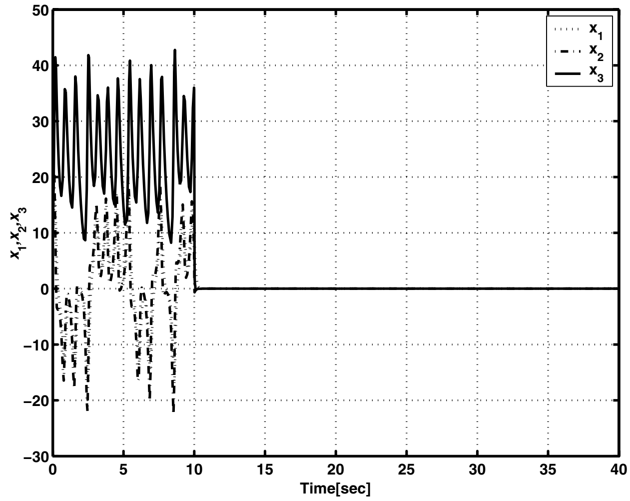


Figure 5 States of the Lorenz chaotic system



5 Conclusion

In this paper a Lyapunov-based method has been proposed to design a fuzzy model based switching controller for non-linear systems. The fuzzy model is represented as a set of uncertain linear systems. A local controller is designed so that the stability region of the corresponding local subsystem is maximised. Under some conditions this switching controller has the ability to stabilise the non-linear system. The control of the chaotic Lorenz system has been used to demonstrate the effectiveness of this approach.

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