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# Effect of the angle of attack on the wind convection coefficient

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#### Abstract

In this study the effect of the positive angle of attack (angle between flat plate surface and incoming uniform flow) on the convective heat transfer coefficient was investigated numerically. In the case of inviscid flow, this effect was also presented analytically and was found to be in good agreement with the corresponding numerical results. From the obtained numerical data, an accurate correlation equation of Nusselt number was proposed by introducing the effect of the angle of attack in terms of a new factor  $A_f$ . The variation of the convective heat transfer coefficient as a function of the angle of attack was found not behaves in the same manner for both small and large values of Prandtl number at small angles of attack.

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Keywords: Wind convection coefficient; Boundary layer equation; Forced convection over a flat plate; Nusselt number; Angle of attack; Correlation equation

## 1. Introduction

The evaluation of heat transfer rate from/to solid bodies with different geometries such as collectors, automobiles, buildings, towers, ... etc. is very important in the optimization of the industrial systems' performance. To help engineers and designers in the research works, convective heat transfer is usually expressed as linear function of temperature via Newton's law. The convective heat transfer coefficient which relates them has been the subject of many theoretical, numerical and experimental investigations.

In solar energy conversion, the heat losses from flat plate collectors' surface to outside winds are strongly depends on wind velocity. This dependence is generally classified under the following three common forms as mentioned by Palyvos (2008) which has reviewed almost existing equations of wind convection coefficient in suitable tabulation with critical discussion and produced simple average correlations for windward and leeward surface:

Linear equation form :  $h_W = a + bV$ 

Power law equation form :  $h_W = a + bV^n$ 

Boundary layer equation form :  $Nu = aRe^{n}Pr^{m} + b$ 

where a, b, n and m are empirical constants depend on the Prandtl number, the type of flow and the geometrical characteristics. Turgut and Onur (2009) have carried out three dimensional numerical and experimental study to determine the average heat transfer coefficients for laminar forced convection air flow over a rectangular flat plate; they observed that as the angle of attack decreases, the average Nusselt number weakly increases. Sparrow et al. (1979) have conducted a series of experiments on rectangular flow in a wind tunnel and found practically no effect of angle of attack on heat transfer. With the aim to arrive at a

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Nomenc	lature
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$egin{array}{c} A_f \ D \ h \end{array}$	angle of attack factor thermal diffusion (m <sup>2</sup> /s) convective heat transfer coefficient (W/m <sup>2</sup> K)	u,v V x,v	velocity components (m/s) wind velocity (m/s) cartesian coordinates (m)
k k	thermal conductivity (W/m K)	Greek	
L	flat plate length (m)	α	angle of attack (°)
$Nu_L$	average Nusselt number $(hL/k)$	$\eta,\eta^+$	dimensionless independent variables
$\text{Pe}_L$	Peclet number $(VL/D)$	v	kinematic viscosity (m <sup>2</sup> /s)
Pr	Prandtl number $(v/D)$	$\theta$	dimensionless temperature
$\operatorname{Re}_L$	Reynolds number $(VL/v)$	Subscr	ipt
Т	temperature (K)	W	wind, wall
U	potential velocity (m/s)		

consensus on which of wind convection coefficient equations is more accurate, Sartori (2006) has carried out various comparisons among well known equations; and he found that the consensual one is that comes from the boundary layer theory. Onur (1993) has investigated experimentally the laminar forced convective heat loss from the surface of flat plate collectors flush mounted on the roof of a model residential house; the results were collected to determine the average heat transfer coefficient over the surface of the collector which was inclined relative to the incoming air flow. Empirical correlations of the convective heat transfer coefficient as a function of wind velocity and direction, and surface to air temperature difference at external building wall surfaces and roofs, have been developed by Clear et al. (2003) and Emmel et al. (2007). Assuming an inviscid flow, analytical solutions for heat and mass transfer to fluids flowing across an isothermal flat plate were obtained by Kendoush (2009), and a new relation of heat transfer coefficient was derived.

Actually, the collector surface that is exposed to the wind flow is usually inclined as shown in Fig. 1 to optimize the reception of incident solar radiation, and this results a pressure gradient along the collector surface. However, the most commonly used equation to predict the heat loss from solar collectors is based on the flow with zero angle of attack which is not appropriate to use when the angle of



Fig. 1. Schematic of flat plate solar collector.

attack increases (Onur, 1993). Therefore, our objective in the present work is to investigate the effect of the pressure gradient due to the increasing angle of attack of flat plate collectors in uniform wind flow on the convective heat transfer coefficient to provide more accurate predictions. The results were correlated in terms of angle of attack factor  $A_f$  which was generalized for various values of Prandtl number. A comparison of the results obtained with those of the previous investigations was presented.

# 2. Mathematical analysis

#### 2.1. Governing equations

As demonstrated by Sartori (2006), the wind convection coefficient equation comes from the boundary layer theory is the most accurate one. Therefore, the present analysis is based on the concept of this theory. The physical problem as shown in Fig. 1 represents forced convection heat transfer over an inclined heated flat plate which could be, in particular, a solar collector surface.

The problem statement concerns a steady laminar incompressible flow of a Newtonian fluid with constant thermo physical properties, over a flat plate with constant temperature distribution, which is placed in different orientations to the incoming wind flow. The contribution of the viscous dissipation to the energy equation can be neglected, except when the flow is at high velocity or above the velocity of sound (Baehr and Stephan, 2006), and this is not the case. Hence, the basic equations govern this problem are:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2}$$
(1)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = D\frac{\partial^2 T}{\partial y^2}$$
(3)

And they have to be solved subject to the following boundary conditions:

$$u = v = 0, T = T_W \text{ at } y = 0, x > 0$$
  

$$u = U, T = T_{\infty} \text{ at } y \to \infty, x > 0$$
(4)

In order to find similarity solution of the Eqs. (1) and (2), the following reduced equation was given by Falkner and Skan, as quoted by Schlichting (1979):

$$f''' + \frac{m+1}{2}ff'' + m(1 - f'^2) = 0$$
(5)

Likewise, the Eq. (3) is reduced to:

$$\theta'' + \frac{m+1}{2} \Pr f \theta' = 0 \tag{6}$$

And the boundary conditions become:

$$f = f' = 0, \theta = 1 \text{ at } \eta = 0, x > 0$$
  

$$f' = 1, \theta = 0 \text{ at } \eta \to \infty, x > 0$$
(7)

In the above relations, f is the dimensionless stream function, defined by  $\psi = (vUx)^{1/2}f(\eta)$ , and m = (x/U)(dU/dx)is a dimensionless parameter related to the geometry, which influences the free stream velocity profile  $U = cx^m$ , which is a general form still allowing self similar solutions. On the other hand, the parameter m is related to the angle of attack formed by the flat plate collector surface and the wind flow. The primes denote differentiation with respect to the similarity variable  $\eta = y\sqrt{U/xv}$ .

Similarly as the surface roughness factor, our purpose here is to introduce an angle of attack factor  $A_f$  which represents the ratio of the average Nusselt number over an inclined flat plate surface to that over a horizontal flat plate surface:

$$A_f = \frac{\mathrm{Nu}_L(m)}{\mathrm{Nu}_L(0)} \tag{8}$$

The average Nusselt number over a flat plate of arbitrary inclination may be deduced from Fourier's law, combined with Newton's Law of Cooling, as:

$$Nu_{L}(m) = \frac{h(m)L}{k} = -\frac{2\theta'_{W}(m)}{m+1}Re_{L}^{0.5}$$
(9)

where  $\theta'_W$  denotes the dimensionless temperature differentiation with respect to  $\eta$  at the wall, where  $\eta = 0$ . Then, from Eq. (8), we find that the angle of attack factor  $A_f$  has as expression:

$$A_f = \frac{1}{m+1} \frac{\theta'_W(m)}{\theta'_W(0)}$$
(10)

# 2.2. Numerical resolution

The solution of the considered problem could be obtained using different mathematical approaches. The integral method using third-order polynomial for velocity and temperature profile which was fist suggested by Pohlhausen may drive us to a simple and closed approximate solution; but this procedure which has lost its importance with the introduction of electronic computers (Baehr and Stephan, 2006) has been excluded from the present study. We use instead the numerical finite difference method with the Tri Diagonal Matrix Algorithm which leads to rapid convergence to solve Eqs. (5)–(7) consecutively. Computations were performed for small and large values of Prandtl number and for  $0 \le \alpha \le 90^{\circ}$ .

Since the thermal boundary layer thickness is inversely proportional to the Prandtl number, it becomes much thinner as the Prandt number increases and consequently a grid refinement near the flat plate surface is required and vice versa. Therefore, different grid sizes were used to ensure the accuracy of the results presented below and a convergence criterion of  $10^{-4}$  was fixed to finish the iteration process.

# 2.3. Analytical equation of $A_f$ for inviscid flow

When the fluid is considered inviscid (Prandtl number is very small) the friction term disappears from the momentum equation, and the flow is called frictionless (Baehr and Stephan, 2006). Therefore, the longitudinal velocity component u is only a function of the distance from the leading edge, and it is equal to the potential velocity U; and the normal velocity component v could easily be determined from continuity equation:

$$u = U \quad v = -mU\frac{y}{x} \tag{11}$$

In this particular case, only the thermal boundary layer can exist; and its equation can be solved analytically. For that we define here a new dimensionless independent variable as:

$$\eta^+ = y \sqrt{\frac{m+1}{4} \frac{U}{Dx}} \tag{12}$$

Hence, the energy Eq. (3) becomes:

$$\theta'' + 2\eta^+ \theta' = 0 \tag{13}$$

The primes here denote the differentiation with respect to  $\eta^+$ . With the following boundary conditions  $\theta(0) = 1$  and  $\theta(\infty) = 0$ , the solution of this equation is the error function complement:

$$\theta = \operatorname{erfc}(\eta^+) = 1 - \operatorname{erf}(\eta^+) \tag{14}$$

As it is defined earlier in Eq. (8), the angle of attack factor  $A_f$  takes in this case the following form:

$$4_f = (1+m)^{-0.5} \tag{15}$$

# 3. Results and discussion

In most of studies (Onur, 1993; Sartori, 2006; Sparrow et al., 1979; Turgut and Onur, 2009), it was observed that heat transfer coefficient at Prandtl number of 0.71 is not strongly sensitive to the angle of attack over the ranges investigated; therefore, its effect has not been taken into account in the derivation of the heat transfer coefficient

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equation which was suggested to be valid for all angles of attack. However, it has been mentioned that at a fixed Reynolds number the highest heat transfer coefficient is obtained at the smallest of the investigated angle of attack (Turgut and Onur, 2009). This conclusion was confirmed in Fig. 2 in which the variation in the relative heat transfer rate with the angle of attack between 0 and 90° was presented for several values of Prandtl number between  $10^{-5}$  and  $10^{6}$ . It can be observed that the angle of attack factor curves increase into maxima before decreasing in quasi linear way for approximately Pr > 0.1, whereas for  $Pr \leq 0.1$  the curves are fully decreased.

Since the angle of attack has not been specified in the suggested correlations in the most previous works, it is hard to carry out a valuable comparison among the results of these works and those obtained in this study. However, the experimental and numerical results of Turgut and Onur (2009) for angles of attack varying from 25° to 90° could provide significant information about the reduction in heat transfer rate relative to its value at angle of attack of 25°. Fig. 3 shows the variation of the average Nusselt number ratio with the angles of attack  $\alpha = 25^{\circ}$ ,  $45^{\circ}$ ,  $65^{\circ}$  and  $90^{\circ}$ . The comparison between the experimental and numerical results of Turgut and Onur (2009) (by circles and squares) and the results of the present study (by solid line) shows a close agreement. However, these authors consider the fact that experimental as well as numerical results are not strong function of angle of attack because the average variation between two successive angles over the considered range is about 6%. What present a lack of accuracy in this conclusion is, although the average variation is of about 6% between successive angles,  $45^{\circ}$  and  $65^{\circ}$  for example, it could reach to about 16.5% between 25° and 90°.

Another investigation which serves to model analytically the convective heat transfer in the considered problem, was carried out by Kendoush (2009). This provides the possibility of making a comparison between our results and those deduced from the new relation derived in the previous



Fig. 2. Variation of  $A_f$  with  $\alpha$  for various values of Prandtl number.



Fig. 3. Average Nusselt number ratio as a function of angle of attack for air (Pr = 0.71).

reference. First of all, we should indicate that although the relation by Kendoush (2009) was derived under the assumption of inviscid flow, it has been used by the author to evaluate the wind convection coefficient; which may conduct into inaccuracies. However, this relation is given by:

$$Nu_L = 1.2(\cos\beta)^{0.5} Pe_L^{0.5}$$
(16)

In which  $\beta$  is the angle between the incoming wind flow and the normal to the flat plate surface. Introducing the angle of attack  $\alpha$ , Eq. (16) becomes:

$$Nu_L = 1.2(\cos(90 - \alpha))^{0.5} Pe_L^{0.5}$$
(17)

Conflicting with what have already been demonstrated in the present study and the earlier investigations (Onur, 1993; Sartori, 2006; Sparrow et al., 1979; Turgut and Onur, 2009), Eq. (17) shows an increasing in the convective heat transfer with the increasing angle of attack. Thus, this equation does not seem to be physically correct. And instead we have derived an analytical solution for inviscid flow (Subsection 2.3), which is in good agreement with the corresponding numerical results as shown in Fig. 2.

### 4. Conclusion

In this study, the effect of angle of attack on laminar forced convective heat transfer over a flat plate surface has investigated numerically. The results were found to be in good agreement with the experimental data of Turgut and Onur (2009) for Pr = 0.71; and in excellent agreement with the analytical solution (15) for inviscid flow.

It has been found that more Prandtl number is small more the effect of angle of attack is preponderant. For air the reduction in convective heat transfer, over the considered range of angle of attack, could reach to about 16.5%, whereas for inviscid flow, it could reach to about C. Mahboub et al. | Solar Energy 85 (2011) 776-780

с

0.00 0.94 0.96

0.99

0.98

0.98

Values of a, b and c for several values of Pr.				
Pr	а	b		
0.00	1.00	0.00		
0.01	1.08	0.91		
0.10	1.21	0.90		
0.71	1.36	0.88		

Table 1

1.58

1.74

29%. It has also been found that the variation in convective heat transfer modifies substantially its behavior for large values of Prandtl number relative to its behavior for small values.

0.87

0.86

From the results obtained previously, the angle of attack factor  $A_f$  could be formulated empirically for different values of Prandtl number and for  $0 \le m \le 1$ , under the following relationship:

$$A_f = \frac{1 + a m^b}{1 + m^c} (1 + m)^{-0.5}$$
(18)

In which the parameter m is, see Schlichting (1979), given by:

$$m = \frac{\alpha}{180 - \alpha} \tag{19}$$

where a, b and c are functions of Prandtl number and are given in the Table 1. Hence, the average Nusselt number is given by:

$$Nu_L(\alpha) = A_f * Nu_L(\alpha = 0)$$
<sup>(20)</sup>

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10.0

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