NUMERICAL STUDY OF DEPTH FACTORS FOR UNDRAINED LIMIT LOAD OF STRIP FOOTINGS

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ABSTRACT

Current studies of bearing capacity for shallow foundations tend to rely on the hypothesis of an isolated footing lying on the ground surface. In practice a footing never lies on the ground surface; it is mostly embedded at a depth *D* below the ground surface. This paper focuses on a numerical study using the finite-difference code Fast Lagrangian Analysis of Continua (FLAC), to evaluate the bearing capacity of embedded strip footings. The effect of the embedment is estimated though a depth factor, defined as a ratio of the bearing capacity of a strip footing at a depth *D* to that of a strip footing at the ground surface. The results presented in this paper show that the size and shape of the shear zone and displacement field defining the undrained capacity of shallow foundations under centred vertical loading are dependent on embedment ratio.

KEY WORDS: bearing capacity; depth factors; vertical loads; numerical modelling; strip footing

1 INTRODUCTION

The bearing capacity of a vertically loaded shallow, rough, rigid, foundation in plane strain conditions is generally evaluated using the superposition equation proposed by Terzaghi [1]. Since Terzaghi's founding work, numerous experimental and numerical studies aiming at estimating the ultimate bearing capacity of shallow foundations have been conducted. Analytical solutions to the bearing capacity problem can be classified into the following categories:

- limit equilibrium method (e.g., Terzaghi [1]; Meyerhof [2]);
- slip line method (e.g., Sokolovskii [3]; Hansen [4]);
- limit analysis method (e.g., Michalowski [5]; Salgado et *al*. [6]);
- finite element or finite difference analyses (e.g., Frydman & Burd [8]; Loukidis & Salgado [9]; Mabrouki et *al*. [10]; Gourvenec & Mana [7]; Zhang et *al*. [11]).
- The undrained bearing capacity equation of foundations embedded in clay has the following expression:

$$q_u = c_u N_c d_c + q \tag{1}$$

where N_c is a bearing capacity factor; c_u is a representative undrained shear strength; $q = \gamma D$ is the surcharge at the footing base level; γ is the soil unit weight; *D* is the distance from the ground surface to the base of the foundation element; d_c is a depth factor. Undrained vertical bearing capacity of shallowly embedded foundations has been addressed extensively, through empirical, analytical and numerical studies for a range of foundation/soil interface conditions (Skempton [12]; Meyerhof [13]; Hansen [4]; Salgado et *al*. [6]; Edwards et *al*. [14]; Gourvenec [15], Gourvenec & Mana [7]). Table 1 summarises the expressions proposed by different authors to evaluate the depth factors for undrained bearing capacity.

In this paper, a series of numerical computations using the finite difference code FLAC are carried out to evaluate the influence of the depth of footing on the undrained bearing capacity. The depth factor was calculated for rigid rough and smooth strip footing, subjected to centred vertical load. The numerical results are compared with the available publications in the literature.

Table 1: Expressions for depth factor dc

Reference	Depth factor
Skempton [12]	$d_c = 1 + 0.2\frac{D}{B}$ $d_c = 1.5$ for $\frac{D}{B} \geq 2.5$
Meyerhof [13]	$d_c = 1 + 0.2\frac{D}{B}$
Hansen [4]	$d_c = 1 + 0.4\frac{D}{B}$ for $\frac{D}{B} < 1$ $d_c = 1 + 0.4\tan^{-1}\left(\frac{D}{B}\right)$ for $\frac{D}{B} \geq 1$
Salgado et *al.* [6]	$d_c = 1 + 0.27\sqrt{\frac{D}{B}}$

2 FINITE DIFFERENCE MODEL

The finite-difference code FLAC was used to estimate the bearing capacity of a strip footing of width B=2m embedded at depth D in clay (D/B ratio was varied from 0 to 4) and subjected to a centred vertical load. The analyses were performed under plane strain conditions. Owing to the symmetry in geometry and loading conditions, only half of the domain was discretised. The boundary conditions are shown in Figure 1. The displacement of the left vertical side is constrained in the horizontal direction only. The base and right side of the model is constrained in all directions. Zero thickness interface elements were placed along the boundary EFG to simulate different interface conditions between the footing and the soil. The soil was considered to be a linearly elastic-perfectly plastic material, obeying Tresca criterion (c_u=20 kPa, υ=0.49, E_u=14 MPa and γ=15 kN/m^3). It is noted that the values of the elastic parameters had a small effect on the value of bearing capacity [10]

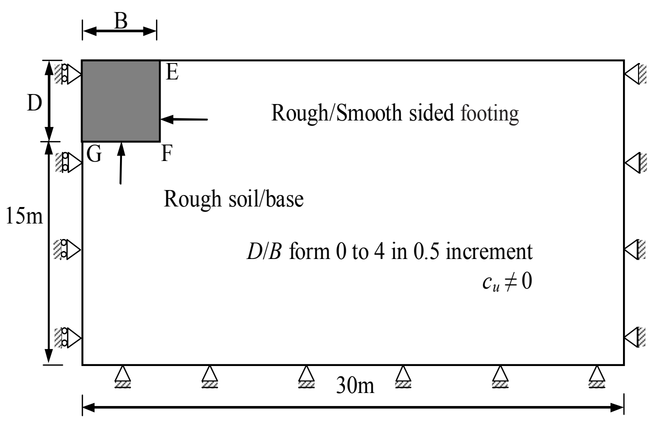

Figure 1 : Problem geometry and boundary conditions

The numerical evaluation of the bearing capacity is based on subdividing the soil into a number of elements. The mesh adopted for all cases studied in this paper, has a depth of 15 m and extends 30 m beyond the line of symmetry, as shown in Figure 2. To simulate the rigid footing, the vertical and horizontal displacements of nodes which discretise the strip footing are constrained in the vertical and horizontal directions.

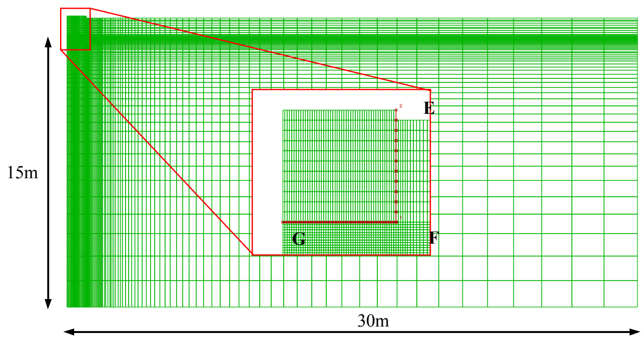

Figure 2: Example of finite difference mesh, D/B=0.5.

The loading of the rigid strip footing is simulated by imposing equal vertical velocities at all nodes representing the footing. The magnitude of chosen vertical velocity is 2×10^{-7}m/s. The rigid footing is connected to the soil via interface elements defined by Coulomb shear-strength criterion. The interface elements along the base FG of foundation always represented a rough interface. They were assigned a cohesion c_u=20 kPa, a normal stiffness k_n=10^9 Pa/m, and a shear stiffness k_s = 10^9 Pa/m. To model the rough interface between the side of the footing and the soil, interface elements along the boundary EF were assigned the same properties as along the base. In the case of a smooth interface, the interface elements along EF were assigned the same normal stiffness and shear stiffness but zero undrained strength.

The progressive movement of the rigid footing induced by the vertical velocity applied at all nodes is accompanied by the increase of the load in the soil. Finally, this load stabilizes for a value that indicates a limit load or bearing capacity. In first case the ultimate load on the footing was calculated as the sum of the vertical reaction forces along the base FG. In the second the resistance from shear stresses mobilised along the side of the footing was included.

3 RESULT AND DISCUSSION

A vertical bearing capacity q_u=5.204c_u was predicted in the present study for the surface footing, representing an overestimation of less than 2% from the analytical solution of q_u=5.14c_u (Prandtl [16]). It should be noted that several preliminary numerical tests have been carried out to study the effect of the mesh size, the refinement of the mesh produce a better results. It means that numerical prediction obtained using FLAC, is in excellent agreement with Prandtl's solution.

Figure 3 shows the result obtained from the finite difference analysis of $N_c d_c = (q_u - q)/c_u$ as a function of the ratio D/B compared with available numerical solutions (Salgado et *al*. [6]; Edwards et *al*. [14], Gourvenec [15]). The results increase substantially with increasing D/B. The values of $d_c N_c$ obtained by the upper-bound approach increasingly diverges from the rough-sided foundations finite difference results predictions with increasing embedment ratio. For $D/B \leq 3$, the result obtained from the present study for rough-sided footing are in excellent agreement with the solutions reported by Edwards et *al.* [14] (using the finite elements analysis) but deviate for $D/B > 3$.

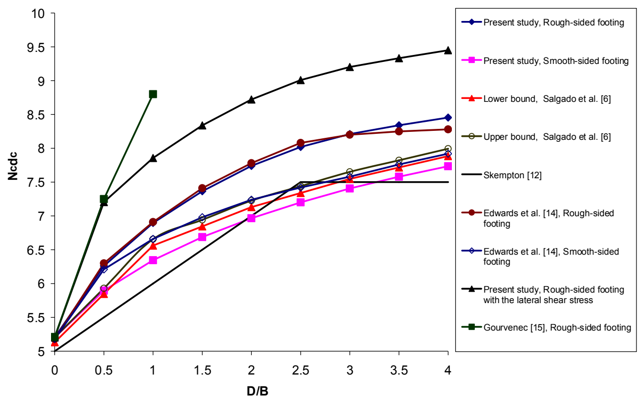

***Figure 3:** Comparison of present $N_c d_c$ values with theoretical and numerical resultsComparison of present $N_c d_c$ values with theoretical and numerical results.*

The values of $d_c N_c$, obtained from the present study for smooth-sided footing solutions are close to the upper- and lower-bound solutions predicted by Salgado et *al*. [6]. Figure 3 also shows the empirical solutions of Skempton [12] for the bearing capacity factors of an embedded strip footing. In the case of a smooth-sided strip footing the present results for $D/B \leq 1.5$ are in good agreement with those obtained by Skempton [12]. It is noted that the results of the bearing capacity obtained from the present study in the case of contribution of both the base of the foundation and its side, increases up to about 30% for $D/B = 4$, when compared with the bearing capacity obtained by the contribution of the base only.

The finite difference results are also used to derive depth factors d_c. This is achieved by dividing the bearing capacities obtained for the footings at depth D by that obtained for the surface footing. The results are presented in Figure 4. The values of d_c proposed by Skempton [12], Meyerhof [13], Hansen [4] and Salgado et *al*. [6] are also presented. The values of d_c obtained from the present study for $D/B \leq 3$ and rough-sided foundations are in good agreement with the results of Edwards et *al*. [14]. For $D/B > 3$, the depth factor obtained with finite difference analyses are slightly greater than those obtained by Edwards et *al*. [14]. The present results of d_c for rough-sided foundations are greater than those obtained by the expressions available in the literature.

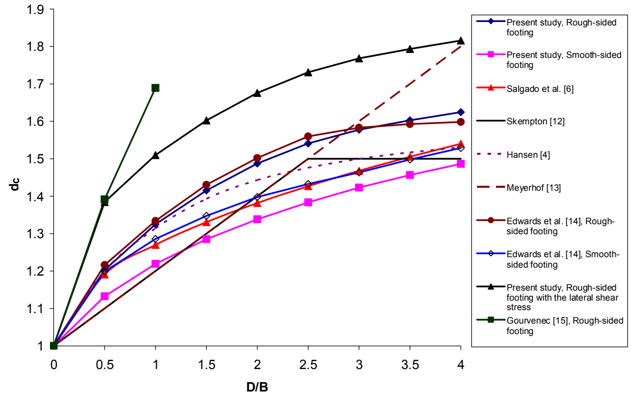

***Figure 4:** Comparison of present d_c values with those obtained from the expressions available in the literature.Comparison of present d_c values with those obtained from the expressions available in the literature.*

Figure 5 shows the contours of maximum shear strain for different embedded footings in the case of a cohesive soil. The plots clearly demonstrate the improved bearing capacity that results from increasing depth. The size of the shear zone increases with increasing value of the depth and a large strain concentration is observed near the corner of the footing with high values of D/B. This behaviour is due to the blocking effect, caused by the weight of the soil situated over the base of the footing. As seen from Figure 5, there is a triangular elastic wedge immediately underneath the rough footing.

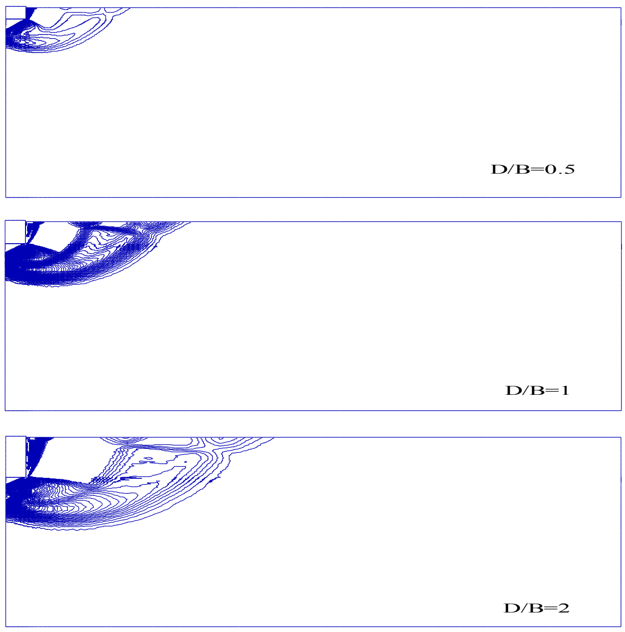

***Figure 5:** Contours of maximum shear strain for rough strip footing: (a) D/B=0.5; (b) D/B=1 and (c) D/B=2.*

Figure 6 shows a comparison of the displacement vector field for footing embedded in soil (D=1m), with footing assumed resting on the soil surface, and the effect of the self-weight of the soil located above the footing base is replaced with a surcharge q=γD. It is noted that the value of the maximum magnitude dmax of the displacement vector of the embedded footing higher than dmax found by replacing the self-weight of the soil located above the footing base by a surcharge q. Also, it is clear from the figure that embedded foundations mobilize a volume of soil larger than mobilize by replacing the self-weight of the soil located above the footing base by a surcharge q.

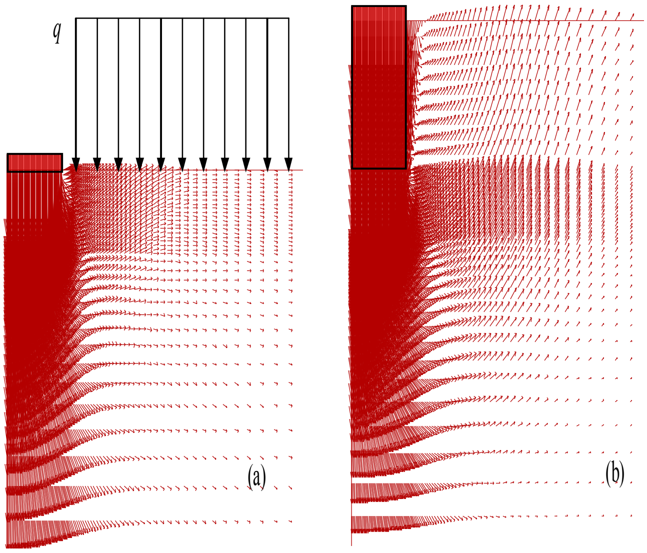

Figure 6: Illustration of difference in displacement vector in two cases: (a) equivalent surcharge use to replace soil above base of footing; (b) footing modelled as an embedded footing

4 CONCLUSIONS

Two-dimensional finite difference analyses of embedded rough and smooth strip footings in clay were performed to study the depth factor. The footings have been considered under centred vertical load. The results of depth factor were compared with existing solutions published in the literature, using the finite elements analysis method and approximations that are commonly used in practice. The results from the finite difference study of the undrained bearing capacity of strip footings embedded in clay have confirmed that the bearing capacity of embedded footing depend on the ratio *D*/*B*, demonstrating that the depth factor depends on the roughness of the vertical sides of the footing.

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