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Face Recognition by Multidimensional Analysis

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In the name of Allah, the Beneficent, the Merciful. Praise and Gratitude be to Allah for giving me strength and guidance, so that this dissertation can be finished accordingly. This thesis is a collection of not only hard work, perseverance and continuous effort over the past years, but also encouragement, cooperation and support from many people who deserve to be acknowledged.

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Abbreviations

MDA: Multilinear disriminant analysis

MPCA: Multilinear principal component analysis

LDA: linear discriminant analysis

PCA: principal component analysis

DNA: Deoxyribonucleic acid

FM: False match

FNM: False non match

FTC: Failure to capture

FTE: Failure to enroll

FAR: False accept rate

FRR: False reject rate

EER: Equal Error Rate

ROC: Receiver Operating Characteristics

LBP: Local binary pattern

PLBP: pyramid of local binary pattern

RI-LBP: rotation invariant local binary pattern

LBPNet: local binary pattern network

CNN: Convolution neural network

MLBP: Multiscale local binary pattern

LTP: Local ternary pattern

LQP: Local quantized pattern

QDA: Quadratic Discriminant Analysis

FDA: Flexible Discriminant Analysis

RDA: Regularized Discriminant Analysis

AFR: Automatic Face Recognition

ATM: Automated teller machine

2-D PCA: two dimensional PCA

ICA: Independent component analysis

SVD: Singular value decomposition

VVP: vector to-vector projection

TTP: the tensor-to-tensor projection

TVP: the tensor-to vector projection

MMD: modified Mahala Nobis distance

MAD: modified angle distance

LFW: Labeled Faces in the Wild

LPQ: Local Phase Quantization

Abstract

There are a lot of biometric techniques, like fingerprints, iris scan, as well as hand geometry, the most efficient and more widely used one is face recognition. This is because it is inexpensive, non-intrusive and natural.

Many algorithms had been proposed for face recognition problems in the last few years, problems such as: lighting, scale and pose at the same time. We will apply a method for improving the solidity of a face recognition algorithm with tensors representation and fusion of the MDA and MPCA these methods are similar to LDA and PCA. A multilinear principal component analysis for tensor objects feature extraction and a multilinear discriminant analysis, also we will use LPQ as feature extraction technique either and it works like LBP, to find the best subspaces. And with this method we are expecting better results with no problems as the other methods.

Keywords: biometrics, Face recognition, multilinear principal component analysis (MPCA), multilinear discriminant analysis (MDA), Local Phase Quantization (LPQ), local binary patterns (LBP), Linear Discriminant Analysis (LDA), Principal Component Analysis (PCA).

الملخص

هناك الكثير من تقنيات القياسات الحيوية ، مثل بصمات الأصابع ، ومسح قزحية العين ، بالإضافة إلى هندسة اليد ، والأكثر كفاءة والأكثر استخدامًا هو التعرف على الوجوه. هذا لأنه غير مكلف وغير تدخلي وطبيعي.

تم اقتراح العديد من الخوارزميات لمشاكل التعرف على الوجوه في السنوات القليلة الماضية ، مشاكل مثل: الإضاءة و المقياس والوضع في نفس الوقت. في هذا البحث ، سنطبق طريقة لتحسين متانة خوارزمية التعرف على الوجوه مع تمثيل الموترات ودمج تحليل المكون الرئيسي متعدد الخطوط و تحليل تمييزي متعدد الخطوط . تم اقتراح تحليل مكون رئيسي متعدد الخطوط لاستخراج ميزة كائن موتر وتحليل تمييزي متعدد الخطوط ، هذه الطرق مشابهة للتحليل التمييزي الخطي و تحليل المكونات الرئيسية, ايضا سوف نستخدم تقنية تكميم الطور المحلي لاستخراج الميزات ايضا و هي طريقة مشابهة لتقنية الانماط الثنائية المحلية,للعثور على أفضل الفراغات الفرعية. وبهذه الطريقة نتوقع نتائج أفضل بدون مشاكل مثل الطرق الأخرى.

الكلمات المفتاحية: القياسات الحيوية ، التعرف على الوجوه ، تحليل المكون الرئيسي متعدد الخطوط ، تحليل التمييز متعدد الخطوط ، تكميم المرحلة المحلية ، الأنماط الثنائية المحلية ، التحليل الخطي المميز ، تحليل المكونات الرئيسية.

General Introduction

The face is our primary focus of attention in social life playing an important role in conveying identity and emotions. We can recognize a few faces learned throughout our lifespan and identify faces at a glance even after years of separation. This skill is quite robust despite of large variations in visual stimulus due to changing condition, aging and distractions such as beard, glasses or changes in hairstyle.

Computational models of face recognition are interesting because they can contribute not only to theoretical knowledge but also to practical applications. Computers that detect and recognize faces could be applied to a wide variety of tasks including criminal identification, security system, image and film processing, identity verification, tagging purposes and human-computer interaction. Unfortunately, developing a computational model of face detection and recognition is quite difficult because faces are complex, multidimensional and meaningful visual stimuli.

Face detection is used in many places now day's especially the websites hosting images like Picasso, photo bucket and Facebook. The automatically tagging feature adds a new dimension to sharing pictures among the people who are in the picture and gives the idea to other people about who the person is in the image. In our project, we have studied and implemented a pretty simple but very effective face detection algorithm which takes human skin color into account. [1]

This project presents a method that can solve the problems in face recognition process and feature extraction. This work will based on multilinear principal component analysis (MPCA) and (MDA) which is multidimensional analysis, to see if there are any effects in our process. And both of the algorithms work with tensor objects so that makes the system doesn't have any issues. Later we will see if there is a progress compare to the other methods. The rest of this project is organized as follows. Section I introduces Facial recognition system as a fast biometric technology. In Section II, I introduce basic multilinear algebra notations and concepts, MPCA and MDA. Then in section III, I applied the fusion of MPCA and MDA algorithms on the face recognition system , after getting results, I will compare it with PCA and LDA algorithms results and see which algorithm is better in recognition accuracy and finally last section conclusion will be presented.

Chapter 1: Face Recognition as a biometric technology

Chapter 1: Face Recognition as a biometric technology

1.1 Introduction

In This chapter I will presents face recognition as a biometric technology; First, I will describe what biometrics is. Next, I will describe Face recognition as biometric software that can identify a person and how does it work, citing their advantages and applications.

1.2 An Overview of the Biometrics System

Biometrics technology shows increasing promise in network security. It's automatically recognizing a person using distinguishing traits. In next year's it will play an increasingly vital role as secure authentication system for different applications. Next, I will explain biometric technology and describes biometric system specifically on face recognition.

1.3 What is Biometrics?

Biometrics is automated methods of recognizing a person based on a physiological or behavioral characteristic. The past of biometrics includes the identification of people by distinctive body features, scars or a grouping of other physiological criteria, such like height, eye color and complexion. The present features are face recognition, fingerprints, handwriting, hand geometry, iris, vein, voice and retinal scan. Biometric technique is now becoming the foundation of a wide array of highly secure identification and personal verification.

The most recognized biometric technologies are fingerprinting, retinal scanning, hand geometry, signature verification, voice recognition, iris scanning and facial recognition. [2]

1.4 Identification and Verification

Biometrics system can be classified into two main categories on the basis of application mode: Verification and Identification In the identification mode, the biometric system identifies an individual by searching the templates of all the individuals whose identification details are stored in the database. In this process, the system conducts a one to many comparisons to prove the identity of a person. In the verification mode, the biometrics information of an individual, who claims certain Identity is compared with his own biometric template stored in the system database. This is also referred as one to one comparison. Biometric technologies find a place in crime detection in a number of ways: (a) The modules and techniques of biometrics help in analyzing the evidence by overcoming the limitations of human cognitive abilities and thus increases efficiency and effectiveness of investigation (b) These methods provide scientific basis (by applying techniques of computer science, applied mathematics and statistics)and standardization for crime investigation procedure by analyzing huge bulk of data which are not humanly possible (c) These techniques provide the advantages of visualizing and documenting the result of analysis.[3]

1.5 Types of Biometrics

Biometric technology is more refined, advanced, and super sensitive. It protects companies and individuals. Most importantly, it is impossible to duplicate biometrics works on biological characteristics of an individual. An individual has two types of biological traits: behavioral and physical [4].

1.5.1 Behavioral Biometrics: Biometrics is the scientific study of how people's and animal's bodies function. It is further divided into:

Signature Recognition, Voice Recognition, Keystroke.

1.5.2 Physiological Biometrics: Physiological biometrics is based on a behavioral trait of an individual. It involves all physical characteristics like facial recognition, ears, eyes, iris, fingerprints, etc..... It is further divided into:

Ear authentication, Eye vein recognition, Facial recognition, Finger vein recognition, Finger print recognition, DNA matching, Footprint and foot dynamics, Gait recognition.

1.6 Biometric Parameters

Biometric system can make two types of errors [5]:

False match (FM): Biometric system which declares incorrectly as a successful match between input pattern and the stored template pattern.

False non match (FNM): Biometric system which declares incorrectly as a failure between input pattern and stored template pattern. Biometric image is first captured by a sensor. Errors encountered at the sensor level can be

Failure to capture (FTC) and Failure to enroll (FTE):

Number of times the user not able to enroll into the system. It gives the degree of accuracy of the system designed. Biometric system performance evaluation is based on FM, FNM, FTC and FTE. False match and false non match are also called as False accept rate (FAR) and false reject rate (FRR) respectively, which can be evaluated as

FAR =Number of false acceptance/Number of impostor attempts

FRR =Number of False rejection/Number of genuine user attempt

A False acceptance allows an impostor to get an access, and a false reject denies an access to the enrolled user. A threshold is set by the system to decide either to accept or to reject the enroller.

Equal Error Rate (EER):

EER is a threshold set to evaluate the performance of the recognition, which is a midpoint region between False accept and false reject ROC plot. It is also a measurement to say how accurate the system is in rejecting an impostor. EER is also called as cross over error rate between FAR and FRR.

Receiver Operating Characteristics (ROC):

A graphical representation giving a relationship between FAR and FRR. It is used as a measure of summarizing the performance of a biometric system. Performance of different biometric system can also be compared by representing on ROC plots with their thresholds made independent.

1.7 Biometric Facial Recognition?

Facial recognition is a biometric software application capable of identifying or verifying a person by comparing and analyzing patterns based on the person's facial contours. The technology is mostly used for security purposes, though there is increasing interest in other areas. Facial recognition technology has received significant attention as it has the potential for a wide range of application related to law enforcement as well as other enterprises [6].

1.8 History of Face Recognition:

Face recognition is an old subject as old as computer vision, both because of the practical importance of the topic and theoretical interest from cognitive scientists. Despite the fact that other methods of identification (such as fingerprints, or iris scans) can be more accurate, face recognition has always remained a major focus of research because of its non-invasive nature and because it is people's primary method of person identification. Perhaps the most famous early example of a face recognition system is due to Kohonen, who demonstrated that a simple neural net could perform face recognition for aligned and normalized face images. The type of network he employed computed a face description by approximating the eigenvectors of the face image's autocorrelation matrix; these eigenvectors are now known as `eigenfaces.' Kohonen's system was not a practical success, however, because of the need for precise alignment and normalization. In following years many researchers tried face recognition schemes based on edges, inter-feature distances, and other neural net approaches. While several were successful on small databases of aligned images, none successfully addressed the more realistic problem of large databases where the location and scale of the face is unknown. Kirby and Sirovich (1989) later introduced an algebraic manipulation which made it easy to directly calculate the eigenfaces and showed that fewer than 100 were required to accurately code carefully aligned and normalized face images. Turk and Pentland (1991) then demonstrated that the residual error when coding using the eigenfaces could be used both to detect faces in cluttered natural imagery, and to determine the precise location and scale of faces in an image. They then demonstrated that by coupling this method for detecting and localizing faces with the eigenface recognition method, one could achieve reliable, real-time recognition of faces in a minimally constrained environment. This demonstration that simple, real-time pattern recognition techniques could be combined to create a useful system sparked an explosion of interest in the topic of face recognition. [7]

1.9 Types of face recognition:

A human being can be identified with the help of different face features, fingerprint, eye/iris, body structure, spot mark and so on. Face is one of the important parts of the body, which plays an important role in recognizing humans. Faces decompose into mainly four features like eye, lip, nose and mouth for recognition. Main theme of face detection is to identify whether there are single faces in the image or more in view of stationary picture or video picture. Faces are mainly having dimension in 2D and 3D with different textures and facial expressions [8].

1.9.1. 2D face recognition: Previously for 2D face recognition following four steps was used: In face recognition first step was to detect face, second step was face alignment, third step was feature extraction and fourth step was feature matching from database of enrolled users to recognize face. Matrix has been computed on the basis of pixel values at corner of face under different illuminations conditions for 2Dface recognition. Normally, face images are represented by a high dimensional vector containing pixel values. Feature matching is done to match the input face in the form of image or video from available database of enrolled images with unique face identity. Various techniques adopted for face detection were based on color, intensity and illumination. It's one of the challenging tasks to recognize who it is? and researcher faces many challenges like facial expression, illumination, poses variations image orientation and occlusion in face recognition. There were some limitations of 2D face recognition. In 2D face recognition system recognition rate and performance are dependent on image capture conditions like head orientation, image quality, lighting conditions, partial occlusion, facial expressions. [8]

1.9.2. 2D -3D face recognition: Andrea F. Abate et al. proposed a reliable technique for collective 2D visual images and 3D model face recognition based on different parameters such as input size, number of addressed tasks and recognition rate. Comparison of different techniques provides future perspective to the researchers for enabling new techniques in the field of face recognition. Eigen faces and stereovision techniques used to improve the performance of 2D face recognition system with 3D information known as disparity of face. Face of a person at different position was matched with the help of scan lined-based neural network. Principal component analysis (PCA) for feature extraction and recognition were effectively used for face recognition. 2D- 3D face recognition rate was improved by adding up information in-depth. Face identification processes shown in figure 1.1.

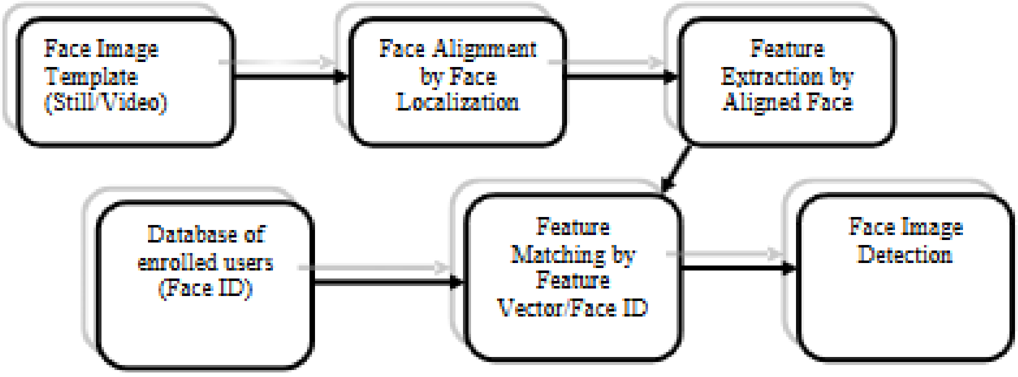


Figure 1.1: Face detection process

1.9.3. 3D face recognition: Faces are in the form of real image, various textures, different framework which convolute in three dimensions. It seems to be more precise recognition of the face image and minimize the problem of pose variations, occlusion and different illumination condition. SimaSoltanpura et al. proposed a survey for 3D face recognition based on local features. They divided the local descriptor into curves, key points and surface. They had applied image acquisition technology on 3D face database to compare under different conditions. Feature extraction is one of the important modules in face recognition which was considered by authors. They studied about different types of face descriptor and feature extractors for 3D face recognition. They also consider the challenges of face recognition with different face expression and occlusions. 3D image and face feature extraction adopted various techniques and methods for efficient and effective recognition. Normalization between probe and gallery texture is done with the help of bidirectional relighting. Introduction of correlation metrics for finding out similarity scores and concept of pose and light normalized signatures for face verifications applied frequently [8]. Motivation to use 3D face recognition technology is to overcome the drawback of 2D face recognition systems. 3D face images were recognized with the help of different augmentation techniques and tested on various database .It was enhanced with the help of experienced sensors camera capturing better 3D face image which can generate 3D face models [8].Various techniques are available to recognize a 3D face from a range of viewing angles. One of the benefits of 3D face recognition system is that it is not affected by light intensity. Several methods were introduced to extract the features so that accuracy and recognition rate will be higher [9]. Ge Wen and al.proposed improved face recognition with domain adaptation. In this author, tried to evaluate face recognition by taking Labeled Faces in the Wild (LFW) dataset as a benchmark. They considered data bias as one of the problems. They had replaced the training data with same distribution because according to them web-collected dataset come from celebrities are quite different from faces of normal people in day to day activities and harder to collect due to privacy concerns. They had applied Face Net triplet loss function. They achieved accuracy rate of 99.33% with single CNN model without face alignment. [8]

1.10 Face Recognition Methods

Face recognition was treated as a 2D pattern recognition problem that was in the beginning of the 1970 [9]. The distances between important points where used to recognize known faces, e.g. measuring the distance between the eyes or other important points or measuring different angles of facial components. But it is necessary that the face recognition systems to be fully automatic. Face recognition is such a challenging yet interesting problem that it has attracted researchers who have different backgrounds: psychology, pattern recognition, neural networks, computer vision, and computer graphics. The following methods are used to face recognition.

1.10.1 Local Approaches: In the context of face recognition, local approaches treat only some facial features. They are more sensitive to facial expressions, occlusions, and pose [10]. The main objective of these approaches is to discover distinctive features. Generally, these approaches can be divided into two categories: (1) local appearance-based techniques are used to extract local features, while the face image is divided into small regions (patches) [11, 12]. (2) Key-points-based techniques are used to detect the points of interest in the face image, after which the features localized on these points are extracted.

1.10.2 Holistic Approach: Holistic or subspace approaches are supposed to process the whole face, that is, they do not require extracting face regions or features points (eyes, mouth, noses, and so on). The main function of these approaches is to represent the face image by a matrix of pixels, and this matrix is often converted into feature vectors to facilitate their treatment. After that, these feature vectors are implemented in low dimensional space. However, holistic or subspace techniques are sensitive to variations (facial expressions, illumination, and poses), and these advantages make these approaches widely used. Moreover, these approaches can be divided into categories, including linear and non-linear techniques, based on the method used to represent the subspace.

1.10.3 Hybrid Approach: The hybrid approaches are based on local and subspace features in order to use the benefits of both subspace and local techniques, which have the potential to offer better performance for face recognition systems.

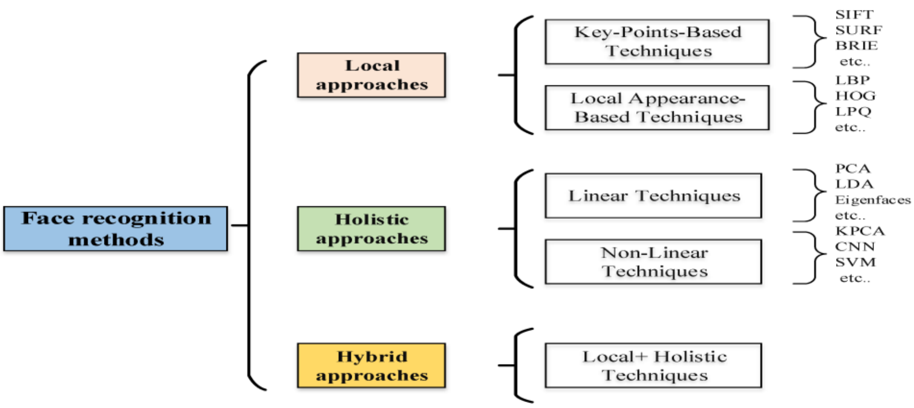


Figure 1.2: Face Recognition Methods

1.11 system architecture

The proposed face recognition system consists of five stages: face detection (localization), face preprocessing includes face alignment/normalization and light correction, feature extraction using each of PCA and LBP, classification using LDA and finally, feature matching displays the five steps of proposed face recognition.

These steps are introduced in this section as follow:

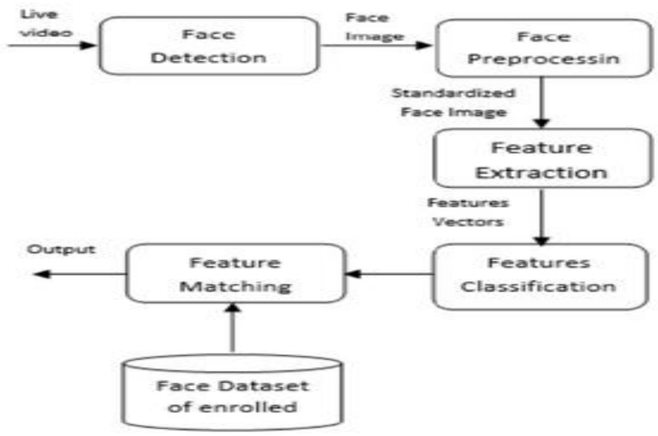


Figure 1.3: The five steps of the face recognition process.

1.11.1 Face Detection: The goal of this stage is to decide the shape of the face in an image. In the case of a video input, the system must have the ability to detect the face in multiple frames. The Viola-Jones face detection algorithm [13] is a very good example of the techniques that are used to detect faces. So, in the face detection stage, this algorithm is used to detect a face in the input image.

1.11.2 Preprocessing: The goal of this stage is to reduce the effect of lighting condition that happened during the face detection stage. It is necessary to implement preprocessing before face detection to obtain the best results. In our proposed system, two steps are applied to remove noises from the image. These two steps are:

Illumination gradient correction: In this step, the best-fit value of brightness is calculated. Then, this value is subtracted from the value of all pixels in the face-image. In the face recognition, illumination gradient correction can be used to reduce the effect of shadows that appear in the image from lighting angles [13].

Histogram equalization: It is considered as a one of image transformation method that can flatten the histogram of the image and recompense the change that caused by an effect of changes in lighting or the changing in response curves of camera [13].

1.11.3 Feature Extraction: Feature extraction stage is applied to obtain the most important feature from the face image. Without this stage, the recognition becomes very complex and it does not give good results. The PCA and LBP methods are used to implement this function.

Feature Extraction Algorithms:

Principal component analysis (PCA):

Principal component analysis (PCA) is a standard tool in modern data analysis - in diverse fields from neuroscience to computer graphics. It is very useful method for extracting relevant information from confusing data sets.

Principal component analysis (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components.

The number of principal components is less than or equal to the number of original variables. [14]

Goals

- The main goal of a PCA analysis is to identify patterns in data
- PCA aims to detect the correlation between variables.
- It attempts to reduce the dimensionality.

Dimensionality Reduction

It reduces the dimensions of a d-dimensional dataset by projecting it onto a (k)-dimensional subspace (where $k<d$) in order to increase the computational efficiency while retaining most of the information.

Transformation

This transformation is defined in such a way that the first principal component has the largest possible variance and each succeeding component in turn has the next highest possible variance.

PCA Approach

- Standardized the data.
- Perform Singular Vector Decomposition to get the Eigenvectors and Eigenvalues.
- Sort eigenvalues in descending order and choose the k- eigenvectors
- Construct the projection matrix from the selected k- eigenvectors.
- Transform the original dataset via projection matrix to obtain a k-dimensional feature subspace. [15]

Applications of PCA:

• Interest Rate Derivatives Portfolios.

• Neuroscience. [16]

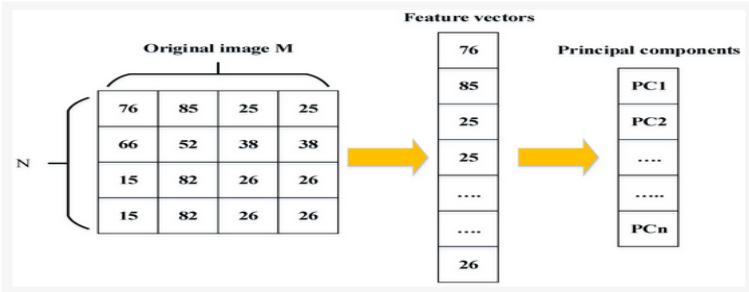


Figure 1.4: Example of dimensional reduction when applying principal component analysis (PCA) [15].

Advantages of PCA

1. it’s the simplest approach which can be used for data compression and face recognition.
2. Operates at a faster rate. [17]

Local binary pattern (LBP): The area binary pattern (LBP) was originally designed for texture description. It's invariant to monotonic grey- scale transformations which are essential for texture description and analysis for the reason of computational simplicity processing of image in real-time is possible.

Local binary pattern (LBP) and its variant: LBP is a great general texture technique used to extract features from any object [18]. It has widely performed in many applications such as face recognition [19], facial expression recognition, texture segmentation, and texture classification. The LBP technique first divides the facial image into spatial arrays. Next, within each array square, a 3×3-pixel matrix (p1……p8) is mapped across the square. The pixel of this matrix is a threshold with the value of the center pixel (p_0) (i.e., use the intensity value of the center pixel i (p_0) as a reference for thresholding) to produce the binary code. If a neighbor pixel’s value is lower than the center pixel value, it is given a zero; otherwise, it is given one. The binary code contains information about the local texture. Finally, for each array square, a histogram of these codes is built, and the histograms are concatenated to form the feature vector. The LBP is defined in a matrix of size 3 × 3, as shown in Equation (1.1).

$$\text{LBP} = \sum_{p=1}^{8} 2^p \, s(i_0 - i_p) \;\; \text{, with } \mathrm{s}(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < \;\; 0 \end{cases} \qquad (1.1)$$

Where i_0 and i_p are the intensity value of the center pixel and neighborhood pixels, respectively. Figure 1.5 illustrates the procedure of the LBP technique.
Khoi et al. [20] propose a fast face recognition system based on LBP, pyramid of local binary pattern (PLBP), and rotation invariant local binary pattern (RI-LBP). Xi et al. [21] have introduced a new unsupervised deep learning-based technique, called local binary pattern network (LBPNet), to extract hierarchical representations of data. The LBPNet maintains the same topology as the convolution neural network (CNN). The experimental results obtained using the public benchmarks (i.e., LFW) have shown that LBP Net is comparable to other unsupervised techniques. Laure et al. [22] have implemented a method that helps to solve face recognition issues with large variations of parameters such as expression, illumination, and different poses. This method is based on two techniques: LBP and K-NN techniques. Owing to its invariance to the rotation of the target image, LBP become one of the important techniques used for face recognition. Bonnen et al. [23] proposed a variant of the LBP technique named “multiscale local binary pattern (MLBP)” for features’ extraction. Another LBP extension is the local ternary pattern (LTP) technique [24], which is less sensitive to the noise than the original LBP technique. This technique uses three steps to compute the differences between the neighboring ones and the central pixel. Hussain et al. [25] develop a local quantized pattern (LQP) technique for face representation. LQP is a generalization of local pattern features and is intrinsically robust to illumination conditions. The LQP features use the disk layout to sample pixels from the local neighborhood and obtain a pair of binary codes using ternary split coding. These codes are quantized, with each one using a separately learned code book.

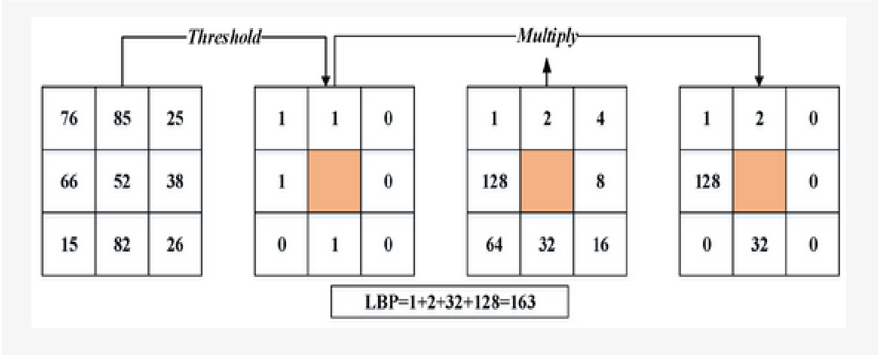
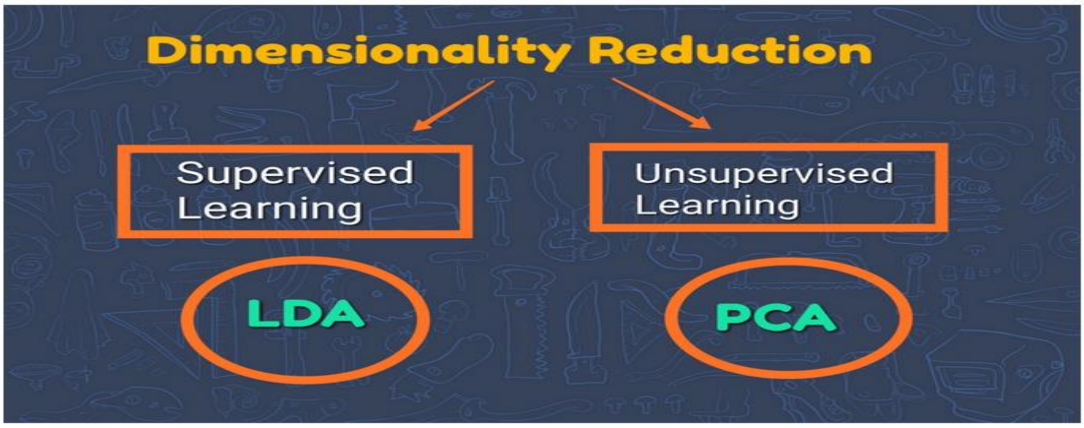


Figure 1.5: The local binary pattern (LBP) descriptor [26].

<u>1.11.4 Classification stage:</u> A classification is the fourth stage of this system. The goal of this stage is to classify the entire face image based on the information that obtained through the training stage. There are many classification methods that can be applied. In this system, LDA is considered as one of the famous classification methods is used to do this job.

Linear Discriminant Analysis (LDA): Linear Discriminant Analysis is a dimensionality reduction technique used as a preprocessing step in Machine Learning and pattern classification applications. The main goal of dimensionality reduction techniques is to reduce the dimensions by removing the redundant and dependent features by transforming the features from higher dimensional space to a space with lower dimensions [27].



Linear Discriminant Analysis is a supervised classification technique which takes labels into consideration. This category of dimensionality reduction is used in biometrics, bioinformatics and chemistry.

How does Linear Discriminant Analysis Work?

The goal of Linear Discriminant Analysis is to project the features in higher dimension space onto a lower dimensional space.

This can be achieved in three steps:

The first step is to calculate the separability between different classes (i.e. the distance between the mean of different classes) also called as between-class variance

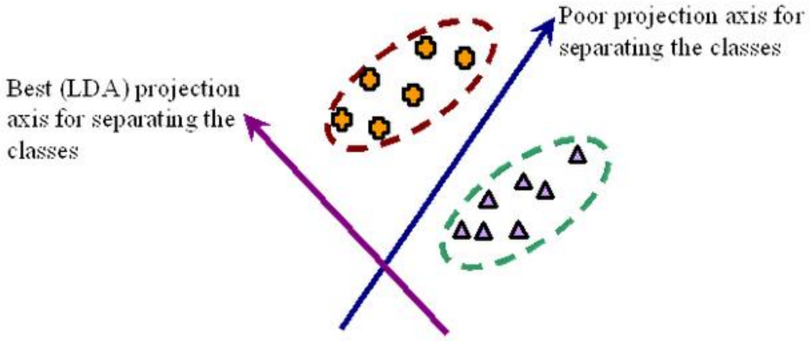
$$S_b = \sum_{i=1}^{g} N_i (\bar{x}_i - \bar{x}) (\bar{x}_i - \bar{x})^T$$

Second Step is to calculate the distance between the mean and sample of each class, which is called the within class variance

$$S_w = \sum_{i=1}^{g} (N_i - 1) S_i = \sum_{i=1}^{g} \sum_{j=1}^{N_i} (\bar{x}_{i,j} - \bar{x}) (\bar{x}_{i,j} - \bar{x})^T$$

The third step is to construct the lower dimensional space which maximizes the between class variance and minimizes the within class variance. Let P be the lower dimensional space projection, which is called Fisher's criterion.

$$P_{lda} = arg_P \max \frac{|P^T S_b P|}{|P^T S_w P|}$$



Extension to LDA: Linear Discriminant Analysis is a simple and effective method for classification. Because it is simple and so well understood, there are many extensions and variations to the method. Some popular extensions include [27]:

Quadratic Discriminant Analysis (QDA): Each class uses its own estimate of variance (or covariance when there are multiple input variables).

Flexible Discriminant Analysis (FDA): Where non-linear combinations of inputs are used such as splines.

Regularized Discriminant Analysis (RDA): Introduces regularization into the estimate of the variance (actually covariance), moderating the influence of different variables on LDA.

1.11.5 Feature Matching: a face matching algorithm is a set of rules that a computer uses to detect a face in an image and then to compare that face to another face (or faces) to determine whether there is a match. [28]

1.12 Face Recognition Challenges:

The study and analysis of faces captured by digital cameras address a wide range of challenges, as detailed in Sections 1.12.1–1.12.5, which all have a direct impact on the computer automated face detection and recognition. [29]

1.12.1. Pose variations : Head's movements, which can be described by the egocentric rotation angles, i.e. pitch, roll and yaw, or camera changing point of views could lead to substantial changes in face appearance and/or shape and generate intra-subject face's variations as illustrated in Figure 1.6, making automated face recognition across pose a difficult task .

Figure 1.6: Illustration of pose variations around egocentric rotation angles, namely (a) pitch, (b) roll and (c) yaw.

<u>1.12.2. Presence/absence of structuring elements/occlusions</u> : The diversity in the intra-subject face's images could also be due to the absence of structuring elements (see Figure 1.7a) or the presence of components such as beard and/or moustache (see Figure 1.7b), cap (see Figure 1.7c), sunglasses (see Figure 1.7d), etc. or occlusions of the face (see Figure 1.7e) by background or foreground objects.

Figure 1.7: Illustration of (a) absence or (b-d) presence of structuring elements, i.e. (b) beard and moustache, (c) cap, (d) sunglasses or (e) partial occlusion.

<u>1.12.3. Facial expression changes</u>: Some more variability in face appearance could be caused by changes of facial expressions induced by varying person's emotional states which are displayed in Figure1. 8.

Hence, efficiently and automatically recognizing the different facial expressions is important for both the evaluation of emotional states and the automated face recognition. In particular, human expressions are composed of macro-expressions, which could express, e.g., anger, disgust, fear, happiness, sadness or surprise, and other involuntary, rapid facial patterns, i.e. micro-expressions; all these expressions generating non-rigid motion of the face. Such facial dynamics can be computed, e.g., by means of the dense optical flow field.

Figure 1.8: Illustration of varying facial expressions that reflect emotions such as (a) anger, (b) disgust, (c) sadness or (d) happiness.

1.12.4. Ageing of the face : Another reason of face appearance's changes could be engendered by the ageing of the human face, and could impact on the entire FR process if the time between each image capture is significant, as illustrated in Figure 1.9.

Figure 1.9: Illustration of the human ageing process, where the same person has been photographed (a) at a younger age and (b) at an older age, respectively.

1.12.5. Varying illumination conditions: Large variations of illuminations could degrade the performance of Face recognition systems. Indeed, for low levels of lighting of the background or foreground, face detection and recognition are much harder to perform, since shadows could appear on the face and/or facial patterns could be (partially) indiscernible. On the other hand, too high levels of lights could lead to over-exposure of the face and (partially) indiscernible facial patterns (see Figure 1.10).

Robust automated face detection and recognition in the case of (close-to-) extreme or largely varying levels of lighting apply to image-processing techniques such as illumination normalization, e.g. through histogram equalization ; or machine-learning methods involving the actual image global image intensity average value. [29]

Figure 1.10: Illustration of camera lighting variations, leading to (a) over-exposure of the face, (b) deep shadows on the face or (c) partial backlight.

1.13 Advantages of face as a biometric:

Facial recognition technology is a fairly new way of identifying people who could be dangerous or need to be located. It works by picking faces out of a crowd, obtaining the measurements necessary and comparing it to the images already in its database. [7]

Safety: Automated security processes also mean that less security personnel would be put in potentially dangerous situations.

Fast and Accurate: With the ever-increasing demand for speed and the growing number of cyber-attacks, having fast and accurate technology is key. Facial recognition technology provides verification that is convenient, quick, and accurate. Although possible, it is very difficult to fool facial recognition technology, which makes it beneficial in helping prevent fraud.

Security: A facial biometric security system can drastically improve your security because every individual who enters your premises will be accounted for. Any trespassers will be quickly captured by the recognition system and you would be alerted promptly. With a facial recognition security system, you can potentially reduce the costs of hiring security staff.

Fully Automated: Instead of manual recognition, which is done by security guards or the official representatives outside of the company's premises, the facial recognition tech automates the identification process and ensures its flawlessness every time without any halting. You won't even need an employee to monitor the cameras 24/7. Automation means convenience and reduces the expenses too. Therefore, any entrepreneur would be fond of the fact that image identification systems are fully automated.

Cost-efficiency: Because facial recognition technology is automated, it also reduces the need for security guards to personally verify a match. This means businesses can save costs on hiring security staff and other security measures.

Easy integration process: Most of the time, integratable facial recognition tools work pretty flawlessly with the existing security software that companies have installed. And they're also easy to program for interaction with a company's computer system [30].

1.14 Facial Recognition Applications:

Based on our assessment of the applications in the field today, a majority of facial recognition use-cases appear to fall into three major categories [31]:

Security: Companies are training deep learning algorithms to recognize fraud detection, reduce the need for traditional passwords, and to improve the ability to distinguish between a human face and a photograph.

Healthcare: Machine learning is being combined with computer vision to more accurately track patient medication consumption and support pain management procedures.

Marketing: Fraught with ethical considerations, marketing is a burgeoning domain of facial recognition innovation, and it's one we can expect to see more of as facial recognition becomes ubiquitous.

1.15 Conclusion

In this chapter we presented biometrics which is a interesting technology that is used in a lot of fields. It can be used for criminal identification and prison security. Biometrics can also apply to check illegal access to ATMs, cellular phones, smart cards, workstations, and computer networks. Biometric technology can make our business and our lives safer and better.

Facial recognition is one of the most successful and famous biometric technologies. The application of this technology is in remarkable progress and it has been used in many fields. Because it is more secure, reliable and fast, its dominance will keep growing until it becomes required in all areas and becomes the best biometric technology ever made

Chapter 2: Multilinear algorithms solution for face recognition

Chapter 2: Multilinear algorithms solution for face recognition

2.1 Introduction

The process of finding the optimal vectors some restrictions will be added to reduce the error rate for face recognition. Most of the previous algorithms for classification and dimensionality reduction transform the input image pixel into a vector, which disregard the underlying data structure so these methods suffer from curse of dimensionality [32], [33], and [34] and often leads us to the small sample size problem. Computationally, handling high dimensional samples is so expensive and many classifiers have a very bad performance in high dimensional spaces when the training set is small. Some recent works have started to consider an object as a 2-D matrix (second order tensors) rather than vectors (first order tensors) for subspace learning. As 2-D PCA algorithm is proposed in [35], where gets the input images as a matrix and compute a covariance matrix. To solve these problems, we proposed a method that is based on tensor representation, so the structures of the objects will not break so it gives us the better performance and also we use 2 different algorithms for classification step. We fuse two multilinear algorithms to cover their defects and increasing the performance of the system. In this paper the MDA and MPCA algorithms have been proposed, and because those algorithms work with tensor objects, those algorithms could give us the better results.

We expect this novel method to be a better choice than LDA and PCA algorithms and a more general algorithm than MDA and MPCA for the pattern classification problems in image analysis and also overcome the small sample sizes and curse of dimensionality dilemma and also more robust on variation of scale, lighting and pose . [36]

2.2 Multilinear basics

Before we discuss the multilinear discriminant analysis MDA and MPCA solutions for biometric signals, it is necessary to review some basic multilinear algebra, including the notations and other basic multilinear operations.

2.2.1 Notations

The notations in this chapter follow the conventions in the multilinear algebra, pattern recognition and adaptive learning literature. Vectors are denoted by lowercase boldface letters, e.g., x; matrices by uppercase boldface, e.g., ***U;*** and tensors by calligraphic letters, e.g., ***A***. Their elements are denoted with indices in brackets.
Indices are denoted by lowercase letters and span the range from 1 to the uppercase letter of the index, e.g., ***n*** = 1***;*** 2 ***,…..., N***. Throughout this chapter, the discussion is restricted to real-valued vectors, matrices and tensors since the biometric applications that we are interested in involve real data only, such as gray-level/color face images and binary gait silhouette sequences.

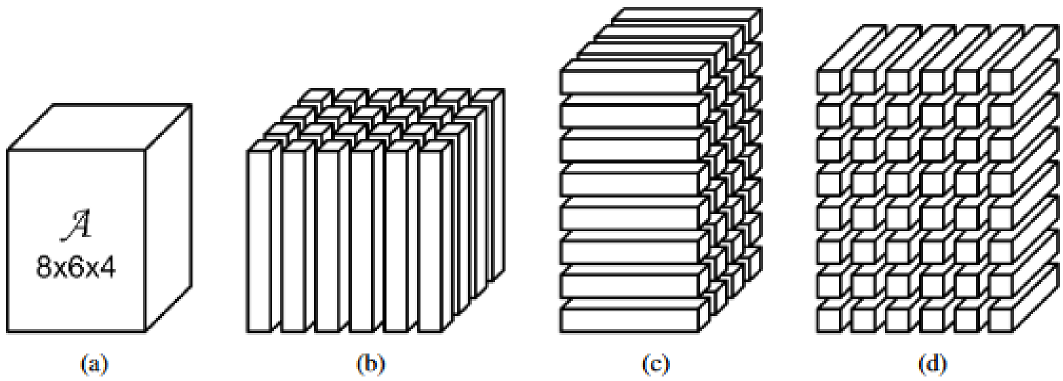


Figure 2.1: Illustration of then-mode vectors: (a) a tensor A ∈ R^ (8×6×4), (b) the*1-modevectors, (c) the 2-mode vectors, and (d) the 3-mode vectors.*

2.2.2 Basic multilinear algebra

The branch of algebra dealing with multilinear mappings (cf. Multilinear mapping) between modules (in particular, vector spaces). The first sections of multilinear algebra were the theory of bilinear and quadratic forms, the theory of determinants, and the Grossmann calculus that extends this (see Exterior algebra; bilinear form; Quadratic form; Determinant). A basic role in multilinear algebra is played by the concepts of a tensor product, a tensor on a vector space and a multilinear form. The applications of multilinear algebra to geometry and analysis are related mainly to tensor calculus and differential forms (cf. Differential form).

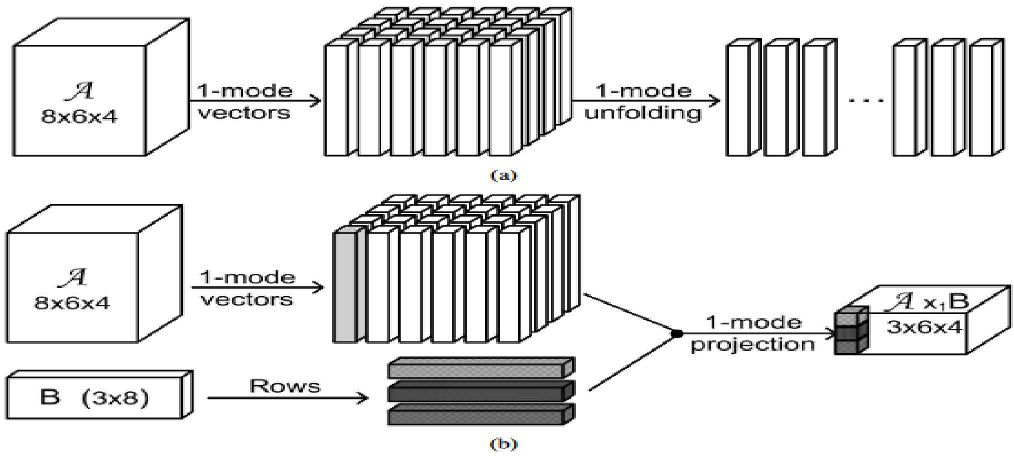


Figure 2.2: Visual illustration of (a) the n-mode (1-mode) unfolding and (b) then-mode (1-mode) multiplication.

2.2.3 Tensor distance measure

To measure the distance between tensors $\boldsymbol{A}$ and $\boldsymbol{B}$, the Frobenius norm is used in [37]:

Dist.(A, B) $=\|\boldsymbol{A}-\boldsymbol{B}\|_{\boldsymbol{F}}$. Let ***vec(A)*** be the vector representation (vectorization) of $\boldsymbol{A}$, 3the nit is straightforward to show that

Proposition 1dist. ***(A, B)*** $=\|\boldsymbol{vec}(\boldsymbol{A})-\boldsymbol{vec}(\boldsymbol{B})\|_{\boldsymbol{2}}$

I.e., the Frobenius norm of the difference between two tensors equals to the Euclidean distance of their vectorized representations, since the Frobenius norm is a point-based measurement as well and it does not take the structure of a tensor into account.

2.3 Multilinear Vs Linear Methods

Ordinary images result from the interaction of multiple factors related to scene, structure, illumination, and imaging. For example, facial images are determined by facial geometry

(Person, expression), the pose of the head relative to the camera, the lighting conditions, and the camera employed. Linear methods, including PCA and ICA, are not well suited to the representation of multifactor image ensembles; they are better treated using nonlinear methods, specifically those based on multilinear algebra. Multilinear algebra involves the natural generalization of matrices. Whereas matrices are linear operators defined over a vector space, these generalizations, referred to as tensors, define multilinear operators over a set of vector spaces. Hence, multilinear algebra, the algebra of higher-order tensors, subsumes linear algebra and matrices/vectors/scalars as a special case. Multilinear algebra serves as a unifying mathematical framework suitable for addressing a variety of challenging problems in image science and visual computing.

The multilinear algebraic frame work can be applied to the synthesis, analysis, and recognition of images. Within this mathematical framework, the image ensemble of interest is represented as a higher-order tensor, which must be decomposed in order to separate and parsimoniously represent the constituent factors [38].

2.4 Multilinear PCA

One extension of PCA is that of applying PCA to tensors or multilinear arrays which results in a method known as multilinear principal components analysis (MPCA) [39].

Since a face image is most naturally a multilinear array, meaning that there are two dimensions describing the location of each pixel in a face image, the idea is to determine a multilinear projection for the image, instead of forming a one-dimensional (1D) vector from the face image and finding a linear projection for the vector. It is thought that the multilinear projection will better capture the correlation between neighborhood pixels that is otherwise lost in forming a 1D vector from the image [40].

2.5 Principle of MPCA

In computer vision, most of the objects are naturally considered as nth-order tensors ($n \geq 2$) [41]. Take Figure 2.3 as an example; the image matrix in (i) is a 2nd-order tensor and a movie clip, while in (ii) it is a 3rd-order tensor. Traditional techniques for subspace dimensionality reduction such as PCA could transform image matrix to vectors with high dimensionality in one mode only, which cannot meet the need of dimensionality reduction. So, such techniques are unable to handle multidimensional objects well and get satisfactory results. Therefore, in order to reduce dimensionality, a reduction algorithm which can directly operate on a high-order tensor object is desirable. Two-dimensional PCA (2DPCA) algorithm is proposed and developed, while researches are using dimensionality reduction solutions which represent facial images as matrices (2nd-order tensors) instead of vectors [41]. However, 2DPCA can only project images in single mode, which results in bad dimensionality of reduction [41]. Thus, a more efficient algorithm MPCA has been proposed to get better dimensionality reduction.

Figure 2.3: 2nd-order and 3rd-order tensor representations samples.

2.6 MPCA Algorithm

MPCA have managed to handle multidimensional objects. According to the above sections, pseudo code for the computation of the MPCA algorithm can be concluded [41] as shown in Figure 2.4.

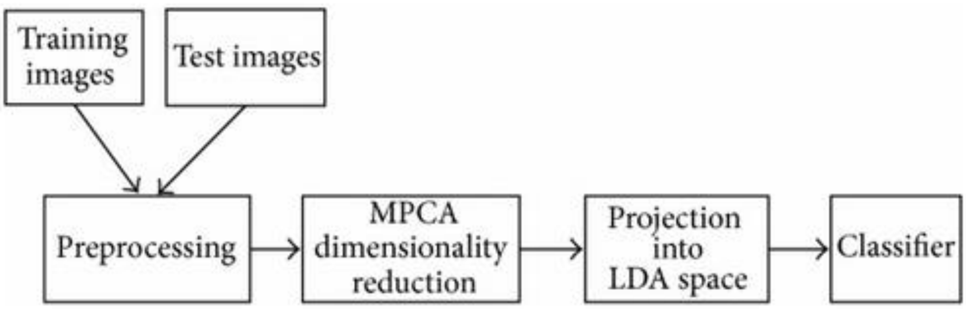


Figure 2.4: Flow chart of face recognition algorithm.

2.7 Multilinear Image Representation

The multilinear analysis framework for appearance-based image representation offers a potent mathematical approach to analyzing the multifactor structure of image ensembles and for addressing the fundamental yet difficult problem of disentangling the causal factors.

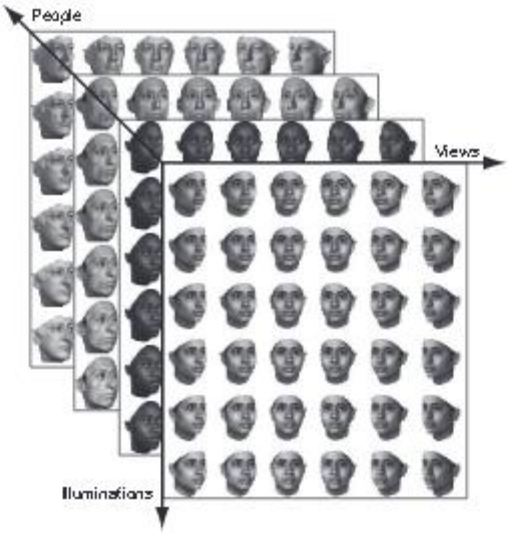


Figure 2.5: A facial image dataset. (a) 3D scans of 75 subjects, recorded using a CyberwareTM3030PS laser scanner as part of the University of Freiburg 3D morphable faces database [42]. Apportion of the 4th-order data tensor *D* of the image ensemble used for training. Only 4 of the 75 people are shown.

Multilinear transformations lead to generative models that explicitly capture how the observed data are influenced by multiple underlying causal factors. A multilinear trans-formation is a nonlinear function or mapping from not just one, but a set of M domain vector spaces R^{m_i}, 1 ≤ i ≤ M, to a range vector space R^n :

$T \ : \ \{R^{m_1} \times R^{m_2} \times .... \times R^{m_M}\} \rightarrow R^n$. (2.1)

Given the data tensor $D$of labeled, vectorized training images d_{pvle} , where the subscripts denote person p, view v, illumination l, and expression e labels, we can apply the MPCA algorithm [42] to compute causal mode matrices U_P , U_V , U_L and U_E as well as the Tensor Faces basis

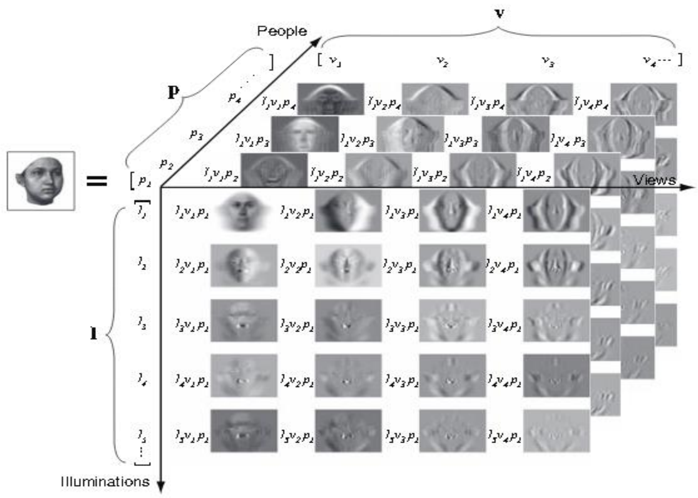
$T = D \times_P U_P^T \times_V U_V^T \times_L U_L^T \times_E U_E^T$ that governs the inter-action between them. Then the method represents an image d_{pvl} by the relevant set of people, view, and illumination coefficient vectors as follows:

$d_{pvle} = \ T \times_P p_p^T \times_V v_v^T \times_L l_l^T \times_E e_e^T$. (2.2)

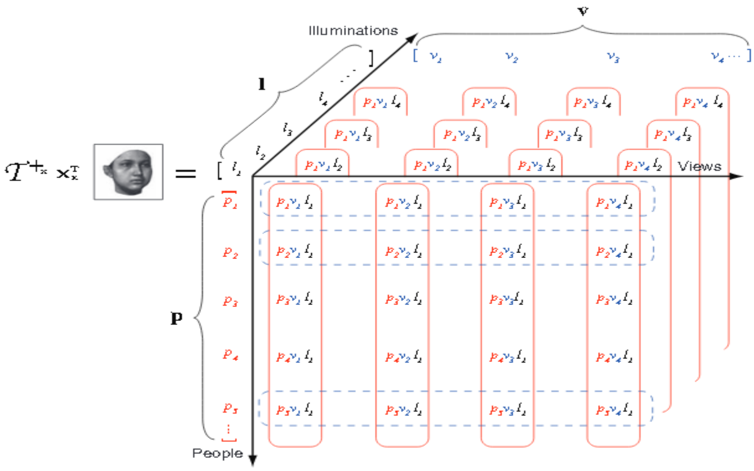
Alternatively, we can apply the MICA algorithm [42], which employs higher order statistics to compute an MICA basis tensor $M = D \times_P C_P^+ \times_V C_V^+ \times_L C_L^+ \times_E C_E^+$. Analogous to the MPCA case, an image can be represented with respect to the MICA basis, as follows:

$d_{pvle} = \ M \times_P p_p^T \times_V v_v^T \times_L l_l^T \times_E e_e^T$. (2.3)

By comparison to linear approaches where an individual has a representation for every image in which they appear, in the multilinear approaches discussed above, such as MPCA, an individual has the same representation regardless of viewpoint, illumination, expression, etc. This is an important advantage of multilinear models over linear ones on which our recognition system capitalizes for superior results.



(a)



(b)

Figure 2.6: (a) MPCA image representation d = $T \times_P p^T \times_V v^T \times_L l^T$. (b) Given an unlabeled test imaged, the associated coefficient vectors p, v, l are estimated by decomposing the response tensor $R = T_x^{+} \times_x^{T} d$ using a multilinear projection algorithm.

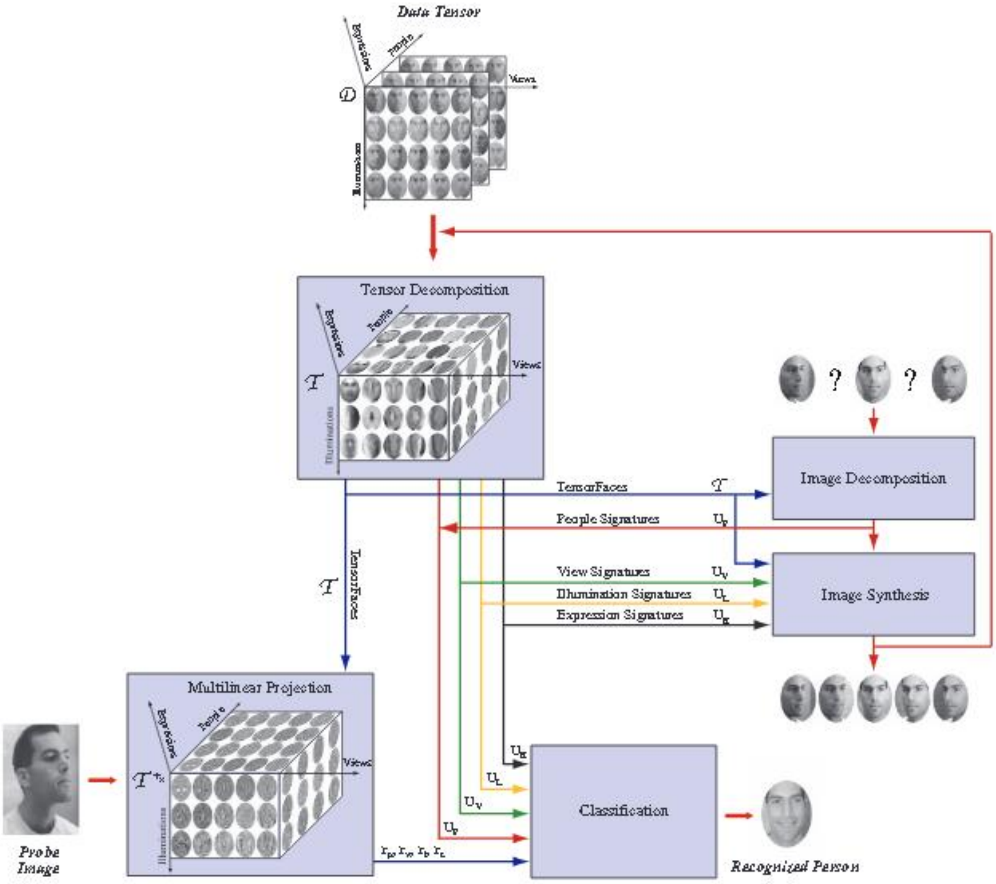


Figure 2.7: Architecture of a multilinear facial image recognition system. A facial training image ensemble including different people, expressions, views, illuminations, and expressions is organized as a data tensor. The data tensor made up of willing participants is de-composed in the (offline) learning phase to train a multilinear model. An uncooperative subject whose face is detected in one or more surveillance images can then be enrolled into the system, by representing his/her facial image(s) relative to the statistics encoded in the learned model. In the (online) recognition phase, the model recognizes a previously unseen probe image as one of the known people in the database. In principle, the trained generative model can also synthesize novel images of known or unknown persons from one or more of their facial images. [42]

2.8 The TensorFaces Representation

We first apply our multilinear analysis technique to ensembles of natural facial images, employing apportion of the Weizmann face image database [43] of 28 male subjects imaged in 15different views, under 4 different illuminations, performing 3 different expressions.1 For the sake of concreteness in the ensuing discussion, consider an image ensemble of the 28 subjects acquired from 5views, under 3 illuminations, and performing 3 expressions, for a total of 1260 images. Using a global rigid optical flow algorithm, we roughly aligned the original 512×352 pixel images relative to one reference image. The images were then decimated by a factor of 3 and cropped, yielding a total of 7943 pixels per image within the elliptical cropping window. Thus, the number of modes is *M*= *5* and our facial image data tensor $D \in \boldsymbol{R}^{7943\times 28\times 5\times 3\times 5}$. Applying the MPCA algorithm to *D*, we obtain the decomposition

$$D = T \times_p U_P \times_V U_V \times_L U_L \times_E U_E. \qquad (2.4)$$

The extended core tensor $T \in \boldsymbol{R}^{7943\times 28\times 5\times 3\times 5}$, which we call TensorFaces, captures the interaction between the four causal factors: The rows of the causal factor mode matrix $U_P \in \boldsymbol{R}^{28\times 28}$ are the representations for different people whose images comprise the data tensor (the subscript “P” denotes the people mode), the rows of the causal factor mode matrix $U_V \in \boldsymbol{R}^{5\times 5}$represent differentviews (the subscript “V” denotes the view mode), the rows of the causal factor mode matrix $U_L \in \boldsymbol{R}^{3\times 3}$represent the illuminations (the subscript “L” denotes the illumination mode), and the rows of the causal factor mode matrix $U_E \in \boldsymbol{R}^{3\times 3}$represent different expressions (the subscript “E” denotes theexpression mode).

(a)

(b)

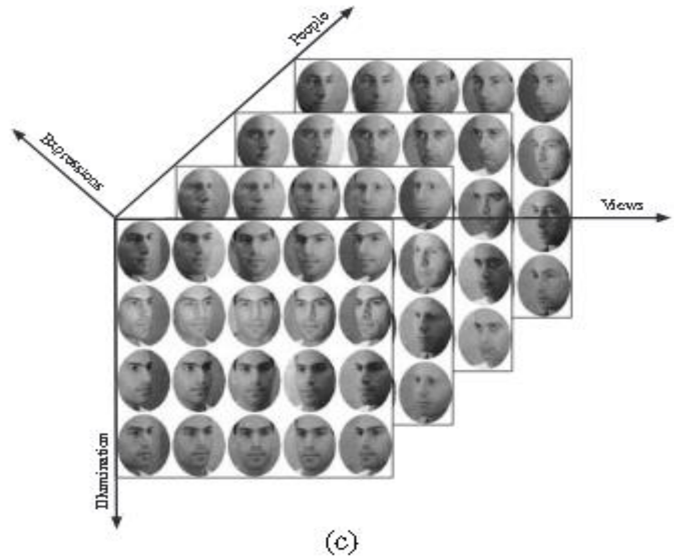


Figure 2.8: The Weizmann facial image database (28 subjects, 45 images per subject). (a) The 28 subjects shown in expression 2 (smile), view 3 (frontal), and illumination 2 (frontal). (b) Part of the image set for subject 1. Left to right, the three panels show images captured in illuminations 1, 2, and3. Within each panel, images of expressions 1, 2, and 3 (neural, smile, yawn) are shown horizontally while images from views 1, 2, 3, 4, and 5 are shown vertically. The image of subject 1 in (a) is the image situated at the center of (b). (c) The5th-order data tensor for the image ensemble; only images in expression 1 (neutral) are shown.

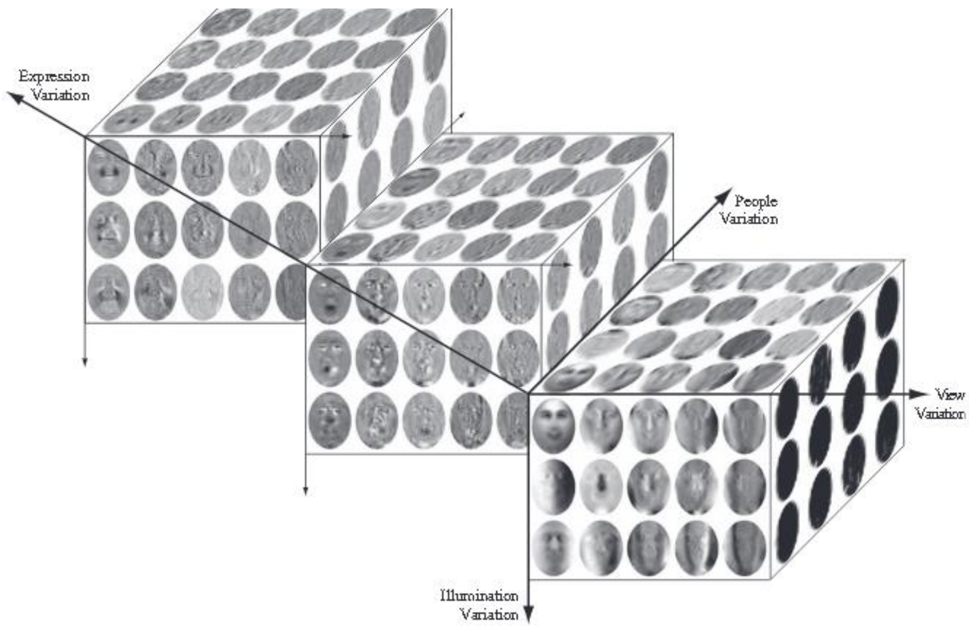


Figure 2.9: A partial visualization of the $T \in \boldsymbol{R}^{7943\times 28\times 5\times 3\times 5}$Tensor Faces representation of D, obtained as $T = D \times_p U_P^T \times_V U_V^T \times_L U_L^T \times_E U_E^T$.

Figure 2.10: The measurement mode matrix U_x contains the conventional PCA eigenvectors ("eigen-faces"), which are the principal axes of variation across all the images. The first 10 eigenvectors are shown.

Tensor Faces is expressed as: $T = Z \times_x U_x$, (2.5)

Where $U_x \in \boldsymbol{R}^{7943\times1260}$is the measurement mode matrix whose orthonormal column vectors spanthe image space (the lower case "x" subscript denotes the pixel mode). Note that neither U_x nor the core tensor $Z \in \boldsymbol{R}^{7943\times28\times5\times3\times5}$ are needed in our multilinear analysis and they are not explicitly computed in practice. Instead, the MPCA algorithm computes Tensor Faces T and the 4 causal factor mode matrices by forming the product

$$T = D \times_p U_P^T \times_V U_V^T \times_L U_L^T \times_E U_E^T. \qquad (2.6)$$

From a computational standpoint, equation (2.5) is preferable to (2.6) since it prescribes computing the relatively small matricesU_P,U_V,U_L, and U_E rather than the generally large matrix U_x that PCA must compute. This efficiency is exploited by the MPCA algorithm. We showed that multilinear analysis subsumes linear, PCA analysis. Each column vector in $U_x$is an "eigen image" (Figure2.10), since U_x is computed by performing an SVD of the matrix$D_{[X]}$, the mode-1 matrixized data tensor D, whose columns comprise all the vectorized facial images. The resulting eigen images are identical to the conventional eigenfaces [44, 45, 46]. Eigen images represent merely the principal axes of variation of the entire ensemble of images in the dataset. The big advantage of our multilinear analysis beyond linear PCA is that Tensor Faces explicitly represent each of the causal factors and encode how they interact to produce facial images.

2.9 A Multilinear Face Recognition System:

Although our multilinear framework is relevant to biometric systems, it has been beyond the scope of this thesis to implement a practical face recognition system. A big challenge is the recognition of an individual under unconstrained imaging conditions. Such a system might be structured as shown in Figure 2.7. First, a multilinear model is learned from a (hopefully large) set of cooperative participants, each of whom supply a complete set of training images acquired from multiple views, under multiple illuminations, in multiple expressions, etc. An uncooperative subject whose face is detected in one or more surveillance images can then be enrolled into the system, by representing his/her facial image(s) relative to the statistics encoded in the learned model. This should make it possible, in principle, to synthesize a complete image set of training images for the subject. The image database can then be augmented with these new training images and the multilinear model incrementally updated. This should end-able the recognition of the subject from an arbitrary, unlabeled image, as illustrated at the bottom of the figure. Implementing a prototype recognition system of this kind, especially one that would support the multilinear fusion of multimodality biometric data (e.g., video and speech) would be a worthy future goal. [47]

2.10 Multilinear Discriminant Analysis

The linear discriminant analysis (LDA) is a classical algorithm that has been successfully applied and extended to various biometric signal recognition problems. The recent advancement in multilinear algebra led to a number of multilinear extensions of the LDA, multilinear discriminant analysis (MDA), being proposed for the recognition of biometric signals using their natural tensorial representation. [39]

In general, MDA and MPCA seek a multilinear projection that maps the input data from one space to another (lower-dimensional, more discriminative) space. Therefore, we need to understand what a multilinear projection is before proceeding to the MDA and MPCA solutions. After this, we propose the categorization of the various multilinear projections.

2.11 Multilinear projection for recognition:

Tensor Faces leads to a multilinear recognition algorithm that projects an unlabeled test image into the multiple causal factor spaces to simultaneously infer its factor labels. In the context of facial image ensembles, where the factors are person, view, illumination, expression, etc., we demonstrate that the statistical regularities learned by MPCA and MICA capture information that, in conjunction with our multilinear projection algorithm, improves automatic face recognition rates. The multilinear projection algorithm employs mode midentity and mode-m Pseudo inverse tensors, concepts that we generalize from matrix algebra [47]. The categorization of the various multilinear projections in terms of the input and output of the projection: the traditional vector to-vector projection (VVP), the tensor-to-tensor projection (TTP) and the tensor-to vector projection (TVP).

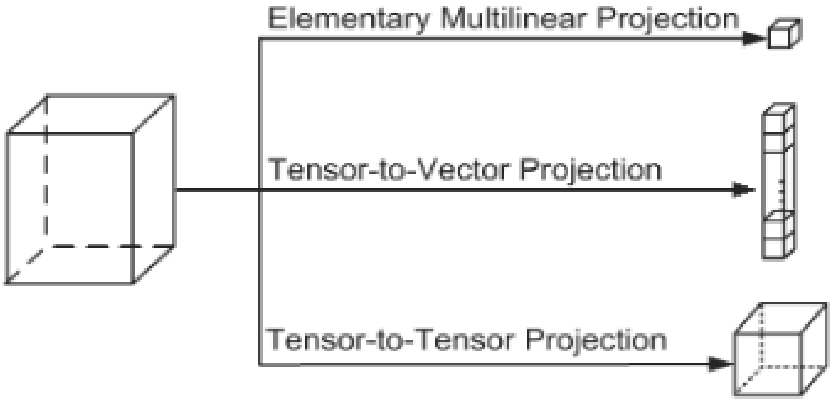


Figure 2.11: Multilinear subspace learning finds a lower-dimensional representation by direct mapping of tensors through a multilinear projection. [48]

2.12 MPCA-Based Tensor Object Recognition

The projection matrices $\{\widetilde{U}^{(n)}, n= 1 , \dots , N \}$, obtained by maximizing the total scatter Ψ_y of the projected samples (in a reduced tensor space) from a set of training tensor samples$\{X_m, m= 1, \dots, M\}$,can be used to extract features for various applications such as data compression, object retrieval, and pattern recognition. This section presents the MPCA-based tensor object recognition framework depicted in Fig 2.12. In typical pattern recognition problems, such as human identification using face signals, there are two types of data sets: the gallery and the probe [49]. The gallery set contains the set of data samples with known identities and it is used for training. The probe set is the testing set where data samples of unknown identity are to be identified and classified via matching with corresponding entries in the gallery set.

2.12.1 Preprocessing

MPCA accepts tensor samples of the same dimensions $I_1 \times I_2 \times \dots \times I_N$ for feature extraction. However, in practice, tensor object samples are often of different dimensions.

Therefore, the input tensors need to be normalized to standard dimensions first, and if they are noisy, a noise-removing preprocessing procedure could follow. The normalized tensor samples are then centered by subtracting the mean obtained from the gallery tensors.

2.12.2 Feature extraction

From the gallery set, a set of eigentensors is obtained, with reduced dimensionality ($P_N \leq I_N$) determined by a user-specified Q, and each entry of a projected tensor feature can be viewed as a (scalar) feature corresponding to a particular eigentensor. Some of the small variation and noise are removed in the projection. For recognition, it should be noted that MPCA is an unsupervised technique without taking class labels into account. As a result, the variation captured in the projected tensor subspace includes both the within-class variation and the between-class variation. In the task of classification, a larger between class variations relative to the within-class variation indicates good class separability, while a smaller between-class variation relative to the within-class variation indicates poor class separability. Hence, a feature selection strategy is proposed to select eigentensors according to their class discrimination power [49], which is defined to be the ratio of, the between class scatter over the within class scatter:

Definition 2: The class discriminability $\Gamma_{p_1 p_2 \dots p_N}$ for an eigentensor $\widetilde{U}_{p_1 p_2 \dots p_N}$ is defined as:

$$\Gamma_{p_1 p_2 \dots p_N} = \frac{\sum_{c=1}^{C} N_c \ . \ [\,\bar{y}_c(p_1, p_2, \dots, p_N) - \bar{y}(p_1, p_2, \dots, p_N)\,]^2}{\sum_{m=1}^{M} \ [\,y_m(p_1, p_2, \dots, p_N) - \bar{y}_{c_m}(p_1, p_2, \dots, p_N)\,]^2} \qquad (2.7)$$

Where C is the number of class is **M** is the number of samples in the gallery set, N_c is the number of samples for class c and c_m is the class label for the m^{th} gallery sample x_m , y_m is the feature tensor of x_m in the projected tensor subspace. The mean feature tensor $\bar{y}$= $\frac{1}{M}\sum_m y_m$ and the class mean feature tensor $\bar{y}_c = \frac{1}{N_c}\sum_{m, c_m = c} y_m$.

For the eigentensor selection, the entries in the projected tensor y_m (from the gallery set) are rearranged into a feature vector $\boldsymbol{y}_m$, ordered according to $\Gamma_{p_1 p_2 \dots p_N}$ in descending order, and only the first H_y most discriminative components of $\boldsymbol{y}_m$ are kept for classification, with H_y

determined empirically or user specified. By this selection, a more discriminating subspace is resulted compared to the MPCA projected tensor subspace that includes both features with good separability and features with poor separability. Next, a weight tensor W is formed with entries defined as $W(p_1, p_2, \ldots, p_N) = \sqrt{\Pi_{n=1}^{N} \lambda_{p_n}^{(n)}}$, Where $\lambda_{p_n}^{(n)}$ denotes the p_n^{th}n-mode eigenvalue corresponding to the projection matrix $\widetilde{U}^{(n)}$, .W is rearranged into a vector w in the same order as $\boldsymbol{y}_m$ and only the first H_y components of w will be used in the next section as weights in measuring distances.

The feature vector $\boldsymbol{y}_m$ can be used directly for recognition, and a classical linear discriminant analysis (LDA) can also be applied to obtain an MPCA+LDA approach for recognition [49], like the popular approach of PCA+LDA. LDA seeks a projection V to maximize the ratio of the between-class scatter matrix $\boldsymbol{S}_B$ to the within-class scatter matrix$\boldsymbol{S}_W$, where

$$\boldsymbol{S}_W = \sum_{m=1}^{M} (\boldsymbol{y}_m - \bar{y}_{c_m}) (\boldsymbol{y}_m - \bar{y}_{c_m})^T \text{ and } \bar{y}_c = \frac{1}{N_c} \sum_{m,c_m=c} y_m, \quad (2.8)$$

$\boldsymbol{S}_B = \sum_{c=1}^{C} N_c (\bar{y}_c - \bar{y}) (\bar{y}_c - \bar{y})^T$, and $\bar{y} = \frac{1}{M} \sum_m y_m$. The solution V_{lda}= arg max V $\frac{|V^T \boldsymbol{S}_B V|}{V^T \boldsymbol{S}_W V}|=$ $[V_1, V_2, \ldots, V_{H_z}]$, where$\{V_{H_z},$ hz = 1, … , $H_z\}$ is the set of generalized eigenvectors of $\boldsymbol{S}_B$ and $\boldsymbol{S}_W$ corresponding to the H_z (≤ C−1) largest generalized eigenvalues$\{\lambda_{H_z},$ hz = 1, … , H_z } [49]: $\boldsymbol{S}_B v_{H_z} = \lambda_{H_z} \boldsymbol{S}_W v_{H_z}$, hz = 1, … , H_z. Thus, the discriminant feature vector z_m is obtained as: $z_m = V_{lda}^T y_m$.

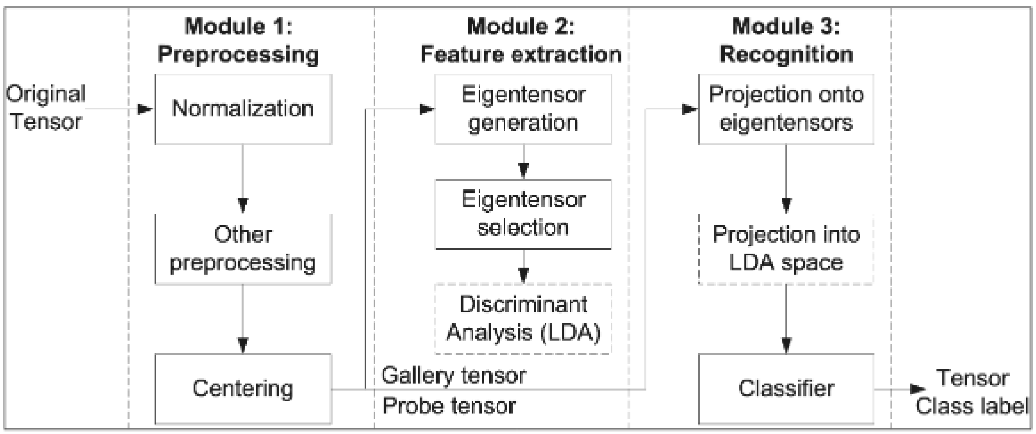


Figure 2.12: Block diagram of the MPCA-based tensor object recognition system.

2.12.3 Feature classification

In classification, the distance between feature vectors is of paramount importance as it determines the performance of the classification module. For the feature vectors discussed above (y_m or z_m), seven distance measures are adapted from [49] and tested: L1, L2, angle, modified Mahala Nobis distance (MMD), modified L1 (ML1), modified L2 (ML2), and modified angle distance (MAD), as listed in Table I, where is a weight vector. The first four measures are commonly used for measuring vector distances and the last three measures can be viewed as the weighted versions of the first three measures. If z_m is the feature vector, $H = H_z$ and g (h) = $\sqrt{\lambda_h}$, which is the simplified Mahala Nobis distance introduced in [49]. If y_m is the feature vector, $H = H_y$ and $g = w$, where w is defined above. To measure the similarity of one test sample feature y (or z) against N_c sample features y_{n_c} (or z_{n_c}) of a class c, the principle of nearest neighbor classifier is applied in this work. The matching score of y (or z) with class is obtained as S(y , c) = $-min_{n_c} d$(y , y_{n_c}) (or S(z , c) = $-min_{n_c} d$(z,z_{n_c})) , using one of the distance measures in Table I. Such a simple classifier is selected to study the performance mainly contributed by the MPCA-based feature extraction algorithm although better classifiers can be investigated.

Table 1: SEVEN DISTANCE MEASURES TESTED FORMPCA-BASED TENSOR OBJECT RECOGNITION.

Distance	L1	L2	Angle	MMD
$d(\mathrm{a},\mathrm{b})$	$\sum_{h=1}^{H} \lvert \mathrm{a}(h) - \mathrm{b}(h) \rvert$	$\sqrt{\sum_{h=1}^{H} [\mathrm{a}(h) - \mathrm{b}(h)]^2}$	$-\frac{\sum_{h=1}^{H} \mathrm{a}(h)\cdot \mathrm{b}(h)}{\sqrt{\sum_{h=1}^{H} \mathrm{a}(h)^2 \sum_{h=1}^{H} \mathrm{b}(h)^2}}$	$-\sum_{h=1}^{H} \frac{\mathrm{a}(h)\cdot \mathrm{b}(h)}{\mathrm{g}(h)}$
Distance	ML1	ML2	MAD	
$d(\mathrm{a},\mathrm{b})$	$\sum_{h=1}^{H} \frac{\lvert \mathrm{a}(h) - \mathrm{b}(h) \rvert}{\mathrm{g}(h)}$	$\sqrt{\sum_{h=1}^{H} \frac{[\mathrm{a}(h) - \mathrm{b}(h)]^2}{\mathrm{g}(h)}}$	$-\frac{\sum_{h=1}^{H} \mathrm{a}(h)\cdot \mathrm{b}(h)/\mathrm{g}(h)}{\sqrt{\sum_{h=1}^{H} \mathrm{a}(h)^2 \sum_{h=1}^{H} \mathrm{b}(h)^2}}$	

2.13 Conclusion

In this chapter, I have presented understanding of the area of multilinear algebra. Then I defined the notations of tensor and tensor distance measure. After this I presented the principle of multilinear PCA and giving a representation of multilinear images, tensor faces multilinear face recognition system. After this, I introduce the MDA algorithm which is related to discriminant analysis, which helps classify a data set, MDA and LDA techniques, but the MDA, but evaluation of MDA is more quality for researchers than the LDA. We discuss also the three basic types of the multilinear projection, and all this studies should give us a result in the end which I will discuss in the next chapter.

Chapter 3: Simulation and Results

Chapter 3: Simulation and Results

3.1 Introduction

There are many algorithms had been proposed to solve the complication in face recognition process but none of them could beat the problems such as lighting, ageing, and the pose. In this chapter, we will apply a method that can solve this which is the fusion of the MDA and MPCA. A multilinear principal component analysis (MPCA) for tensor object feature extraction and a multilinear discriminant analysis (MDA), and using LPQ for feature extraction which is has the same job as LBP. Finally, it will be a comprehensive experiment in MATLAB to demonstrate that these fusion experimental results will have a progress results compare to the traditional methods or not.

3.2 Fusion of MDA and MPCA for face recognition

Here, we will present our new method that is fusing the MPCA and the MDA for recognition rules. We already saw the fusion of LDA and PCA for face recognition problems with good results. The procedure of our approach will be shown here:

First, we applied the MPCA and the MDA algorithms separately on target database and train these algorithms and find the projection matrix for each of them in each mode. After that we project each of the training set to MPCA and MDA area, for recognition the new given image of face we also should project that into those area and compute the distance between this image to all of the other faces.

All the reduction algorithms that had been proposed for recognition such as LDA and PCA had been applied separately to project the image to the lower space for face recognition problems and never used together. Because of the correlation between these two algorithms We inspired to mixture the multi linear form of these two. And because of the difference at their projection space we interested to try mixing them and get the better recognition accuracy. [50]

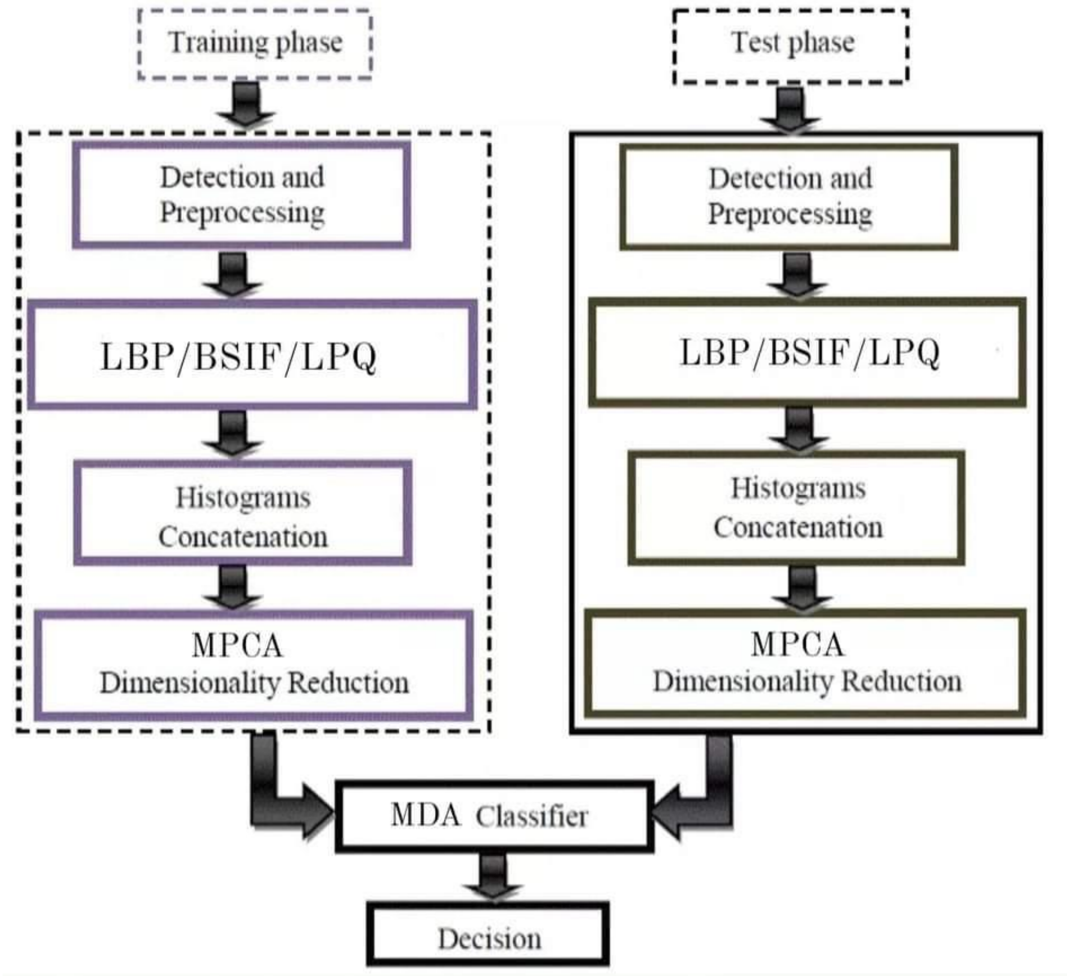


Figure 3.1: the diagram of our algorithm (MPCA/MDA/LPQ).

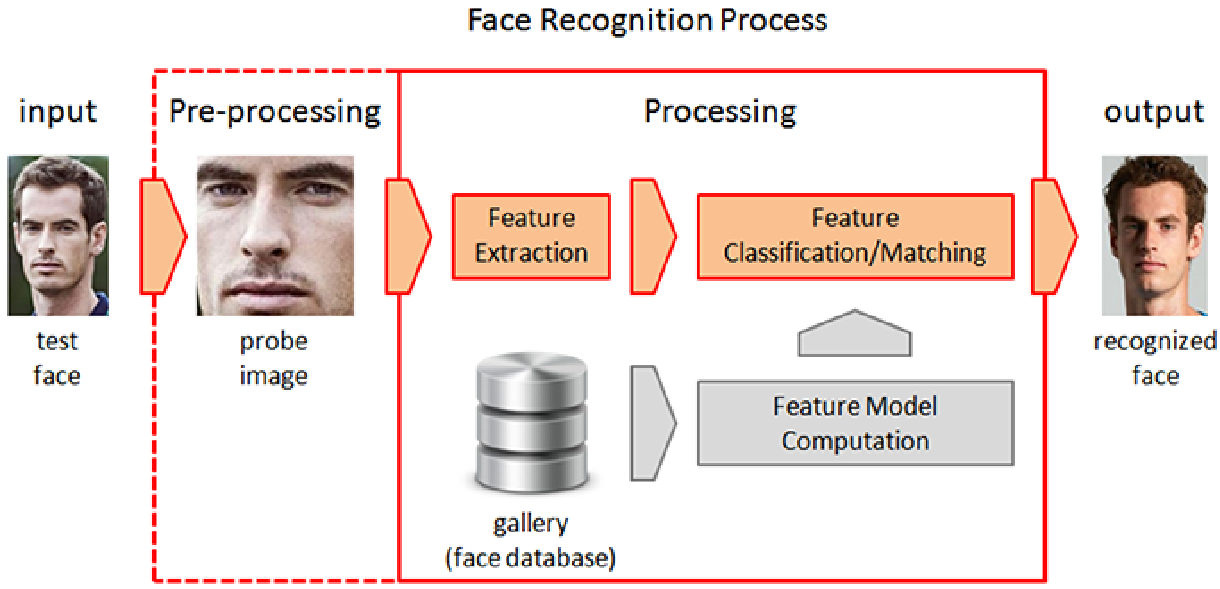


Figure 3.2: representation of face recognition process. [29]

Here in the figure explain how we use face detection algorithm to crop the face region from the whole frame. Face detection makes the features extraction easy to apply because we take only the features of the face and we don't need what left in the next steps of the process. After applying the face detection algorithm we do next step which is feature extraction using LPQ and next step is dimensionality reduction using MPCA and after this classification using MDA and last step is decision or matching in the output

3.2.1 Feature Extraction Based on Local Phase Quantization (LPQ)

In this work and in the feature extraction step we used the local phase quantization (LPQ) as descriptor of features extraction.LPQ descriptor proposed by Ojansivu and Heikkila has gained a lot of popularity in recent years due to its outstanding performance in image texture analysis [51]. LPQ method is based on quantized phases of the Discrete Fourier Transform (DFT). This descriptor utilizes phase statistical feature computed locally in a window for each pixel from the image, generating a code and finally making a histogram from the codes. The codes produced by the LPQ are insensitive to centrally symmetric blur, which includes motion, atmospheric turbulence blur and etc. [51]. Due to the images limited size, in practice, it is impossible to achieve absolute stability against the blur in images. Convolution of an ideal image with spread point blur function is much more than what we see in images therefore, some information is lost when the blur boundaries is much larger than the image size.

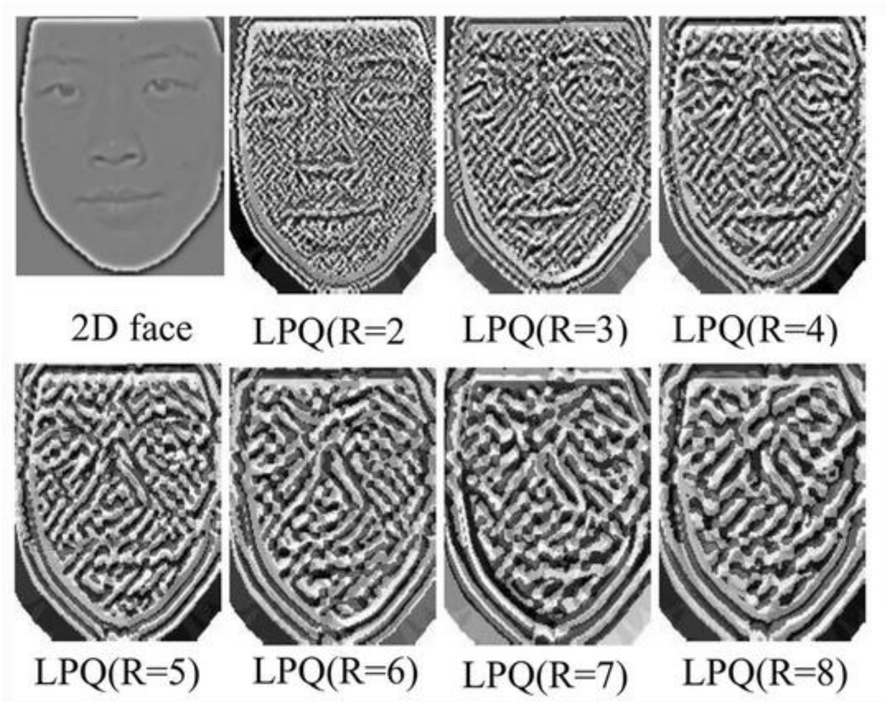


Figure 3.3: feature extraction using LPQ by changing in the parameters of the Rayon.

3.3 Experiments and results

In this section, several experiments are carried out to show the efficiency and effectiveness of our proposed algorithm for face recognition. We will compare our algorithm with LDA and PCA methods , the most known linear methods in face recognition.

We used a standard face databases, which is LFW Face Database to evaluate the effectiveness of our proposed algorithm. These algorithms were contrasted with other popular algorithms. In this work, we report the best result on different test. The performances on the problems with low number of training samples were also appraised to demonstrate their robustness in the small sample size cases.

3.3.1 LFW face database

The LFW dataset, released in 2007, is the most popular academic dataset for face verification because of the availability of detailed evaluation standards and the diversity of evaluation protocols. The LFW dataset contains 13,233 pictures of 5749 different persons, 4069 of them have just one face image, with massive variations in expressions, illuminations, and poses. All of the images are gathered from the internet. Figure 3.2 presents examples of LFW images.

Figure 3.4: Face images in Labeled Faces in the Wild (LFW) dataset.

Two training approaches have been proposed by the LFW dataset:

- The restricted training approach where just a small number of determined pairs are available for training.
- The unrestricted approach where the formation of extra training pairs is allowed by combining images in LFW.

Lately, new approaches were developed to keep up equitable comparisons among methods that start to use additional training data from outside LFW for the sake of performance improvement. The proposed model is evaluated while using the standard unrestricted approach with labeled outside data protocol that includes 3000 positive pairs and 3000 negative pairs. The standard unrestricted approach divides pairs into ten disjoint subsets for cross-validation. In every subset, the identities are mutually exclusive, 300 positive, and 300 negative pairs are provided in every subset (examples are shown in Figure 3.3). Ten-fold cross-validation is applied for performance evaluation. [52]

Figure 3.5: Example of matched/mismatched pairs in LFW.

3.3.2 Results and discussion

As result shows in figure 3.6, it is worth trying our method for face recognition and the results are worth comparing to the traditional algorithms like LDA and PCA. These results presented the process of multi classifier method for face recognition on the LFW face dataset. The combination of MPCA and MDA gives us the better performance, and a better robustness.

The results to my work can be best explained through a bunch of XY Graphs as shown in figure 3.6, this graphs is about an axis label which is a text string aligned with the *x* or *y* axis in a graph. Axis labels can help explain the meaning of the units that each axis represents. The axis in this work represents False Positive Rate and True Positive Rate.

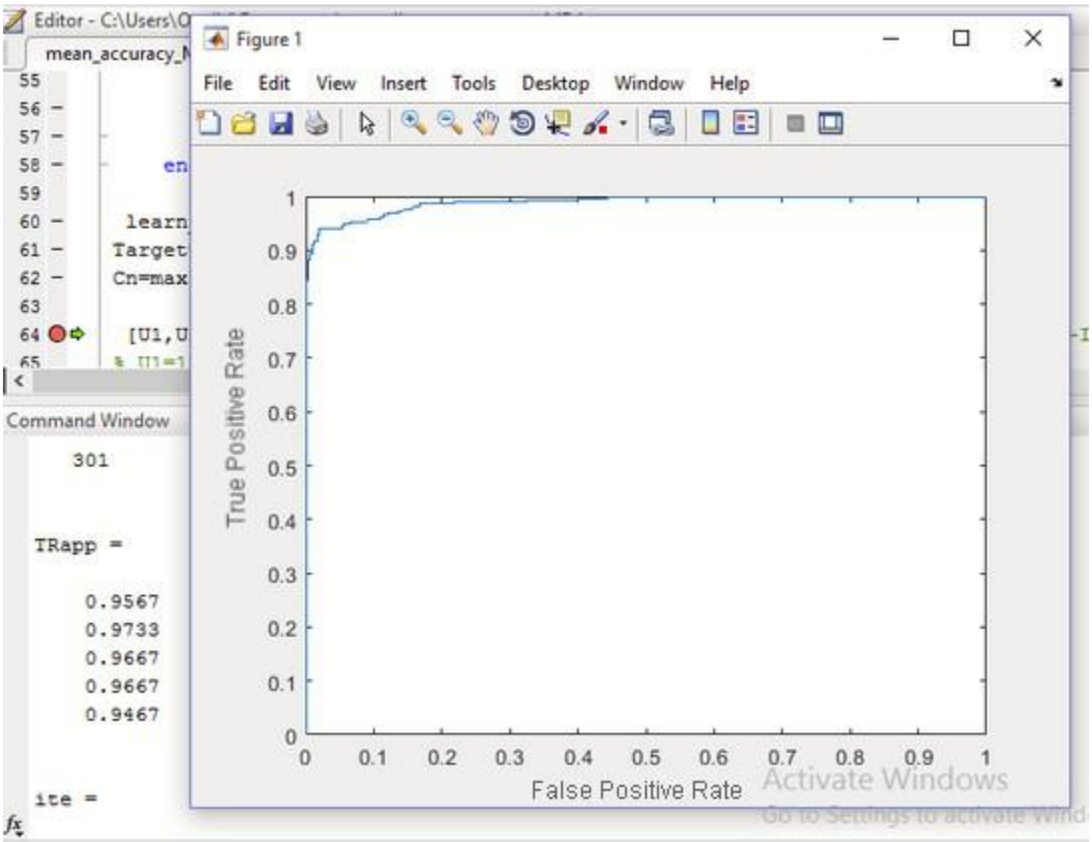


Figure 3.6: Results of Face recognition accuracy using MATLAB based on MPCA/MDA algorithms on LFW face database.

3.4 Comparison between MPCA/MDA and PCA/LDA

3.4.1 Recognition Accuracy comparison:

As we can see in figure 3.7 in the first set and by using the same database the results shows that face recognition based on PCA and LDA methods got 91.13% of recognition accuracy with using LBP for feature extraction, we conclude that this method which is fusion of MPCA/MDA clearly got a better result. We are giving a comparison between MPCA/MDA and PCA/LDA in the first five sets in each program and the results are shown in the table 2

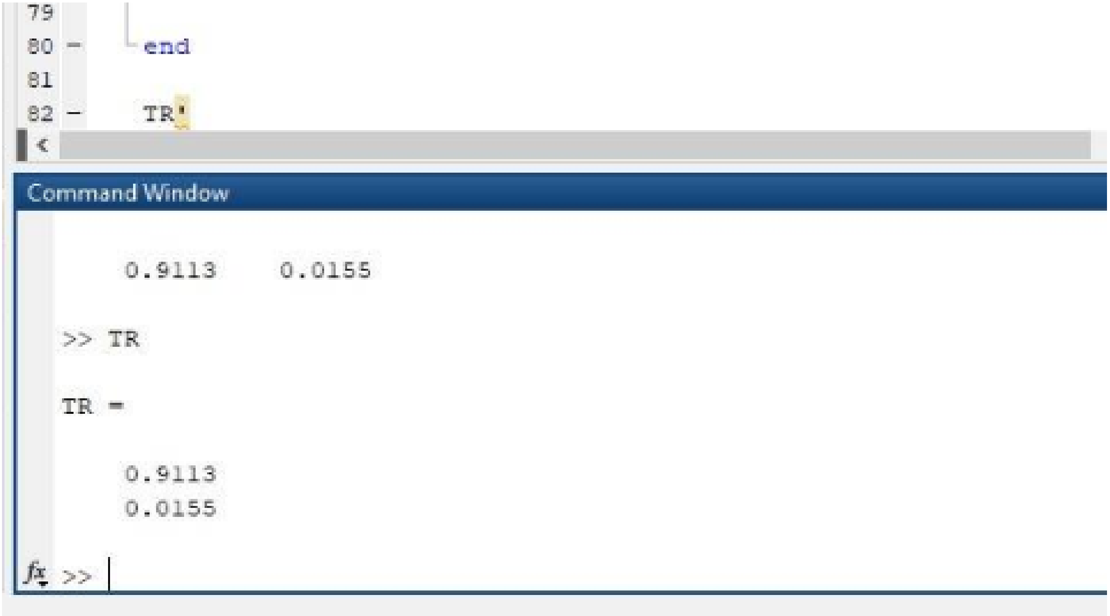


Figure 3.7: Results of Face recognition accuracy using MATLAB based on PCA/LDA algorithms on LFW face database.

Recognition accuracy of MPCA/MDA	**Recognition accuracy of PCA/LDA**
95.67%	**91.13%**
97.33%	**91.57%**
96.67%	**91.18%**
96.67%	**91.67%**
94.67%	**91.67%**

Table 2: Recognition accuracy (%) comparison in the first five set of each method

3.4.2 Comparison between PCA and MPCA:

PCA

•Requires full frontal display.

•Operates at a faster rate.

•Each face is a single entity in the database.

•Little sensitive to lighting, position of faces.

•Perform a linear map of the data to a low dimensional space.

MPCA

•Works well with different viewpoints, expressions and lighting conditions.

•Faces of same person are grouped together classes [53].

•MPCA is the extension of PCA.

•Not sensitive to lighting, position of faces.

•Perform a multilinear map of the data to a higher dimensional space.

3.5 Design of the tensor based on local characteristics:

The facial tensor data is constructed using local features over three steps; image preprocessing, feature extraction based on the local LPQ (Local Phase Quantization) descriptor and tensor representation. First, the face pretreatment includes cropping and trimming centered on the tip of the nose. [54]

Second, for feature extraction, the images are described using the LPQ descriptor. We use this descriptor because it achieved the best performance in previous work.

3.5.1 LPQ (Local Phase Quantization) descriptor:

In order to maintain the spatial structure of faces, face images are subdivided without

Overlap into P regions of faces in which the histograms of these rectangular blocks are concatenated to form a vector of characteristics v of size n = P × 256. To further improve the description of faces, we use the multi-scale LPQ (MSLPQ) for extracting features from images. Note that LPQ is actually a special case of MLPQ with one scale, the LPQ multi-scale representation can be obtained by varying the size of the M window. Therefore, the LPQ characteristics are extracted at different scales. The characteristic vectors of different scales are arranged in a matrix forming a representation of the second order tensor. Finally, all of the faces candidates are arranged to create the third order face tensor. [54]

Therefore, the three modes of the tensor are defined as follows:

i1: The characteristic vectors at different scales used for face images

i2: The concatenated histograms.

i3: The different people in the database.

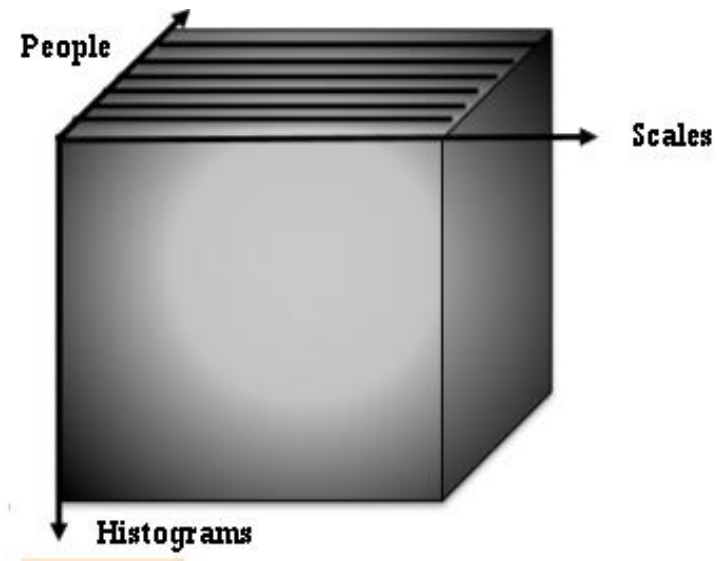


Figure 3.8: illustrates the shape of the tensor used in our verification system cube [55].

3.5.2 Dimensionality reduction and classification:

In this step, the learning tensor data is projected into a lower discriminating subspace (based on MPCA-MLDA). In the training phase, the optimal multilinear projection matrices are estimated, and in the testing phase, new samples are projected using these optimal matrices and compared.

Finally, for classification, the reduced test characteristics VI are compared to the training tensor data V using the comparison based on cosine similarity. [54]

3.6 Conclusion

In this chapter, we present the fusion of two algorithms which is MPCA and MDA in face recognition system. And how this algorithms works together in MATLAB, Then we Compared it with common algorithms, such as PCA and LDA, the algorithm has no problem with the light, pose and age.

MPCA and MDA give us the better recognition accuracy as we expected because it compare the image with all the training images, we used LFW Face database as an example and we used either LPQ as a feature extraction technique, we discuss also how tensors analysis is important in our work, As we can see from the results this fusion method gives us the better recognition than the other traditional algorithms.

General conclusion

In this work, we presented face recognition technology and how important this technology is in our life, Face recognition is an emerging technology that can provide many benefits. Face recognition can save resources and time, and even generate new income streams, for companies that implement it right.

The advances in camera's and image processing technologies and computers are getting better and better to make face recognition performances working well and make images look clear , research's are still looking for better ideas for a great success and make life more easier.

We presented also how this technology works, when the computer takes an image and calculates the distance between major structural pieces like nose and eyes, it could also take into account the width and curves of your face, these measurements are converted to a numerical code called the face print, one the computer knows your face print it looks in its database of images that have already gone through the process to see if it can find any matching. We discussed either the problems and challenges that can disrupt the face recognition performance like the age, pose, lighting, facial expression, viewing distance, make-up, beard, or glasses.

But we tried a solution for this problem which is the fusion of MPCA and MDA algorithms. the algorithm has no problem with the illumination or poses or any problems that can disable our face recognition system performance, and to confirm that this method has better results , because we did tests using the LFW face database in MATLAB With some algorithms applied like LPQ, at the end we confirmed that with this method the face recognition system will always get good recognition accuracy results it is dependent on which database we use , compare to other algorithms like PCA and LDA in results for sure our method has the better recognition accuracy for sure .

Face recognition will change people's lives by tracking people's movements and machines can follow you around, and technology companies are trying to make facial recognition an everyday part of our lives. The technology behind facial recognition has been around for years but recently it's grown more its applications have expanded greatly, and it's no longer just humans who are the targets.

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