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Design of Pressurized Vaulted Rectangular Conduits Using the Rough Model Method

Bachir ACHOUR

Research Laboratory in Subterranean and Surface Hydraulics
University of Biskra, PO Box 145 RP 07000, Biskra, Algeria

bachir.achour@larhyss.net

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Abstract. A new theory is presented to design pressurized conduits, particularly vaulted rectangular conduit. This theory is based on a referential rough conduit model characterized by an arbitrarily assigned relative roughness taken in the fully turbulence flow regime. Thus, the geometric elements of the chosen model are well defined. These allow a direct determination of the required geometric elements of the conduit by the use of a non-dimensional correction factor. A practical example is presented to explain the procedure of calculation.

Introduction

Three great categories of problems are encountered in the field of turbulent conduit-flow. The first one is to compute the discharge capacity of the conduit. The second one is to determine the geometric elements of the conduit in order to design it. The third one is to evaluate the energy slope in order to adjust the slope if necessary. These three categories of problems are often solved using three basic relationships, namely Darcy-Weisbach, Colebrook-White and Reynolds number [1, 2, 3]. For pipes and conduits, only discharge can be explicitly computed when combining Colebrook-White and Darcy-Weisbach relationships that express the friction factor and the energy slope respectively [4, 5, 6]. Two other parameters influence turbulent conduit-flow, namely the absolute roughness which characterizes the state of the inner wall of the conduit and the kinematic viscosity. These are measured in practice and rarely cause any particular problem. When it comes to answer the last two categories of problems, the difficulty lies in assessing the friction factor since the Colebrook-White relationship is implicit. Moreover, the three basic equations of turbulent flow do not allow expressing the geometric element of the conduit in an explicit form. The solution involves many trials and tedious computations. For pipe-flow problems, some authors have proposed approximate relations or graphical solutions for friction factor, diameter of the pipe and energy slope as well [7, 8]. Referring to the literature, many studies have been performed on the circular pipe but no study has found on pressurized vaulted rectangular conduit despite its extensive use in practice as water supply lines, sanitary sewers, culverts and storm drains or penstocks as well. For this reason, the main objective of this paper is to contribute to enrich the literature by studying this type of conduit under pressurized condition of the flow. The study is based on a new theoretical approach known as the referential Rough Model Method (RMM) [9]. Our attention is focused exclusively on the computation of the minimum height of the conduit. The three basic equations of turbulent flow are applied to a rough conduit model characterized by an arbitrarily assigned relative roughness value. This is a referential rough model from which the required minimum conduit height is directly deduced. Resulting RMM equations are not only explicit but also cover the entire domain of Moody diagram [2], corresponding to Reynolds number $R \geq 2300$ and relative roughness varying in the wide range [0; 0.05].

Geometric and hydraulic characteristics of the vaulted rectangular conduit

Fig.1 is a schematic representation of the pressurized vaulted rectangular conduit. This is characterized by the two geometric elements D and Y corresponding to the diameter of the vault at the top of the conduit and the minimum height respectively. Diameter D is also the width of the rectangular bottom of the conduit.

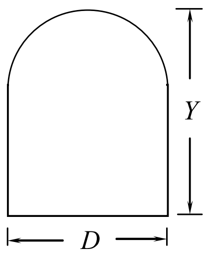

Fig. 1: Geometric elements of the pressurized vaulted rectangular conduit

Let us assume $\eta = Y / D$ as the aspect ratio of the area. Thus, the water area A can be expressed as:

$$A = D^2 \left(\eta - C_0\right) \tag{1}$$

where C_0 is a constant equal to :

$$C_0 = \frac{1}{2}\left(1 - \frac{\pi}{4}\right) = 0{,}107300918 \tag{2}$$

The wetted perimeter P can be written as:

$$P = 2D\left(\eta + \frac{\pi}{4}\right) \tag{3}$$

The hydraulic diameter $D_h = 4A / P$ is thus:

$$D_h = 2D \frac{\left(\eta - C_0\right)}{\left(\eta + \frac{\pi}{4}\right)} \tag{4}$$

The energy slope J is given by the Darcy-Weisbach relationship as:

$$J = \frac{f}{D_h} \frac{Q^2}{2gA^2} \tag{5}$$

where Q is the discharge, g is the acceleration to gravity and f is the friction factor given by the well known Colebrook-White formula as :

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon / D_h}{3.7} + \frac{2.51}{R\sqrt{f}}\right) \tag{6}$$

ε is the absolute roughness and R is the Reynolds number which can be expressed as :

$$R = \frac{4Q}{P\nu} \tag{7}$$

where ν is the kinematic viscosity.

Referential rough model

Both of geometric and hydraulic characteristics of the rough model are distinguished by the symbol "¯". The rough model we consider is a pressurized vaulted rectangular conduit characterized by $\overline{\varepsilon} / \overline{D_h} = 0.037$ as the arbitrarily assigned relative roughness value. The chosen relative roughness value is so large that the prevailed flow regime is fully rough. Thus, the friction factor is $\overline{f} = 1/16$ according to Eq. 6 for $R = \overline{R}$ tending to infinitely large value. Applying Eq. 5 to the rough model leads to:

$$\overline{J} = \frac{\overline{f}}{\overline{D_h}} \frac{\overline{Q}^2}{2g\overline{A}^2} \tag{8}$$

Eq. 8 can be rewritten as:

$$\overline{J} = \frac{1}{128g} \frac{\overline{P}}{\overline{A}^3} \overline{Q}^2 \tag{9}$$

Introducing Eq. (1) and Eq. (3) into Eq. 9, one may write:

$$\overline{J} = \frac{1}{64} \frac{\left(\overline{\eta} + \frac{\pi}{4}\right)}{\left(\overline{\eta} - C_0\right)^3} \left(\frac{\overline{Q}^2}{g\overline{D}^5}\right) \tag{10}$$

Let us assume $\overline{Q} = Q$, $\overline{J} = J$ and $\overline{D} = D$, implying $\overline{Y} \neq Y$ and obviously $\overline{\eta} \neq \eta$. One can deduce from Eq. 10:

$$D = \left[\frac{\overline{\eta} + \frac{\pi}{4}}{64\left(\overline{\eta} - C_0\right)^3}\right]^{1/5} \left(\frac{Q^2}{gJ}\right)^{1/5} \tag{11}$$

Introducing the relative conductivity $\overline{Q}_D^* = Q_D^* = \dfrac{Q}{\sqrt{gJ\overline{D}^5}}$, Eq. 11 becomes:

$$\frac{\overline{\eta} + \frac{\pi}{4}}{64\left(\overline{\eta} - C_0\right)^3} Q_D^{*2} = 1 \tag{12}$$

By adopting the following change in variables

$$x = \overline{\eta} - C_0 \tag{13}$$

Eq. 12 is reduced to:

$$x^3 - \frac{Q_D^{*2}}{64} x - \frac{\left(1 + \pi/4\right)}{128} Q_D^{*2} = 0 \tag{14}$$

Eq. 14 is a cubic equation without second order. Its discriminant is:

$$\Delta = 3\left(\frac{Q_D^*}{48\sqrt{2}}\right)^4 \left[\left(1 + \pi/4\right)6\sqrt{3} - Q_D^*\right]\left[\left(1 + \pi/4\right)6\sqrt{3} + Q_D^*\right] \tag{15}$$

Eq. 15 shows that two cases arise:

1. $Q_D^* \leq (1 + \pi/4)6\sqrt{3}$, then $\Delta \geq 0$. The real root of eq. 14 is:

$$x = \frac{Q_D^*}{4\sqrt{3}} ch(\beta/3) \tag{16}$$

Taking into account Eq. 13, the aspect ratio $\overline{\eta}$ of the rough model is expressed as:

$$\overline{\eta} = C_0 + \frac{Q_D^*}{4\sqrt{3}} ch(\beta/3) \tag{17}$$

where the angle β is as :

$$ch(\beta) = \frac{(1+\pi/4)6\sqrt{3}}{Q_D^*} \tag{18}$$

2. $Q_D^* \geq (1+\pi/4)6\sqrt{3}$, then $\Delta \leq 0$. The real root of Eq. 14 is expressed as:

$$x = \frac{Q_D^*}{4\sqrt{3}}\cos(\beta/3) \tag{19}$$

According to Eq. 13, the aspect ratio of the rough model is then:

$$\overline{\eta} = C_0 + \frac{Q_D^*}{4\sqrt{3}}\cos(\beta/3) \tag{20}$$

where the angle β is given by what follows :

$$\cos(\beta) = \frac{(1+\pi/4)6\sqrt{3}}{Q_D^*} \tag{21}$$

Correction factor of linear dimension

The RMM states that the linear dimension *D* of the conduit and that of its rough model $\overline{D}$ are related by the following equation:

$$D = \psi\overline{D} \tag{22}$$

where ψ is a non-dimensional correction factor of linear dimension, less than unity, which is governed by the following relationship [9]:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D_h}}{4.75} + \frac{8.5}{\overline{R}}\right)\right]^{-2/5} \tag{23}$$

Computation steps of the linear dimension *Y*

Knowing the discharge *Q*, the diameter *D* of the vault, the energy slope *J*, the absolute roughness ε and the kinematic viscosity ν , the following steps are recommended to compute the required linear dimension *Y*:

1. Compute the relative conductivity $Q_D^* = Q/\sqrt{gJD^5}$ for $\overline{D} = D$.
2. Compute the aspect ratio $\overline{\eta}$ of the rough model using one of the Eq. 17 or Eq. 18 in accordance with the sign of the discriminant Δ .
3. Knowing *D* and $\overline{\eta}$, compute the wetted perimeter $\overline{P}$, the hydraulic diameter $\overline{D_h}$ and the Reynolds number $\overline{R}$ using Eq. 3, Eq. 4 and Eq. 7 respectively.
4. Applying Eq. 23, the dimensionless correction factor ψ is then worked out.
5. Assign to the rough model the linear dimension $\overline{D} = D/\psi$ according to Eq. 22.
6. Compute the new value of the relative conductivity $Q_D^* = Q/\sqrt{gJ(D/\psi)^5}$

7. Applying then one of the Eq. 17 or Eq. 18, in accordance with the sign of the discriminant Δ, results in $\overline{\eta} = \eta$.
8. The required value of the conduit height Y is finally $Y = D\eta$.

Example

Compute the minimum height of the pressurized vaulted rectangular conduit for the following data:

$Q = 4\,m^3/s$, $D = 1.45\,m$, $J = 6.10^{-4}$, $\varepsilon = 0.0015\,m$, $\nu = 10^{-6}\,m^2/s$

1. For $\overline{D} = D$, the relative conductivity $Q_D^* = Q/\sqrt{gJD^5}$ is:

$$Q_D^* = \frac{Q}{\sqrt{gJD^5}} = \frac{4}{\sqrt{9.81\times6\times10^{-4}\times1.45^5}} = 20.59348543 > (1+\pi/4)6\sqrt{3}$$

2. The discriminant Δ being negative, the aspect ratio $\overline{\eta}$ of the rough model is then given by both Eq. 20 and Eq. 21. Hence:

$$\cos(\beta) = \frac{(1+\pi/4)6\sqrt{3}}{Q_D^*} = \frac{(1+\pi/4)\times6\times\sqrt{3}}{20.59348543} = 0.900984054$$

or $\beta = 0.44876395$ radian

$$\overline{\eta} = C_0 + \frac{Q_D^*}{4\sqrt{3}}\cos(\beta/3) = 0.107300918 + \frac{20.59348543}{4\times\sqrt{3}}\times\cos(0.44876395/3)$$
$$= 3.046520272$$

3. According to Eq. 3, Eq. 4 and Eq. 7, the hydraulic diameter, the wetted perimeter and the Reynolds number in the rough model are respectively:

$$\overline{D_h} = 2D\frac{\left(\overline{\eta} - C_0\right)}{\left(\overline{\eta} + \frac{\pi}{4}\right)} = 2\times1.45\frac{\left(3.046520272 - 0.107300918\right)}{\left(3.046520272 + \frac{\pi}{4}\right)} = 2.224404373\,m$$

$$\overline{P} = 2\overline{D}\left(\overline{\eta} + \frac{\pi}{4}\right) = 2\times D\times\left(\overline{\eta} + \frac{\pi}{4}\right) = 2\times1.45\times\left(3.046520272 + \frac{\pi}{4}\right) = 11.11256346\,m$$

$$\overline{R} = \frac{4\overline{Q}}{\overline{P}\nu} = \frac{4Q}{\overline{P}\nu} = \frac{4\times4}{11.11256346\times10^{-6}} = 1439811.8$$

4. According to Eq. 23, the non-dimensional correction factor ψ is then:

$$\psi = 1.35\times\left[-\log\left(\frac{0.0015/2.224404373}{4.75} + \frac{8.5}{1439811.8}\right)\right]^{-2/5} = 0.788948655$$

5. Let us assign to the rough model the following linear dimension, according to Eq. 22:

$$\overline{D} = \frac{D}{\psi} = \frac{1.45}{0.788948655} = 1.83788893\,m$$

6. The new value of the relative conductivity is then:

$$\overline{Q}_D^* = \frac{Q}{\sqrt{gJ\left(D/\psi\right)^5}} = \frac{4}{\sqrt{9.81\times 6\times 10^{-4}\times 1.83788893^5}} = 11.38548952$$

7. The new value of the relative conductivity is less than $(1+\pi/4)6\sqrt{3}$ and the discriminant Δ is then positive. Thus, the aspect ratio $\overline{\eta}$ is given by Eq. 17, along with Eq. 18. Hence:

$$ch(\beta) = \frac{(1+\pi/4)6\sqrt{3}}{\overline{Q}_D^*} = \frac{(1+\pi/4)\times 6\times\sqrt{3}}{11.38548952} = 1.629653425$$

or $\beta = 1.070357112$ radian

$$\overline{\eta} = \eta = C_0 + \frac{\overline{Q}_D^*}{4\sqrt{3}}ch(\beta/3) = 0.107300918 + \frac{11.38548952}{4\times\sqrt{3}}ch(1.070357112/3)$$
$$= 1.856365272$$

8. Finally, the required value of the linear dimension Y is:

$Y = D\eta = 1.45\times 1.856365272 = 2.691729644\,m \cong 2.692\,m$

Conclusions

The rough model method was applied to design a pressurized vaulted rectangular conduit. The main objective was to compute the minimum height of the conduit. Two non-dimensional parameters were highlighted, namely the aspect ratio and the relative conductivity. These were related by a cubic equation which was analytically solved using hyperbolic and trigonometric functions. The explicit steps of calculation were clearly presented and one can observe the degree of their simplicity. It has been observed that the dimensionless correction factor of linear dimension plays a fundamental role in determining the aspect ratio.

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