

Discussion

Exact solutions for normal depth problem

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The dimensionally consistent uniform flow relationship $Q = \varphi(S_o, \varepsilon, A, R, \nu)$ can be established by the only use of the Darcy–Weisbach formula, thereby, accounting for the friction factor f according to Colebrook–White equation. Nevertheless, as it will be shown, the discussers’ relation (1.19) differs from the authors’ equation (5) in the first term in parenthesis. This may be explained by the restrictions which led to the establishment of Hager’s inequality, numbered (2) by the authors, along with 1.5% of deviation between friction factor values for complete turbulent state and transitional state flow was assumed. Moreover, the same inequality (2) is theoretically valid as long as Reynolds number Re varies within the confined range 1×10^4 to 1×10^7 . Furthermore, relating Chezy’s constant C to the known characteristics of a chosen referential rough channel, the relative normal depth can be determined using authors’ explicit equations.

Basic equations

The Darcy–Weisbach formula and the Colebrook–White equation are expressed, respectively, by:

$$S_o = \frac{f}{8g} \frac{P}{A^3} Q^2 \quad (1.1)$$

$$f^{-1/2} = -2 \log \left(\frac{\varepsilon}{14.8R} + \frac{2.51}{Re\sqrt{f}} \right) \quad (1.2)$$

where Q is the discharge, S_o is the energy slope, ε is the absolute roughness, P is the wetted perimeter, A is the water area, R is the hydraulic radius, ν is the kinematic viscosity, g is the acceleration due to gravity, and Re is the Reynolds number. The latter is defined by:

$$Re = \frac{4Q}{P\nu} \quad (1.3)$$

For any shape of channel section, the geometric elements A and P can be written, respectively, as:

$$A = L^2 A^* \quad (1.4)$$

$$P = LP^* \quad (1.5)$$

in which L is the linear dimension such as the bed width b of a rectangular channel or the diameter D of a circular section, etc. Both of the non-dimensional parameters A^* and P^* depend solely on the relative normal depth. Inserting Eqs (1.4) and (1.5) into Eq. (1.1) and rearranging, results in:

$$L = (f/8)^{1/5} (Q/\sqrt{gS_o})^{2/5} (P^*/A^{*3})^{1/5} \quad (1.6)$$

whereas the combination of Eqs (1.3) and (1.5) leads to:

$$Re = \frac{4Q}{LP^*\nu} \quad (1.7)$$

Referential rough channel

With the subscript “ r ” we refer to a referential rough channel characterized by $\varepsilon_r/R_r = 0.148$ as the arbitrarily assigned relative roughness value. Moreover, assuming a complete turbulent state flow, the friction factor f_r is given by Eq. (1.2) for $R_r \rightarrow \infty$, implying $f_r = 1/16$. Thus, with the aid of Eqs (1.4) and (1.5), Eq. (1.1) gives the following expression of the linear dimension L_r :

$$L_r = (128)^{-1/5} (Q_r/\sqrt{gS_{o,r}})^{2/5} (P_r^*/A_r^{*3})^{1/5} \quad (1.8)$$

According to (1.7), the Reynolds number Re_r can be expressed as:

$$Re_r = 4Q_r/(L_r P_r^* \nu) \quad (1.9)$$

On the other hand, eliminating Q_r from Eqs (1.8) and (1.9), leads to:

$$Re_r = 32\sqrt{2} \frac{\sqrt{gS_{o,r}} L_r^3}{\nu} (A_r^*/P_r^*)^{3/2} \quad (1.10)$$

which can be rewritten, with the aid of Eqs (1.4) and (1.5), as follows:

$$Re_r = 32\sqrt{2} \frac{\sqrt{gS_{o,r}} R_r^3}{\nu} \quad (1.11)$$

Dimensionally consistent uniform flow relationship

Let us assume $S_{o,r} = S_o$, $L_r = L$, $\beta_{n,r} = \beta_n$ ($\eta_n = \eta_{n,r}$ for the circular section) for $Q_r \neq Q$ and obviously $Re \neq Re_r$. Consequently, one may write $A_r^* = A^*$, $P_r^* = P^*$ and $R_r = R$. Combining Eqs (1.6) and (1.8), we thus derive:

$$Q = \psi_Q Q_r \quad (1.12)$$

where ψ_Q is expressed by:

$$\psi_Q = \frac{1}{4\sqrt{f}} \quad (1.13)$$

whereas Q_r is given by:

$$Q_r = 8\sqrt{2g}\sqrt{S_o}L^5(A^*/P^*)^{1/2} \quad (1.14)$$

In view of Eq. (1.12), the discharge Q is equal to the corrected discharge Q_r for effect of ψ_Q . The latter must be then considered as a non-dimensional correction factor of discharge. Moreover, Eq. (1.14) can be simply rewritten as follows:

$$Q_r = 8\sqrt{2g}A\sqrt{RS_o} \quad (1.15)$$

which has the form of Chezy's equation with $C_r = 8\sqrt{2g}$ as the Chezy's constant.

On the other hand, Eqs (1.7), (1.9), and (1.12) lead to:

$$Re = \psi_Q Re_r \quad (1.16)$$

Inserting Eqs (1.13) and (1.16) into Eq. (1.2), we derive the following $\psi_Q(\varepsilon/R, Re_r)$ explicit relationship:

$$\psi_Q = -\frac{1}{2} \log \left(\frac{\varepsilon}{14.8R} + \frac{10.04}{Re_r} \right) \quad (1.17)$$

With the aid of Eqs (1.16) and (1.17), we derive the following Reynolds number equation:

$$Re = -\frac{1}{2} Re_r \log \left(\frac{\varepsilon}{14.8R} + \frac{10.04}{Re_r} \right) \quad (1.18)$$

From Eqs (1.12), (1.15), and (1.17), the following dimensionally consistent uniform flow equation is deduced:

$$Q = -4\sqrt{2g}A\sqrt{RS_o} \log \left(\frac{\varepsilon}{14.8R} + \frac{10.04}{Re_r} \right) \quad (1.19)$$

in which Re_r is given by either Eq. (1.9) for $Q_r = Q$, $L_r = L$ and $P_r^* = P^*$, or Eq. (1.11) for $S_{o,r} = S_o$ and $R_r = R$, whence:

$$Re_r = 32\sqrt{2} \frac{\sqrt{gS_o}R^3}{\nu} \quad (1.20)$$

Inserting Eq. (1.17) into Eq. (1.13), the friction factor relationship is obtained as:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon}{14.8R} + \frac{10.04}{Re_r} \right) \quad (1.21)$$

When S_o , ε , ν , and R are given, Eq. (1.21) allows, then, a direct determination of the exact friction factor value for $R \geq 2300$.

Comparing Eq. (1.19) with Chezy's relation, the Chezy's constant C can be written as:

$$C = -4\sqrt{2g} \log \left(\frac{\varepsilon}{14.8R} + \frac{10.04}{Re_r} \right) \quad (1.22)$$

or as:

$$C = C_r \psi_Q \quad (1.23)$$

On the other hand, comparing Eq. (1.19) with Manning's equation, the roughness coefficient n is obtained as:

$$\frac{1}{n} = -4\sqrt{2g}R^{1/6} \log \left(\frac{\varepsilon}{14.8R} + \frac{10.04}{Re_r} \right) \quad (1.24)$$

or as:

$$n = n_r \psi_Q^{-1} \quad (1.25)$$

in which:

$$n_r = (8\sqrt{2g})^{-1} R^{1/6} \quad (1.26)$$

Computation of the relative normal depth

The relative normal depth can be worked out, once the following improved friction factor relationship is assumed:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\varepsilon/R_r}{14.8\psi} + \frac{10.04}{\psi^{3/2}Re_r} \right) \quad (1.27)$$

In which:

$$\psi \cong 1.35 \left[-\log \left(\frac{\varepsilon/R_r}{19} + \frac{8.5}{Re_r} \right) \right]^{-2/5} \quad (1.28)$$

For $R \geq 2300$ and for the wide range $0 \leq \varepsilon/R \leq 0.20$, it was observed that the deviation between Eqs (1.2) and (1.27) is negligible. Thus, applying Eq. (1.27), one may consider the obtained values of f as practically exact. Furthermore, the Chezy's constant $C = \sqrt{8g/f}$ is obtained as:

$$C = -4\sqrt{2g} \log \left(\frac{\varepsilon/R_r}{14.8\psi} + \frac{10.04}{\psi^{3/2}Re_r} \right) \quad (1.29)$$

Thus, to compute the relative normal depth for the given values of Q , S_o , L , ε , and ν , the following steps are recommended, assuming $Q_r = Q$, $S_{o,r} = S_o$, and $L = L_r$:

1. Knowing Q , S_o , L , and C_r , authors' explicit equations (21), (22), (40), (43), and (58) give the value of the corresponding normal depth of the flow in the considered referential rough channel. Furthermore, the hydraulic radius R_r can be, then, worked out.
2. With S_o and R_r , Eq. (1.11) gives the value of Re_r .
3. Knowing R_r , Re_r , and ε , the parameter ψ is obtained using Eq. (1.28).
4. Inserting the values of R_r , ε , and ψ into Eq. (1.29), gives the value of Chezy's constant C .
5. Knowing Q , S_o , L , and C , authors' explicit equations (21), (22), (40), (43), and (58) give finally the corresponding relative normal depth.

Reply by the Authors

The authors are very much grateful to Achour and Bedjaoui for their discussion. The derivation of Eq. (1.19) by the discussers uses Colebrook equation, which is valid for pipe flow. For open-channel flow, the corresponding resistance equation given by Anonymous (1963) is

$$f = 1.325 \left[\ln \left(\frac{\varepsilon}{12R} + \frac{0.625\nu}{VR\sqrt{f}} \right) \right]^{-2} \quad (1.30)$$

Equation (5) of the paper is based on Eq. (1.30). Thus, the discussers' Eq. (1.19) is not applicable to open-channel flow.

For derivation of Eq. (5) of the paper, the discussers may see Swamee (1994), which is on similar lines as given by the discussers.

References

1. Anonymous (1963) "Friction Factors in Open Channels: Progress Report of Task Force on Friction Factors in Open Channels". *J. Hydraul. Eng. Div.*, ASCE 89(2), 97–143.
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